Pion-Nucleon Processes above the First Maximum According to the Strong-Coupling Method*

HELMUT JAHN†
Department of Physics, Brandeis University, Waltham, Massachusetts
(Received October 9, 1961)

A rough estimate of pion production by pion-nucleon collisions, in the range of 300–600 Mev incident kinetic energy in the lab system, is carried through on the basis of the strong-coupling method. This method, explicitly introducing the nucleon isobars, gives as the main contribution the pion production via isobaric excitation, in agreement with the conclusions of Lindenbaum and Sternheimer. Especially, a mechanism for pion productions via isobaric excitation follows without an extra π-π interaction term, using only the fixed-source theory. Because of the heavier mass of the isobar and the participation of one more particle in the pion production process, the neglect of nucleon recoil in the fixed-source theory should not be taken too seriously for pion production below 1 Bev. Our results for the magnitude and energy dependence of the pion production cross section are within the same order of magnitude as the experimental results, in contradiction with the conclusions of Rodberg and Kazes who use a different kind of approximation based on the usual Chew-Low formalism and not on the strong-coupling method. We also find, as a typical result of the strong-coupling approximation, that the contributions to the isospin-$\frac{3}{2}$ states are about 4 times smaller than those to the isospin-$\frac{1}{2}$ states, which is just what the experimental results show in the energy range concerned. Thus it is shown that these important properties of the pion production cross section for pions on nucleons can already be explained by the strong-coupling approximation of the fixed-source theory without an extra π-π term. Therefore, these contributions from the fixed-source theory without an extra π-π term should be taken into account in addition to the π-π mechanism which was considered as the only mechanism by Dyson, Takeda, Rodberg, Goebel and Schnitzer, Peierls, and Carruthers and Bethe. Indications are presented that maxima could arise in the production as well as in the elastic scattering cross section as a consequence of pion production via isobaric excitation and pion scattering on isobars in the intermediate states. The latter effect of pion resonance scattering on isobars also has been considered by Wong and Ross and by Tomozawa in a different kind of approach. But we did not include these last-mentioned effects in this first rough estimate.

1. INTRODUCTION

ONE of the remarkable properties of the pion-nucleon processes is that the pion production by pion-nucleon collisions gives, according to the experiments,[1] a considerable contribution only above 300 Mev (kinetic energy in the lab system) and not yet between 140 Mev and 300 Mev as is already energetically possible. As a reasonable explanation of this effect, Lindenbaum and Sternheimer[2] have suggested that pion production should take place mainly via the excitation of the $\frac{3}{2}$, $\frac{3}{2}$ isobar which causes the first resonance maximum at 180 Mev. This would give in fact a pion production threshold energy of about 180 Mev+140 Mev=320 Mev, since 140 Mev is the pion rest energy. On the basis of this idea Sternheimer and Lindenbaum[3] have developed a phenomenological model for pion production which greatly helped to explain the shape of the momentum distribution of the outgoing pions.[3,4]

But then the question arises as to what the mechanism is, which leads to such a behavior of the pion production cross section by pions on nucleons, and how it can be derived from field theory in order to calculate, not only the shape of the momentum distribution of the outgoing pions, but also the magnitude and energy dependence of the total cross section showing the further experimental observed maxima at 615, 950, and 1300 Mev.

As such a mechanism, Dyson[5] and Takeda[6] first suggested a strong pion-pion interaction giving a pion-pion isobar and thus leading to resonance scattering of the incident pion on a pion of the pion cloud around the nucleon followed[6] by a $\frac{3}{2}$, $\frac{3}{2}$ resonance scattering of that cloud pion on the nucleon. The original purpose of these suggestions was to explain the above-mentioned maxima for which in this way it was shown that they also should involve pion production via isobaric excitation. This picture has been extended in different ways by Goebel,[7] Rodberg,[8] Goebel and Schnitzer,[9] Peierls,[10] Itabashi, Kato, Nakagawa, and

* Work performed under a grant from the National Science Foundation and a Fulbright travel grant.

† Present address: Department of Physics and Astronomy, State University of Iowa, Iowa City, Iowa.

1 W. A. Perkins, J. C. Caris, R. W. Kenney, and V. Perez-Mendes, Phys. Rev. **118**, 1364 (1960); J. H. Foote, O. Chamberlain, E. H. Rogers, H. M. Steiner, C. Wiegand, and T. Ypsilantis, Phys. Rev. Letters **4**, 30 (1960); R. R. Crittenden, J. H. Scandrett, W. D. Shephard, D. Walker, and J. Ballam, *ibid.* **2**, 121 (1959); S. M. Korenchenko and V. G. Zinow, Soviet Phys.—JETP **6**, 1006; **34**(7), 210 (1958); J. Ashkin, J. P. Blaser, F. Feiner, and M. O. Stern, Phys. Rev. **105**, 724 (1957).

2 S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. **106**, 1107 (1957).

3 R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. **109**, 1723 (1958); S. J. Lindenbaum and R. M. Sternheimer, *ibid.* **105**, 1874 (1957); S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. Letters **5**, 24 (1960).

4 Alles-Borelli, S. Bergia, E. Perez-Ferreira, and P. Waloschek, Nuovo cimento **14**, 211 (1959); I. Derado and N. Schmitz, Phys. Rev. **118**, 309 (1960); F. Bonsignori and F. Selleri, Nuovo cimento **15**, 465 (1960); I. Derado, *ibid.* **15**, 853 (1960).

