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Interpretation of Range Measurements for Kilovolt Electrons in Solids

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The maximum range of electrons, as distinguished from the extrapolated or practical range, has been measured in Al, Ni, Ag, and Au for electron energies between 1.0 and 10.0 kev. The technique utilized involves the detection of the onset of penetration through freely supported evaporated films by observation of the current collected in a 2π geometry. The range defined in this manner may be compared with the predictions of stopping theory, with little influence of scattering processes. The results show no significant Z dependence of the exponent in the range-energy relation, in agreement with results at higher energies. Some indications exist for an increasing value of the absolute range with increasing Z when expressed in mg/cm^2, in accordance with the theoretically predicted decrease in stopping power for heavier elements. The results are compared with recent measurements using the onset and decline of luminosity in a phosphor layer to define the range, which leads to a strong Z dependence of the range-energy relation in the oppasite sense to that observed by absorption techniques. It is shown that the differences in results may be traced to the nature of the detecting technique. The luminescence detector is sensitive to the total energy carried by the penetrating electrons in excess of a certain critical value rather than to their number.

I. INTRODUCTION

WHILE the penetration of fast electrons in the energy range from 10 kev to 1 Mev is readily studied by well-known absorption techniques,[1] the difficulty of preparing thin foils only a few hundred angstrom units thick has until now made it necessary to resort to other methods of determining the penetration depth of electrons below 10 kev. These latter methods involve the use of phosphors as detectors[2,3] or utilize the back-scattered electrons from composite layers to detect the onset of penetration.[4,5] Due to the fact that some of the results obtained by these alternative methods are in serious disagreement with each other, especially as far as the dependence on atomic number is concerned, it seemed desirable to extend the usual absorption method down to the very lowest energies made possible by recent advances in the preparation of freely supported thin films.

Aside from the difficulties arising out of the use of different methods for the determination of the range, there is also the problem of the precise definition of the range in any given measuring technique which makes comparison with theory difficult. The present paper will therefore present new data on the penetration of 1–10 kev electrons through thin foils of carbon, aluminum, nickel, silver and gold, using current-collection from the exit side in a 2π geometry. Both "practical" and "maximum" ranges as defined below will be compared with data at higher energies using the same technique, as well as with experimental results using other techniques in the energy region below 10 kev. Finally, the physical reasons for the disagreement between the data obtained by different methods will be analyzed and shown to result from the difference between the characteristics of an energy sensitive detector and one responsive to the number of penetrating electrons irrespective of their energy.

II. EXPERIMENTAL TECHNIQUE AND RANGE DEFINITION

The absorption method used in the present experiments involves a measurement of the electron current penetrating thin foils of various thicknesses by collecting them in a spherical electrode arrangement provided with a suppressor screen as described in detail elsewhere.[6] For each foil the total number of electrons penetrating the foil with energies in excess of 45 ev was plotted as a fraction of primary energy from zero to 10 kev by means of an X–Y recorder.

A typical absorption curve obtained in this manner for Al is shown in Fig. 1, corrected for the effect of secondary electrons emitted from the collector and sup-

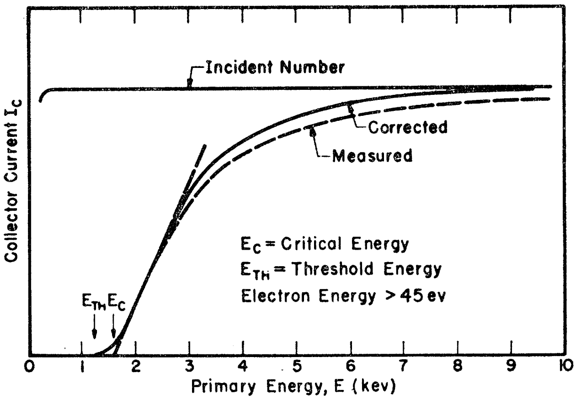

FIG. 1. Typical electron absorption curve for an aluminum film of 17 μg/cm^2 thickness.

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[1] For a summary of the theory and experimental results on the range of electrons in matter, see the review article by H. A. Bethe and J. Ashkin in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley & Sons, Inc., New York, 1953), Vol. I.

[2] J. R. Young, J. Appl. Phys. **27**, 1 (1956).

[3] C. Feldman, Phys. Rev. **117**, 455 (1960).

[4] J. E. Holliday and E. J. Sternglass, J. Appl. Phys. **30**, 1428 (1959).

[5] I. M. Bronshtein and R. B. Segal, Soviet Phys.-Solid State **1**, 1365 (1960).

[6] H. Kanter, Phys. Rev. **121**, 465 (1961).

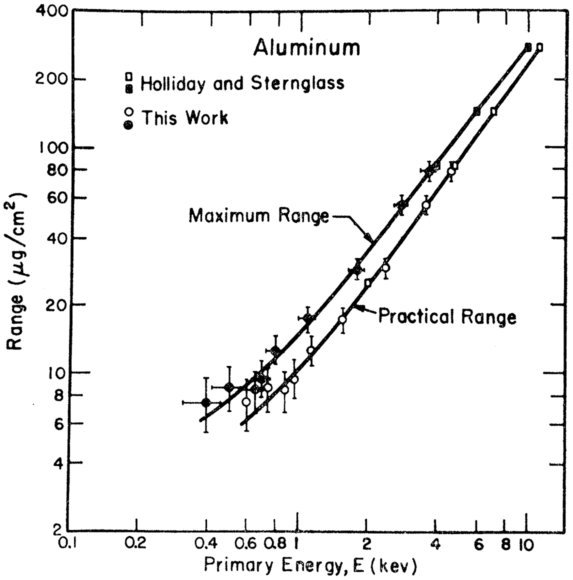

FIG. 2. Maximum range and practical range data for 0.4–10.0 kev electrons in aluminum.

pressor grid. Also shown on this plot are the two methods of defining the range commonly used in the literature. The energy marked E_c is often termed the "critical energy," defined by extrapolating the linear portion of the absorption curve. The critical energy corresponds to the so-called "practical range." The energy marked E_{Th} is the "threshold energy," more nearly representative of the maximum range of the electrons in the material, or the primary energy for which the average energy of the emerging electrons is reduced to zero.

Although the maximum range is believed to equal closely the range defined by stopping theory,[7] the practical range characterized by E_c is somewhat more uniquely determined experimentally, independent of the sensitivity of the detector. It has been found empirically that E_c provides a very convenient measure in terms of which all the absoprtion curves as well as the mean and most probable energies of the emerging electrons may be normalized.[6,8] Physically, E_c may be characterized as the energy below which all the penetrating electrons are emitted with a cosine distribution, indicative of a state of complete diffusion. On the other hand, the practical range suffers from the disadvantage that it is relatively more strongly dependent upon scattering, especially for high energies and high atomic numbers where elastic scattering by the screened nucleus plays an important part. In order to clarify the relation between these two types of ranges, both were determined in all cases and compared with each other over the energy range from 0.6 to 10.0 kev.

III. TARGET PREPARATION

The metal films studied in the present investigation were all prepared by rapid vacuum evaporation onto nitro-cellulose film, stretched across a tube of 6-mm inside diameter as described in detail elsewhere.[6] The supporting film was removed by baking in air at 200°C, leaving the target material stretched across the target holder supported only at the edges. Thickness determinations by multiple beam interferometry were carried out on samples simultaneously evaporated onto glass slides. These measurements are believed to be accurate to $\pm 5\%$ or 25 Å, whichever is larger. For the reasons discussed in reference 6, actual target thicknesses measured in mass per unit area are believed to be accurate to only about $\pm 10\%$ or $\pm 2\ \mu$g/cm^2, respectively.

In the case of aluminum, where oxidation is especially severe, a correction for the presence of an oxide was made as discussed in reference 6. Since the theoretical stopping power of aluminum and aluminum oxide differs only very little, the presence of this oxide is not believed to represent a serious source of error.

The carbon films were prepared by vacuum evaporation from carbon electrodes. Due to uncertainty in the actual density of the resulting layers, only relative measurements could be made on these films.