5 F. J. Dyson, Phys. Rev. **99**, 1037 (1955).

6 G. Takeda, Phys. Rev. **100**, 440 (1955).

7 C. Goebel, Phys. Rev. Letters **1**, 337 (1958).

8 L. S. Rodberg, Phys. Rev. Letters **3**, 58 (1959).

9 C. Goebel and H. J. Schnitzer, Phys. Rev. **123**, 1021 (1961).

10 R. F. Peierls, Phys. Rev. **111**, 1373 (1958).

Takeda,[11] and Carruthers[12–14] and Bethe[12] based on the assumption of a sharp pion-pion resonance as has been found in the Chew-Mandelstam[15] approximation of the π-π double dispersion relations and used by Frazer and Fulco[16] to explain the experimental observed behavior of the electromagnetic form factors of the nucleon.

However, because of the difficulties of direct measurements of the π-π cross section, all experimental information has been obtained until now only indirectly, via the measurements of either the electromagnetic nucleon form factors or the pion production by pion-nucleon collisions. But the conclusions about the π-π interaction drawn from these experiments already involve special assumptions about the role of the π-π interaction in these processes, like the Dyson-Takeda mechanism mentioned just now or like the single-meson exchange approximation discussed by Goebel,[7] Chew and Low,[17] and Salzman and Salzman.[18] The difficulties arising from this procedure have been emphasized by these authors[17] as well as by Erwin, March, Walker, and West[19] and by Anderson, Bang, Burke, Carmony, and Schmitz.[20] The latter group has pointed out the restrictions of the Chew-Low[17] procedure using their experimental results. From the viewpoint of these results they come to the conclusion that the experimental evidence for the π-π resonance is not yet very strong. Furthermore, experimental investigation concerning the sticking together of the pions in the pion multiple production processes did not as yet give a strong evidence for a pion-pion resonance.[21]

One theoretical argument used very often in favor of the π-π mechanism is that the calculations of Rodberg,[22] Kazes,[22] and Omnès[23] using the pseudoscalar model of the fixed-source theory without an extra π-π term in the Hamiltonian have given pion-production by pion-nucleon collisions which is too small by one order of magnitude compared with the experiments, whereas, after inclusion of the extra π-π term, Rodberg,[8] Goebel and Schnitzer,[9] and Carruthers[13] got the same order of magnitude as the experiments.

But using a different approach, Barshay[24] and Franklin[25] already got the experimental order of magnitude from the fixed-source theory without the extra π-π term, and, recently, Wong and Ross[26] have given an indication that Rodberg's[22] kind of approach could be completed to give larger contributions showing the peaks at 950 Mev and 1.3 Bev. These peaks could also be explained in a different approach by Tomozawa[27] using the fixed-source theory only without an extra π-π term. Barshay[28] has introduced a general decomposition of the transition amplitude of which one part is mainly a π-N part besides the specific π-π part. So it seems that one has to conclude that the mechanism based on the extra π-π interaction is not the only one which has to be taken into account to describe the pion production by pions on nucleons, but that the fixed-source theory alone without an extra π-π interaction already gives considerable contributions.

In the following we will affirm this conclusion in the framework of the strong-coupling method which treats the pion production via isobaric excitation most directly and without an extra π-π interaction term, using only the fixed-source theory.

The field-theoretical model of the symmetric pseudoscalar fixed-source theory which, as well known, gives the 3, 3 resonance behavior of the pion-nucleon elastic scattering shown by Chew and Low,[29] contains the pion part in a full relativistic field-theoretical manner. That means it contains not only pion-nucleon elastic scattering but also pion-production processes. Since, in the meson production processes, the energy transfer is distributed between the two pions and the nucleon, with the mesons more favored than the nucleon, according to phase space considerations and in agreement with experimental results,[30] the neglect of nucleon recoil in the fixed-source theory should not be taken too seriously for pion production below 1 Bev. This argument should especially be valid for meson production via isobaric excitation because of the heavier mass of the isobar. The same conclusion is also the basis of the work of Rodberg,[8] Goebel and Schnitzer,[9] Carruthers,[12,13] and Bethe.[12]

The strong-coupling method was introduced a long time ago by Wentzel,[31,32] Oppenheimer and Schwinger,[33]

[11] K. Itabashi, M. Kato, K. Nakagawa, and G. Takeda, Progr. Theoret. Phys. (Kyoto) **24**, 529 (1960).

[12] P. Carruthers and H. A. Bethe, Phys. Rev. Letters **4**, 536 (1960).

[13] P. Carruthers, Ann. Phys. **14**, 229 (1961).

[14] P. Carruthers, Phys. Rev. **122**, 1949 (1961).

[15] C. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960); M. Baker and F. Zachariasen, *ibid.* **118**, 1659 (1960).

[16] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1609 (1960); M. Gell-Mann and F. Zachariasen, Synchrotron Laboratory, California Institute of Technology Report CTSL-26, Pasadena, 1961 (unpublished); S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961); R. M. Littauer, H. F. Schopper, and R. R. Wilson, *ibid.* **141**, 144 (1961).

[17] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

[18] F. Salzman and G. Salzman, Phys. Rev. **120**, 599 (1960); F. Salzman and G. Salzman, Phys. Rev. Letters **5**, 377 (1960). The introduction of these approximations into the framework of the dispersion theory with general unitarity has been discussed by K. Meetz, Phys. Rev. **125**, 714 (1962).