IV. EXPERIMENTAL RESULTS

The results for the maximum and practical ranges as defined by Fig. 1 are show nfor the two typical cases of Al and Au in Figs. 2 and 3. Also shown for comparison are the recent data of Kanitcheva and Burtsev[9] for Au and those of Holliday and Sternglass for Al and Au[4],

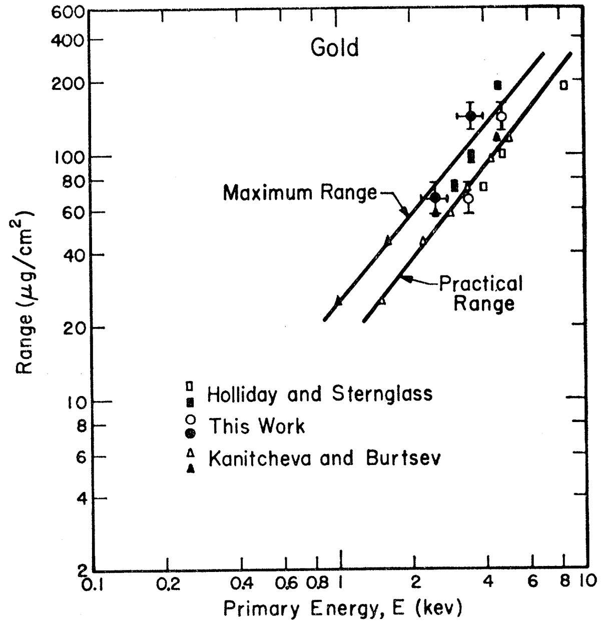

FIG. 3. Maximum range and practical range data for 1–10 kev electrons in gold.

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[7] For a review on stopping theory see W. Brandt, Health Phys. 1, 11 (1958).

[8] F. Wecker, Ann. Physik 40, 405 (1941).

both of which used techniques in which the number of electrons are used to detect the onset of penetration, either by transmission[9] or by backscattering.[4]

It is seen that good agreement between the results of the different authors exists in all cases both for the maximum and the practical ranges wherever the regions investigated join or overlap. It is also of interest to note that whereas the two types of ranges approach each other more closely for Al as the energy or the foil thickness is increased, the two curves remain nearly parallel on the log-log plot for Au, indicating a nearly constant percentage difference.

This is more graphically brought out by the plot of E_c/E_{Th} vs film thickness D in Fig. 4. The measurements on carbon have been added also since these are independent of the precise absolute value of D, together with the results for Ni and Ag. It appears that for elements of atomic number greater than about 28, the ratio of the two ranges is almost independent of foil thickness, such that the maximum range is some 40% greater than the practical range determined by E_c. Thus, although the absolute values of the different types of ranges will differ substantially, the slopes of the corresponding range-energy curves should be fairly independent whether maximum or practical ranges are determined. Only at low energies and for low Z elements will the slope of the practical range-energy curves be somewhat higher than for the maximum range-energy curves.

In order to see whether there is a definite Z dependence of the maximum ranges for different materials, the maximum range curves drawn through the experimental points in Figs. 2 and 3 were replotted in Fig. 5, together with the maximum range data for Ag and Ni. It is apparent that there is a definite and consistent dependence on atomic number such that the materials of lower Z show a lower range at a given energy throughout the region investigated. This is to be expected on the basis of the Bohr–Bethe Theory[10] according to which the mean excitation potential increases with Z, leading to a greater stopping power per unit mass for low Z materials and, therefore, to a shorter range.

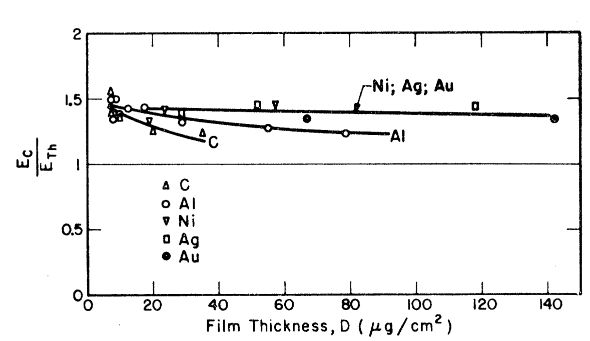

FIG. 4. Ratio of critical energy E_c to threshold energy E_{Th} as a function of thickness for elements of different atomic number.

It is seen that this difference tends to decrease with increasing energy, a fact which is more clearly brought out in Fig. 6, where the present data for Al and Au has been combined with all available measurements on these materials of similar type all the way to 1 Mev. In the region of 40 to 120 kev, the results of Gubernator[11] and Flammersfeld[12] show that the range becomes almost independent of Z, although Al continues to show a slightly steeper slope than Au well past 120 kev. Beyond this point, the data of Seliger[13] and Fleeman[14] actually show a reversal of the situation holding at lower energies, with Al having a slightly larger range than Au beyond 200 kev.

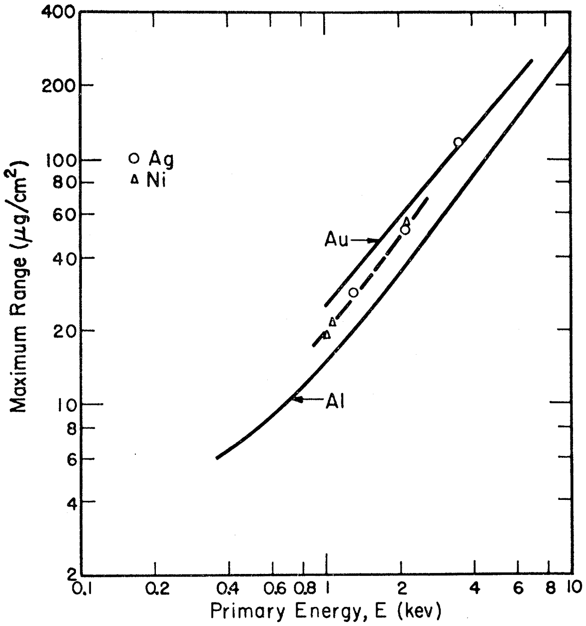

FIG. 5. Maximum range-energy data for elements of different atomic numbers below 10 kev.

This reversal may be understood in terms of a number of factors. First of all, as the energy increases past the K-shell binding energies, the effect of elastic scattering in the higher Z material tends to decrease the relative penetration depth, an effect that is even more strongly noticeable in a plot of the practical range as given by Holliday and Sternglass, Fig. 6 of reference 3. Secondly, relativistic effects on the stopping become important at high energies, which effectively decrease the mean excitation potential in the stopping equation by a factor $(1-\beta^2)$. Since the mean excitation potential for gold is much larger than for aluminum, the relative

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[9] I. R. Kanitcheva and V. A. Burtsev, Fiz. Tverdogo Tela **1**, 1250 (1959).

[10] H. A. Bethe, Ann. Physik **5**, 325 (1930); N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, No. 8 (1948).

[11] V. Gubernator, Z. Physik **152**, 183 (1958).

[12] V. Gubernator and A. Flammersfeld, Z. Physik **156**, 179 (1959).

[13] H. H. Seliger, Phys. Rev. **100**, 1029 (1955).

[14] T. Fleeman, National Bureau of Standards Circular 527, 1954 (unpublished), p. 91.

effect on the logarithmic term is greater for Au than for Al, causing the range for Au to be depressed still further. Finally, at the upper energy of Fig. 6, such phenomena as bremsstrahlung and pair formation begin to become important, which again tend to affect the range in higher Z materials more strongly than in low Z materials.

It is important to note that due to the curvature of the range-energy relation at both extreme ends of the region shown in Fig. 6, there really is no single straight line that can be fitted to all the data over the entire region from 1 to 1000 kev. Thus, all empirical range relations of the form

$$R=bE^n \tag{1}$$

can at most be useful over a narrow energy region and for a limited range of Z values. Furthermore, at no point can one fit a straight line with n as large as 2, which is the form of the classical Whiddington law.[15] This is illustrated by the line of slope 2 drawn through the upper end of the solid line for the Al data near 100 kev. It approximates the data for Al only over a very narrow range between about 40 and 80 kev, but it fails completely at energies much higher or lower than this value. It may, in fact, be regarded as the ideal limiting case to which the Bohr–Bethe theory would lead at high energies if it were not for the onset of relativistic effects and elastic scattering. In the limit of very low energies, the logarithmic term in the Bohr–Bethe expression for the rate of energy loss becomes dominant, leading to a substantial decrease in the effective value of n in Eq. (1) from its classical maximum of 2 closer to a value of 1.5.