[19] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961).

[20] J. A. Anderson, Vo X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters **6**, 365 (1961).

[21] S. Goldhaber and G. Goldhaber, Seminar talk at Massachusetts Institute of Technology, summer, 1961 (unpublished).

[22] L. S. Rodberg, Phys. Rev. **106**, 1090 (1957); E. Kazes, *ibid.* **107**, 1131 (1957).

[23] R. Omnès, Nuovo cimento **6**, 780 (1957).

[24] S. Barshay, Phys. Rev. **103**, 1102 (1956).

[25] J. Franklin, Phys. Rev. **105**, 1101 (1957).

[26] W. N. Wong and M. Ross, Phys. Rev. Letters **3**, 398 (1959).

[27] Y. Tomozawa, University College, London, 1961 (to be published).

[28] S. Barshay, Phys. Rev. **111**, 1651 (1958).

[29] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

[30] I. Derado, Nuovo cimento **15**, 853 (1960).

[31] G. Wentzel, Helv. Phys. Acta **13**, 169 (1940).

[32] W. Pauli and M. Dancoff, Phys. Rev. **62**, 85 (1942); G. Wentzel, Helv. Phys. Acta **26**, 222, 551 (1943).

[33] J. R. Oppenheimer and J. Schwinger, Phys. Rev. **60**, 150 (1941).

and Pauli and Dancoff,[32] and further investigated in the meantime by several authors[34]; the latest contributions were those of Kaufman,[35] Pais and Serber,[36] Goebel,[37] Landovitz and Margolis,[38] the author,[39–41] Nickle and Serber,[42] and Chun.[43] Stable nucleon isobars have been obtained as a solution of the fixed-source theoretical models for values of the coupling constant above a certain limit. Scattering states orthogonal to the nucleon isobars have been found. As an important property these scattering states contain pion scattering connected with isobaric excitation. If extended to the case of unstable isobars, this would give pion production via isobaric excitation. Such an extension has not been treated until now. But a first survey of the results can be obtained by considering the strong-coupling Low equation. This is a Low equation, in which the isobars are taken into account explicitly as investigated by Goebel and the author for the charged scalar[37,40] and the symmetric pseudoscalar theory.[41] Starting with the results of this last-mentioned work, we give in the following this first survey and an estimate for the pion production via isobaric excitation in pion-nucleon collisions resulting from the strong-coupling method.

2. STRONG-COUPLING LOW EQUATION

As has been pointed out,[40,41] the strong-coupling Low equation can be obtained by considering that the Wick expression[44] for the S matrix of the fixed-source meson theory,

$$S_{N\mathbf{k}}=\delta_{N\mathbf{k}}-2\pi i\delta(E_N-E_\mathbf{k})T_\mathbf{k}(N), \tag{1}$$

with

$$T_\mathbf{k}(N)=(\chi_N^-,V_\mathbf{k}\chi_{0T}), \tag{2}$$

and in the symmetric pseudoscalar case

$$V_\mathbf{k}=-\frac{ig}{2\pi\kappa}\frac{v(k)}{(2\epsilon)^{\frac{1}{2}}}\sum_i k_i\sigma_i\tau_\rho, \tag{3}$$

can also be written down for the case of stable isobars. Then the meson vacuum state χ_{0T} contains, not only the nucleon ground state, but also the isobaric states, as indicated by the index T, so that

$$H\chi_{0T}=E_T\chi_{0T}, \tag{4}$$

E_T being the isobaric energy eigenvalues with

$$|\Delta_{TT'}|=|E_T-E_{T'}|>0. \tag{5}$$

In the same manner the ingoing scattering states χ_N^- also can contain isobars in the ingoing as well as in the outgoing asymptotic configuration. k_i are the three components of the ingoing meson momentum $\mathbf{k}$ with the energy $\epsilon=(k^2+\kappa^2)^{\frac{1}{2}}$, where κ is the meson rest mass. $v(k)$ is the Fourier transform of the source function $[v(0)=1]$, and σ_i, τ_ρ are spin and isospin of the bare nucleon.

Then the strong-coupling Low-equation can be obtained from (2) and (3) in the same way as the usual Low-equation in the Chew-Low[29] work. One obtains[40,41]

$$T_\mathbf{k}(N)$$
$$=-\sum_{N'}\left\{\frac{T_{\mathbf{k}_N}{}^*(N')T_\mathbf{k}(N')}{\epsilon_{N'}-\epsilon_N+\Delta_{T_{N'}T_N}-i\alpha}+\frac{T_\mathbf{k}{}^*(N')T_{\mathbf{k}_N}(N')}{\epsilon_{N'}+\epsilon_N+\Delta_{T_{N'}T_\mathbf{k}}}\right\}, \tag{6}$$

where ϵ_N, $\epsilon_{N'}$ are the total meson energies in the concerning states and $\Delta_{T_{N'}T_N}$, $\Delta_{T_{N'}T_\mathbf{k}}$ are the isobaric energy differences as defined by (5), whose appearance is an important difference from the usual Low equation. The indices N, N', $\mathbf{k}$ distinguish the outgoing, intermediate, and ingoing states, respectively. Equation (6) holds if the ingoing and outgoing states $\chi_\mathbf{k}$ and χ_N are only one-meson states, whereas the intermediate states $\chi_{N'}$ have to contain the full spectrum. For more than one meson in the outgoing state one gets, instead of (6), a more complicated equation as can be seen for the usual Low equation from the work of Omnès.[23]