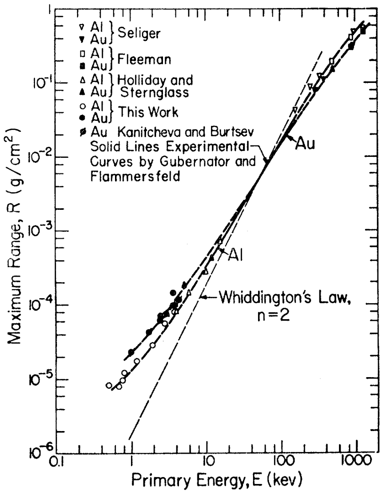

FIG. 6. Comparison of maximum range-energy data for electrons in Al and Au obtained by absorption techniques in the region from 1 kev to 1.5 Mev.

It appears, therefore, that the techniques which measure the number of penetrating electrons lead to a consistent picture all the way from 600 ev to above 1 Mev, in good agreement with the expectations of the Bohr–Bethe theory.

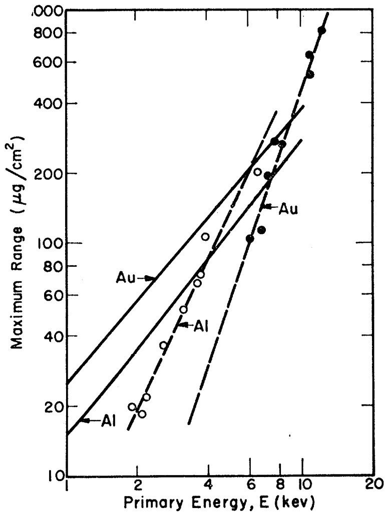

FIG. 7. Comparison of maximum ranges in Al and Au taken from Figs. 2 and 3 with range data obtained by Feldman[3] using the onset of luminescence.

V. DISCUSSION OF OTHER TECHNIQUES

In view of the above results, it becomes particularly important to understand the physical origin of the very divergent results recently obtained by Feldman using the light-output of a phosphor layer to detect the onset of penetration.[3] The disagreement obtained is all the more surprising when one considers that Young,[2] who also employed a phosphor for observing the penetration though in a somewhat different manner, arrived at good agreement with all the above data on aluminum obtained by absorption techniques.

The extent and nature of the disagreement between the data obtained using the onset of luminescence and the data from the number of particles penetrating are illustrated for the two typical cases of aluminum in gold in Fig. 7. The data points are those of Feldman, while the solid lines represent the average of the data for the maximum ranges taken from Figs. 3 and 4.

Inspection of Fig. 7 reveals the following features:

1. The relative positions of aluminum and gold are reversed, such that in the luminescent detector case, electrons show a greater range in aluminum than in gold.
2. The slopes are considerably greater than when measured with an absorption technique, the luminescence methods giving too small a range at low energies, and somewhat too large a range at higher energies.
3. The slope for the higher Z material is larger than for the lower Z material in the case of the phosphor detector, or just the reverse of the results obtained with the absorption technique at all energies greater than 1 kev (see Fig. 6).

Since the metals of intermediate Z fit quite well to this general trend, Feldman was led to conclude that both the constant term and the exponent in the range-energy relation of Eq. (1) are relatively strongly Z dependent. This is in direct contradiction with the results obtained by earlier investigators using absorption

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[15] R. Whiddington, Proc. Roy. Soc. (London) A86, 360 (1912).

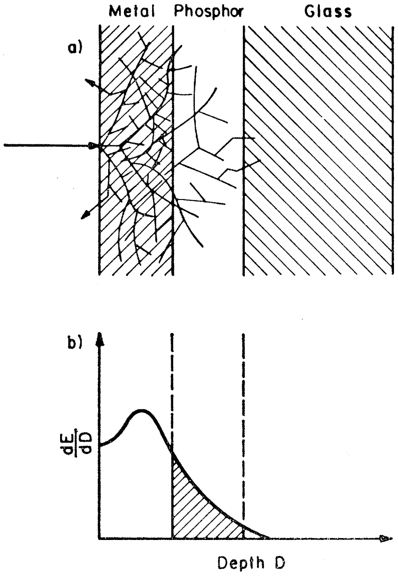

FIG. 8. Electron paths and energy dissipation in a metal and phosphor sandwich layer.

techniques as indicated by Fig. 6. Thus, to the extent that Gubernator and Flammersfeld[11,12] did observe a small Z dependence, they found decreasing values of the exponent in going from aluminum to copper, silver and gold in that order, or the opposite trend noted by Feldman.

In order to resolve this contradiction, it is necessary to examine in greater detail the physical process of electron scattering and energy loss, and the nature of the response of thin phosphor layers covered by conducting films.

The physical arrangement of the metallic layer and phosphor used by Feldman is shown in Fig. 8(a). The metal films used ranged from about 500 to 10 000 Å, as did the transparent phosphor layers deposited on a Vycor glass substrate. A photomultiplier is used to measure the luminescence as a function of primary energy, as shown by a typical response curve reproduced from Feldman's paper in Fig. 9.

The basic assumptions fundamental to this technique are: (a) that the onset of penetration through the metallic film corresponds to the point where the luminescence rises above the background (point 1 in Fig. 9), and (b) that the onset of penetration through the phosphor occurs at the point where a deviation from a linear increase in light output occurs (point 2 in fig. 9). In order to examine these assumptions, it is necessary to consider the energy of the penetrating electrons since it is this quantity to which the phosphor responds.

Information on the energy of the penetrating electrons for films of this thickness has recently been obtained by Kanter.[6] In Fig. 10(a), the average fractional energy of an electron E_a/E_p emerging from thin films of C, Al, Al_2O_3, Ni, Ag, and Au has been plotted as a function of primary energy E_p expressed in units of E_c, taken from Fig. 8 of Kanter's paper. Also shown in Fig. 10(a) is a plot of E_a in units of E_c plotted vs E_p/E_c. The latter is seen to be very nearly linear for all materials with a slope of almost unity, tending towards an intercept with the abcissa close to the threshold energy E_{Th} corresponding to the "maximum range" at approximately 0.7 E_c. Thus it is seen that for all atomic numbers, the electrons which have just enough energy to penetrate a given film have essentially zero average energy (actually, less than 45 v). The average energy of any penetrating electron E_a is therefore expressed by the simple empirical relation

$$E_a = 0.92(E_p - E_{Th}), \qquad (2)$$

valid for $E_p \lesssim 10E_{Th}$.

To obtain the total energy carried through the film into the phosphor by the sum of all penetrating electrons, E_a must be multiplied by the fraction of all incident electrons transmitted, η_T. This quantity was also measured by Kanter (Fig. 4 of reference 6), and has been combined with E_a in Fig. 10(b). Since η_T as a function of (E_p/E_c) is dependent upon atomic number due to the strong effect of scattering, the curves for $(\eta_T E_a)$ in Fig. 10(b) are also strongly Z dependent, the curves for higher Z elements rising less steeply than those for the low Z materials.

With the aid of Figs. 10(a) and (b), it is now possible to understand the results obtained by Feldman. Consider first the case of a typical low atomic number element such as Al. In Fig. 11 have been plotted the quantities η_T and E_a for two films of different thickness. Also plotted in each case is the quantity $\eta_T E_a^2$, which is to a good approximation a measure of the total light output for the $Zn_2SiO_4(Mn)$ phosphor used when metal films were studied. The dashed horizontal line in Figs. 11(c) and (f) represents the combined effect of the existence of a dead-layer and finite noise in the detector, both of which require a certain minimum value of $\eta_T E_a^2$ before the onset of luminescence can be observed at a threshold energy denoted by the symbol E_{Th}^*.