3. STRONG-COUPLING ONE-MESON APPROXIMATION

As a counterpart to the one-meson approximation of the usual Low equation, one can introduce the one-meson approximation of the strong-coupling Low equation, in which the isobars are taken into account explicitly in the asymptotic configurations. It has been shown[40,41] for the fixed-source meson-theoretical models that this strong-coupling one-meson Low equation has a finite limit for infinite g, which at the same time is exactly the limit of the whole Low equation. This Low equation for infinite g has been given and investigated by the author for the charged scalar,[40] symmetric scalar,[39] and symmetric pseudoscalar[41] cases. It plays the role of a consistency equation for the transformation procedure of the strong-coupling method and has, as a solution, the scattering solution for the infinite g case of the transformation procedure, for which case the pion production vanishes.

So the strong-coupling one-meson Low equation has the advantage that there is a case, the infinite g case, for which it is exactly valid whereas no such case exists for the usual Low equation except for the trivial $g=0$ case. Since this strong-coupling one-meson approximation includes isobaric excitation, an extension to the case of unstable isobars would mean that pion production via isobaric excitation has been taken into

[34] See survey given in reference 39.
[35] A. N. Kaufman, Phys. Rev. **92**, 468 (1953).
[36] A. Pais and R. Serber, Phys. Rev. **105**, 1636 (1957); Phys. Rev. **113**, 955 (1959).
[37] C. J. Goebel, Phys. Rev. **109**, 1846 (1958).
[38] L. Landovitz and B. Margolis, Phys. Rev. Letters **1**, 206 (1958); **2**, 318 (1959), Ann. Phys. **7**, 52 (1959).
[39] H. Jahn, Nuclear Phys. **26**, 353 (1961); Phys. Rev. **124**, 280 (1961).
[40] H. Jahn, Fortsch. Physik **7**, 452 (1959); Z. Physik **156**, 633 (1959).
[41] H. Jahn, lecture notes, Oberwolfach, 1959 (unpublished).
[42] H. Nickle and R. Serber, Phys. Rev. **119**, 449 (1960).
[43] K. W. Chun, Phys. Rev. **112**, 973 (1961).
[44] G. C. Wick, Revs. Modern Phys. **27**, 339 (1955).

account. But this is just the approximation corresponding to the explanation of the late rise in the production cross section given by Lindenbaum and Sternheimer[2] which is referred to at the beginning of this paper. Thus the strong-coupling one-meson approximation has important support by the experiments.

The procedure of calculation is first to solve the stable case and then to continue the solution regarding to the coupling constant until the experimentally observed unstable case is reached. This would mean that, if we continue the solution analytically into the complex energy plane, the poles and thresholds move from the stable region of the first Riemann sheet into the unstable complex region of the second Riemann sheet. This would correspond to the behavior investigated by Blankenbecler, Goldberger, MacDowell, and Treiman[45] for the pion production via unstable deuteron excitation in nucleon-nucleon collisions.

The details of the formalism in our case are as follows:

By introducing in (6) the scattering phase shift $\delta_\alpha{}^{T'T}(\epsilon)$ according to

$$T_{\mathbf{k}}(N)=-\frac{v(k)v(k_N)}{(2\pi)^2(\epsilon\epsilon_N)^{\frac{1}{2}}}\sum_\alpha P_\alpha{}^{T'T}h_\alpha{}^{T'T}(\epsilon_N), \qquad (7)$$

with

$$h_\alpha{}^{T'T}(\epsilon)=\exp[i\delta_\alpha{}^{T'T}(\epsilon)]\sin[\delta_\alpha{}^{T'T}(\epsilon)]/[v(k)]^2k_{T'T}{}^3, \quad (8)$$

we obtain for the strong coupling one-meson approximation

$$h_\alpha{}^{T'T}(\epsilon)=h_\alpha{}^{0T'T}(\epsilon)+\sum_{T''}\frac{1}{\pi}P\int_\kappa^\infty d\epsilon' \times\left\{\frac{k_{T''T}{}'^3[v(k_{T''}{}')]^2h_\alpha{}^{T'T''*}(\epsilon)h_\alpha{}^{T''T}(\epsilon')}{\epsilon'-\epsilon-i\alpha}+\text{cross}\right\}. \quad (9)$$

Here, as in the Chew-Low paper,[29] $\alpha=(2I,2J)$ indicates the pair of total isospin and angular momentum quantum numbers to which the phase shift concerned belongs. The same pair of quantum numbers is indicated by T' and T for the outgoing and incoming isobaric states, respectively, thus describing isobaric excitation by means of pion scattering and pion scattering on isobars as well as on nucleons. In the Chew-Low[29] case there was only elastic meson scattering on nucleons with $T'=T=(1,1)$. $P_\alpha{}^{T'T}$ are the concerning projection operators (see Appendix), which in the case of $T'=T=(1,1)$ are given by Wick[44] and Chew and Low.[29] "Cross" on the right-hand side of (9) indicates the crossed term corresponding to the second term on the right-hand side of (6). We do not want to write it down here for our following estimate. But for $T'=T=(1,1)$, we also have the same crossing matrix $A_{\alpha T}$ as in the Wick[44] and Chew and Low[29] papers if the inelastic processes are included on the right-hand side by carrying out the summation over T''.