Inspection of the two sets of diagrams makes it clear that the relative difference between E_{Th} and E_{Th}^* is much greater for a very thin film than it is for a heavy

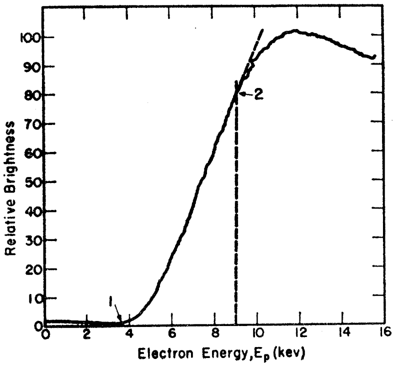

FIG. 9. Typical luminescent brightness vs electron energy recording as obtained by Feldman (Fig. 2 of reference 3).

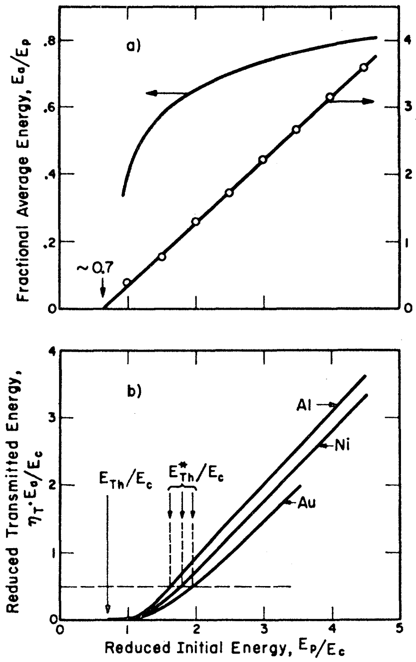

FIG. 10. (a) Fractional and reduced average energy and (b) reduced transmitted energy of electrons having traversed thin films of Al, Ni, and Au. Note the actual particle penetration threshold energy at $E_p/E_c \sim 0.7$ as compared to the "luminescence threshold energy" $E_{Th}{}^*$ at 1.5 to 2.

film. Thus, for a given known thickness of Al, penetration will appear to occur at too high an incident energy in all cases, but more so in the case of a very thin layer. But this is precisely the direction in which Feldman's data points for Al in Fig. 7 are displaced relative to the data obtained from charge-detecting techniques. The greatest deviation occurs for the thinnest films studied, thus giving rise to a range-energy curve with too great a slope or exponent n in Eq. (1).

Turning next to the Z dependence, Fig. 10(b) shows the $\eta_T E_a$ curves for three layers of different atomic number but the same value of E_p/E_c. (The curves for $\eta_T E_a{}^2$ were omitted, since they show only a somewhat larger curvature than the $\eta_T E_a$ curves.) It is seen that the curves will give rise to different "luminescence thresholds" $E_{Th}{}^*$, depending on the atomic number. In fact, it is clear that for a given thickness measured in μg/cm^2, the elements with higher Z will lead to higher values of the primary energy required to penetrate, which is exactly the trend observed by Feldman as shown for Au and Al in Fig. 7.

That the percentage difference between $E_{Th}{}^*$ and E_{Th} can be very large for thin layers is indicated by Fig. 7, particularly in the case of a high Z element where scattering is very strong. Thus, a linear extrapolation of Feldman's Au data down to 25μ g/cm^2 would predict a threshold energy of 3.8 kev, compared with a value of 1.0 kev given by absorption measurements.

Turning next to the range measurements on insulators, Feldman found a still larger slope as compared to metals, with n ranging from 2.4 for ZnS to 3.0 for Zn_2SiO_4. Since the theoretical maximum for n is always less than the classical limiting value of $n=2$ (see Fig. 6), this anomalous result must again be sought in the nature of the technique employed.