4. ZERO-MESON PART

In (9) the inhomogeneity $h_\alpha{}^{0TT'}(\epsilon)$ represents the zero-meson part which has to be obtained from (6) as the summand with $N'=0$, $\epsilon_{N'}=0$. This summand is

$$T_{\mathbf{k}}{}^0(N)=-\left(\frac{g}{2\pi\kappa}\right)^2\frac{v(k)v(k_N)}{2(\epsilon\epsilon_N)^{\frac{1}{2}}}\sum_{i',i,T''}k_{Ni'}k_i \times\left\{\frac{(\chi_{0T'},\sigma_{i'}\tau_{\rho'}\chi_{0T''})(\chi_{0T''},\sigma_i\tau_\rho\chi_{0T})}{\Delta_{T''T}-\epsilon_N} +\frac{(\chi_{0T'},\sigma_i\tau_\rho\chi_{0T''})(\chi_{0T''},\sigma_{i'}\tau_{\rho'}\chi_{0T})}{\Delta_{T''T}+\epsilon_N}\right\}. \quad (10)$$

The isobaric spectrum of Wentzel[32] and Pauli and Dancoff[32] contains only states with equal spin and isospin. It could be verified[41] that using only these states the transformation procedure of the strong-coupling method can be carried through consistently. Whether the states with unequal spin and isospin calculated additionally by Landovitz and Margolis[38] can be included remains an open question.

With only the use of the isobaric states with equal spin and isospin, it has been shown[41] that, in the zero-order strong-coupling approximation, the product $\sigma_i\tau_\rho$ in (10) can mainly be replaced by operators $S_{i\rho}$ already introduced by Pauli and Dancoff[32] and Wentzel.[32] These $S_{i\rho}$ can be understood as the coefficients of an orthogonal transformation leading from a room-fixed to a body-fixed coordinate system of a symmetric top whose angular momenta measured in the body-fixed and the room-fixed system are identical with the isospin and the usual angular momentum, respectively. In this picture the equality,

$$L=T, \qquad (11)$$

of the magnitudes of the angular momentum L and the isospin T of the isobaric states is in fact verified with a $T(T+1)$-dependent mass spectrum of the isobars.[33]

The matrix representation of the $S_{i\rho}$ operators with regard to the isospin and the angular momentum quantum numbers of the isobars has been investigated by Fierz[46] (see Appendix). One important result is that there are only matrix elements with $\Delta T=1,\ 0,\ -1$ but for any of the infinite isobaric quantum numbers T of the $T(T+1)$ spectrum. So if we represent (10) by

[45] R. Blankenbecler, M. L. Goldberger, S. W. MacDowell, and S. B. Treiman, Phys. Rev. **123**, 692 (1961). A survey of the papers about the analytic behavior of the scattering amplitudes in the second sheet and a general presentation of this topic is given in the article by R. Oehme, "The Compound Structure of Elementary Particles," *Werner Heisenberg und die Physik unserer Zeit*, (Verlag Friedr. Vieweg u. Sohn, Braunschweig). Early discussions were already given by V. Glaser and G. Källén, Nuclear Physics **2**, 706 (1956/57) and G. Höhler, Z. Physik **152**, 546 (1958) and M. Lévy, Nuovo cimento **15**, 13 (1959).

[46] M. Fierz, Helv. Phys. Acta **17**, 181 (1944).

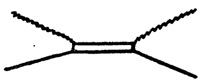


FIG. 1. Strong-coupling graph for the $\frac{3}{2}$, $\frac{3}{2}$ pion-nucleon resonance scattering.

means of graphs, then there are only transitions with $\Delta T=1, 0, -1$ for any quantum number T at any vertex of any graph, at least in that approximation in which we replace $\sigma_i\tau_\rho$ by $S_{i\rho}$.

For the elastic scattering the only pion-nucleon graph giving a resonance denominator is shown in Fig. 1, thus corresponding to the first 3, 3 resonance of the pion-nucleon scattering. The jagged lines are the pion lines, and the single and twofold or manifold straight lines are the nucleon and isobar lines, respectively.

For the inelastic scattering, the graphs giving resonance denominators are given in Fig. 2 for the case of excitation of the 3, 3 isobar, and given in Fig. 3 for the case of 5, 5 excitation. The denominators of the graphs of Figs. 2(a), (b) and Fig. 3(b) α have the same zeros as the denominator of the preceding elastic 3, 3 graph corresponding to the excitation of the 3, 3 isobar. For the graphs Figs. 3(a) and (b) β, the zeros of the denominators are determined by the energy difference between the 5, 5 isobar and the 3, 3 isobar. If we call the latter Δ_{53} and the 3, 3 excitation energy Δ, then we get $\Delta_{53}=(5/3)\Delta$ as a consequence of the $T(T+1)$ dependence of the Wentzel and Pauli-Dancoff[32] spectrum of the isobars.

But these poles of Figs. 2(a), (b) and Figs. 3(a), (b) lie altogether below the thresholds of the inelastic processes concerned, which lie at $\Delta+\kappa$ for Figs. 2(a), (b)

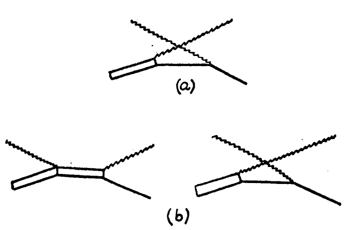


FIG. 2. Strong-coupling graphs for the excitation of the $\frac{3}{2}$, $\frac{3}{2}$ isobar by means of pion-nucleon collision, (a) for total isospin $I=\frac{1}{2}$; (b) for total isospin $I=\frac{3}{2}$.

and at $\Delta_{51}+\kappa=(8/3)\Delta+\kappa$ for Figs. 3(a), (b). So we get not just resonance maxima but another kind of contribution arising from the product between the decreasing branch of the resonance factor and the rising threshold factor of the isobaric excitation.

5. SURVEY OF FURTHER EFFECTS

In contrast to the usual one-meson Low equation, Eq. (9) represents an infinite set of coupled integral equations because of the infinite number of isobars of the $T(T+1)$ spectrum whose elastic as well as excitation amplitudes are multiplied with each other in the summation over T'' on the right-hand side of (9). In this way contributions arise on the right-hand side of (9), giving further effects beyond those indicated by the graphs previously cited. For instance, the graphs in Fig. 4 indicate effects with the resonance energies

$$\Delta_{53}+\Delta_{31}=\Delta_{51}=(8/3)\Delta,\quad \Delta_{53}+\Delta_{51}=(13/3)\Delta,$$

and

$$\Delta_{51}=(8/3)\Delta,$$

of the amplitudes $h_\alpha^{35}(\epsilon)$ and $h_\alpha^{55}(\epsilon)$ which occur in the summation on the right-hand side of the equations for $h_\alpha^{31}(\epsilon)$ and $h_\alpha^{51}(\epsilon)$ multiplied by the threshold factor $k_{51}^3=[(\epsilon-\Delta_{51})^2-\kappa^2]^{3/2}$. Similar contributions can also be obtained by the elastic scattering amplitudes on the 3, 3 isobar with the resonance energies Δ or 2Δ and the

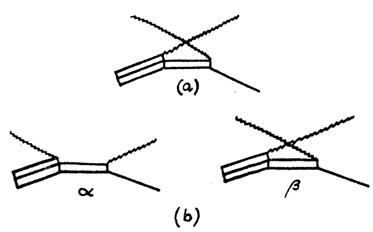


FIG. 3. Strong-coupling graphs for the excitation of the $\frac{5}{2}$, $\frac{5}{2}$ isobar by means of pion-nucleon collision, (a) for total isospin $I=\frac{1}{2}$; (b) for total isospin $I=\frac{3}{2}$.

threshold factor $k_{31}^3=[(\epsilon-\Delta)^2-\kappa^2]^{3/2}$ on the right-hand sides of the equations for $h_\alpha^{31}(\epsilon)$ and $h_\alpha^{51}(\epsilon)$. The corresponding graphs would be as shown in Fig. 5. They should have some relation to the approach of pion scattering on 3, 3 isobars considered by Tomozawa.[27] Finally we get contributions from the inelastic processes represented by the graphs 1 and 2 with the threshold factor k_{31}^3 and k_{51}^3, respectively, to the right-hand side of the equations for the elastic pion-nucleon scattering.

Thus the mechanism of the strong-coupling one-meson Low-equation presents, because of isobaric excitation, several possibilities of getting maxima of the pion-nucleon cross section beyond the first maximum.

6. ROUGH ESTIMATE

In this paper we only want to carry through a rough semiphenomenological estimate which shall show us whether the results for the cross section of pion production via isobaric excitation from the strong-coupling method are in the same order of magnitude as the experimental results. For this purpose we realize that the zero-meson parts $h_{33}^{011}(\epsilon)$, $h_{11}^{031}(\epsilon)$, and $h_{33}^{031}(\epsilon)$ which belong to the graphs of Fig. 1, Fig. 2(a), and Fig. 2(b) differ by constant factors only because, according to (7), (8), (9), and (10), we have

$$h_{33}^{011}(\epsilon)=\lambda_{33}^{11}/(\Delta-\epsilon),\quad h_{11}^{031}(\epsilon)=\lambda_{11}^{31}/(\epsilon-\Delta),\quad h_{33}^{031}(\epsilon)=\lambda_{33}^{31}/(\epsilon-\Delta),\tag{12}$$

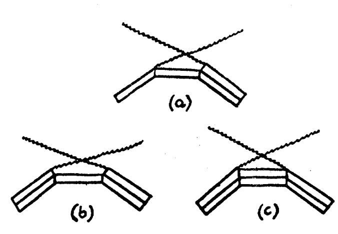
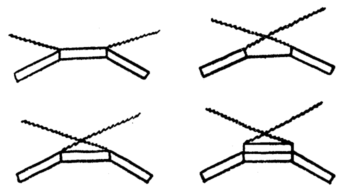


FIG. 4. Strong-coupling graphs for (a) excitation of the $\frac{5}{2}$, $\frac{5}{2}$ isobar by means of pion collision with the $\frac{3}{2}$, $\frac{3}{2}$ isobar; (b) and (c) pion scattering on the $\frac{5}{2}$, $\frac{5}{2}$ isobar.

FIG. 5. Strong-coupling graphs for the scattering of pions on the $\frac{3}{2}$, $\frac{3}{2}$ isobar.



where the numbers $\lambda_\alpha^{T'T}$ follow from applying to Eq. (10) the projection operators $P_\alpha^{T'T}$ introduced in (7). From (12) it follows that

$$|h_{11}^{031}(\epsilon)|^2=(\lambda_{11}^{31}/\lambda_{33}^{11})^2|h_{33}^{011}(\epsilon)|^2,$$
$$|h_{33}^{031}(\epsilon)|^2=(\lambda_{33}^{31}/\lambda_{33}^{11})^2|h_{33}^{011}(\epsilon)|^2. \qquad (13)$$

Now one would expect that these relations (12) would hold approximately for $h_{33}^{11}(\epsilon)$, $h_{11}^{31}(\epsilon)$, and $h_{33}^{31}(\epsilon)$ as well as for $h_{33}^{011}(\epsilon)$, $h_{11}^{031}(\epsilon)$, and $h_{33}^{031}(\epsilon)$ if one is not too far away from the 3, 3 resonance maximum since because of (12) all three amplitudes have the same resonance energy. Then this extension of (13) would give, with the cross-section formula[39]

$$\sigma_\alpha^{T'T}=4\pi[v(k)]^2[v(k_N)]^2k\cdot k_N^3|h_\alpha^{T'T}(\epsilon)|^2, \qquad (14)$$

the relations

$$\sigma_{11}^{31}(\epsilon)=(\lambda_{11}^{31}/\lambda_{33}^{11})^2(k_{31}/k)^3\sigma_{33}^{11}(\epsilon),$$
$$\sigma_{33}^{31}(\epsilon)=(\lambda_{33}^{31}/\lambda_{33}^{11})^2(k_{31}/k)^3\sigma_{33}^{11}(\epsilon), \qquad (15)$$

by which we have expressed the inelastic cross sections for 3,3 excitations by means of the 3, 3 elastic cross section. In (15) we have set $v(k)=1$. $k_{31}=[(\epsilon-\Delta)^2-\kappa^2]^{\frac{1}{2}}$ is the threshold factor for excitation of the 3, 3 isobar with the excitation energy Δ.

For a first rough estimate it may probably be not too bad to use (15) also for the case of unstable isobars as is experimentally observed. Then (15) would give us the relations between the pion production cross section via the excitation of the 3, 3 isobar and the cross section of the 3, 3 elastic resonance scattering. If for the latter we insert the experimentally observed shape and for Δ the experimentally observed resonance energy on the right-hand side of (15) and take the $\lambda_\alpha^{T'T}$ from the strong-coupling approximation, then we obtain a semiphenomenological estimate of the pion production via excitation of the 3, 3 isobar following from the strong-coupling method.

The evaluation of the $\sigma_i\tau_\rho$ and $\sigma_{i'}\tau_{\rho'}$ matrix elements in the zero-order strong-coupling approximation has to be carried through by inserting the $S_{i\rho}$ operators using the matrix-representation of Fierz[46] as mentioned in connection with Eq. (11) (see Appendix). This gives, after considering (7), (8), (10), and (12),

$$\lambda_{11}^{31}/\lambda_{33}^{11}=(8/g)\sqrt{2}, \quad \lambda_{33}^{31}/\lambda_{33}^{11}=\sqrt{2}/2. \qquad (16)$$

That means [see Eq. (15)] that this inelastic contribution is much smaller (about 4 times) to the 3, 3 state [graph, Fig. 2(b)] than to the 1, 1 state [graph, Fig. 2(a)]. A similar result also holds for the contributions corresponding to the graphs Fig. 3(a) and (b). This is very satisfactory with regard to the experimental results, showing much more contribution to the inelastic pion-nucleon cross section below 1 Bev for isospin $\frac{1}{2}$ than for isospin $\frac{3}{2}$.

Now the problem arises how the threshold has to be taken into account for the excitation of such short-lived states as our isobars are. In this case the resonance width Γ is in the same order of magnitude as the resonance energy Δ and can no longer be neglected. The threshold is at the complex energy $\Delta-i\Gamma/2$. That such complex thresholds should occur has been pointed out by Blankenbecler, Goldberger, MacDowell, and Treiman[45] for the case of pion production by nucleon-nucleon collisions if the unstable deuteron singlet state is taken into account. For our rough estimate of the pion production cross section via the excitation of the 3, 3 isobar we insert simply $(\epsilon-\Delta)^2+\Gamma^2/4$ instead of $(\epsilon-\Delta)^2$ in k_{31}. The results following from (15) and (16) using the experimental results for $\sigma_{33}^{11}(\epsilon)$ are shown in Fig. 6. They are compared with the experimental results of Crittenden, Scandrett, Shephard, Walker, and Ballam[1] and the much smaller results of Rodberg[22] and Kazes[22] from a different approach using the fixed-source theory only. So our estimate using the one-meson strong-coupling Low equation gives an indication that pion production via isobaric excitation gives a comparable contribution already from the fixed-source theory only and therefore should be taken into account besides the additional π-π contribution. Also the contributions corresponding to the graphs Fig. 3(a) and Figs. 4(a), (b), (c) might be considerable in addition to our estimate using Fig. 2(a) only. So a more extended calculation on the basis of the strong-coupling method would be important.

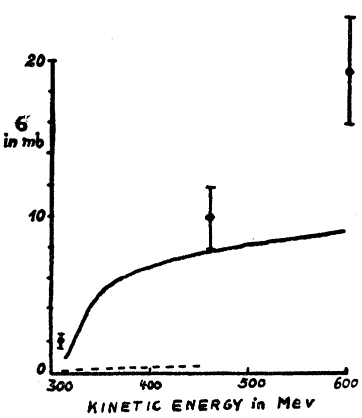


FIG. 6. Pion-nucleon cross section above 300 Mev. Experimental results for inelastic isospin $\frac{1}{2}$ cross section according to Crittenden, Scandrett, Shephard, Walker, and Ballam.[1] The solid curve represents our estimate for $\sigma_{\frac{1}{2}\mathrm{inel.}}$ from the fixed-source theory using only Fig. 2(a) of the strong-coupling approximation without an extra π-π term. The dashed curve represents the fixed-source theory prediction of the total $\pi^-+p\rightarrow\pi^++\pi^-+n$ cross section made by Rodberg[22] and Kazes[22] from the fixed-source theory without an extra π-π term.

ACKNOWLEDGMENTS

I am greatly indebted to Professor S. S. Schweber and the Physics Department of Brandeis University for their kind hospitality during the summer half-year 1961 and to Dr. S. Barshay, Dr. M. Chretien, Dr. R. Drachman, Dr. A. Grossman, Dr. D. Kleitman, Dr. H. Pendleton, Dr. R. Rockmore, and Dr. K. Wilson for interesting conversations. I wish especially to thank Dr. O. W. Greenberg for a stimulating discussion, Dr. I. Derado, with whom the subject of this paper was discussed over a period of several years, and Dr. N. Schmitz who called my attention to certain experimental results.

APPENDIX

Matrix representation of the $S_{i\rho}$ operators explained in connection with Eq. (11) and used to get the results of Eq. (16) and of the projection operator introduced with Eq. (7).

The result obtained by Fierz for the matrix representation of the $S_{i\rho}$ operators with respect to the isospin and angular momentum quantum numbers can be presented as follows:

$$S_{i\rho}\equiv\begin{bmatrix} \cdots & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \dfrac{B_i^{(T-1,T)}C_\rho^{(T-1,T)}}{T[(2T+1)(2T-1)]^{\frac12}} & 0 & 0 \\ 0 & \dfrac{B_i^{(T,T-1)}C_\rho^{(T,T-1)}}{T[(2T+1)(2T-1)]^{\frac12}} & \dfrac{\mathcal{L}_i^{(T)}\mathcal{T}_\rho^{(T)}}{T(T+1)} & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \cdots \end{bmatrix}$$

$$\equiv\begin{bmatrix} \dfrac{\bar\sigma_i\bar\tau_\rho}{3} & \dfrac{B_i^{(\frac12,\frac32)}C_\rho^{(\frac12,\frac32)}}{3\sqrt2} & 0 & 0 & 0 \\ \dfrac{B_i^{(\frac32,\frac12)}C_\rho^{(\frac32,\frac12)}}{3\sqrt2} & \dfrac{\mathcal{L}_i^{(\frac32)}\mathcal{T}_\rho^{(\frac32)}}{15/4} & \cdots & 0 & 0 \\ 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots \end{bmatrix},$$

where the lowest submatrices are:

$$\mathcal{L}_i^{(\frac32)},\ \mathcal{T}_\rho^{(\frac32)}=\tfrac12\begin{bmatrix} 0 & \sqrt3 & 0 & 0 \\ \sqrt3 & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt3 \\ 0 & 0 & \sqrt3 & 0 \end{bmatrix},\quad \tfrac12 i\begin{bmatrix} 0 & -\sqrt3 & 0 & 0 \\ \sqrt3 & 0 & -2 & 0 \\ 0 & 2 & 0 & -\sqrt3 \\ 0 & 0 & \sqrt3 & 0 \end{bmatrix},\quad \begin{bmatrix} \frac32 & 0 & 0 & 0 \\ 0 & \frac12 & 0 & 0 \\ 0 & 0 & \frac12 & 0 \\ 0 & 0 & 0 & \frac12 \end{bmatrix};$$

$$B_i^{(\frac32,\frac12)},\ C_\rho^{(\frac32,\frac12)}=\tfrac12\begin{bmatrix} 0 & -\sqrt6 & 0 & 0 \\ 0 & 0 & -\sqrt2 & 0 \\ 0 & \sqrt2 & 0 & 0 \\ 0 & 0 & \sqrt6 & 0 \end{bmatrix},\quad \tfrac12 i\begin{bmatrix} 0 & \sqrt6 & 0 & 0 \\ 0 & 0 & \sqrt2 & 0 \\ 0 & \sqrt2 & 0 & 0 \\ 0 & 0 & \sqrt6 & 0 \end{bmatrix},\quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sqrt2 & 0 & 0 \\ 0 & 0 & \sqrt2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$B_i^{(\frac12,\frac32)},\ C_\rho^{(\frac12,\frac32)}=\tfrac12\begin{bmatrix} 0 & 0 & 0 & 0 \\ -\sqrt6 & 0 & \sqrt2 & 0 \\ 0 & -\sqrt2 & 0 & \sqrt6 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ -\tfrac12 i\begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt6 & 0 & \sqrt2 & 0 \\ 0 & \sqrt2 & 0 & \sqrt6 \\ 0 & 0 & 0 & 0 \end{bmatrix},\quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sqrt2 & 0 & 0 \\ 0 & 0 & \sqrt2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$