More particularly, the anomalous result may be traced to the assumption that the onset of penetration through the phosphor layer occurs at point 2 of Fig. 9, especially in the case of very thin phosphor layers. Consider for instance the physical situation existing for two layers each of the order of 500 Å thick when the true maximum range is almost 1000 Å as shown in Fig. 8. The scattering of the incident beam is illustrated schematically as discussed theoretically by Fano[16] and Sternglass[17] and observed experimentally by Ehrenberg and Franks[18] and Castaign and Descamps.[19]

In Fig. 8(b) the resulting energy-density distribution as a function of depth is indicated, which shows a pronounced peak at about $\frac{1}{4}$ to $\frac{1}{2}$ of the maximum range, depending on the atomic number. The shaded portion in Fig. 8(b) is responsible for the light output. If now the primary energy is increased, both the peak and the tail of the energy distribution curve will advance to greater depths. However, since so much less energy is contained in a unit layer near the tail than near the peak, the resultant light output will continue to increase essentially at the rate it did at a slightly lower energy. As a result, there will be no sharp break in the luminosity vs primary energy curve at the true threshold energy E_{Th}, and not until the peak has advanced substantially farther will a noticeable change in curvature occur.

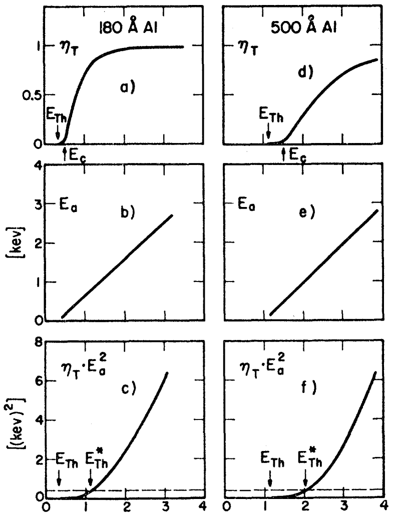

FIG. 11. The transmitted fraction η_T of electrons (a; d), their average energy E_a (b; e) and their light-output $\eta_T E_a{}^2$ (c; f) as a function of primary energy E_p for two Al films, 180 Å (left side) and 500 Å (right side) thick.

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[16] V. Fano, Phys. Rev. **70**, 44 (1946).

[17] E. J. Sternglass, Westinghouse Research Laboratories Scientific Paper No. 1772, 1954 (unpublished).

[18] W. Ehrenberg and J. Franks, Proc. Phys. Soc. (London) **66**, 1051 (1953).

[19] R. Castaign and J. Descamps, J. Phys. radium **16**, 304 (1955).

Thus, the apparent threshold as defined by point 2 of Fig. 9 will in general be too high, giving rise to a significantly high apparent penetration energy for thin phosphor layers. This, in turn, leads to an excessively steep slope for the resulting range-energy curves, just as observed by Feldman.

By contrast, the technique employed by Young[20] does not suffer from these difficulties. Young employed a thick phosphor layer and plotted light-output curves for energies well beyond the threshold for a given atomic number material. He then shifted the whole curve so as to reduce all curves to the same E_c, thus in fact utilizing the scaling law noted by Wecker[8] and Kanter.[6] In this manner he eliminated the effect of any dead-layer, and obtained excellent agreement with results using absorption techniques, both for metals[20] and insulators.[2]

VI. SUMMARY AND CONCLUSION

The above considerations show that when charge-detection or absorption techniques are used, good agreement exists among the various investigators for practical and maximum ranges of electrons in solids, all the way from 0.6 to 1000 kev. Only a weak Z dependence exists at the upper and lower ends of this energy region, so that the maximum penetration is essentially only a function of mass per unit area of the material, independent of the state of aggregation or electrical conductivity.

On the other hand, when penetration is measured by the threshold response of an energy sensitive detector such as a phosphor, an apparent Z dependence of the range may easily arise which has its origin in the scattering properties of the material. The onset of luminescence leads to an apparent threshold energy E_{Th}^* which, especially for small thichnesses, lies at higher energies than the true particle penetration threshold E_{Th} that corresponds to the range calculated from stopping theory.

The present results also indicate that the physical significance of E_{Th} is, in fact, the maximum range along the track of an electron as calculated from stopping theory. This follows from the close agreement of the point where the experimentally observed average energy of the penetrating electrons extrapolates to zero and the point where the tail of the particle absorption curve merges into the background. On the other hand, the so-called practical range corresponding to a primary energy E_c as defined by extrapolation of the linear portion of the absorption curve appears to be associated with the onset of complete diffusion. For materials with $Z \gtrsim 25$, there exists a nearly constant relation between the two energies thus defined, such that $E_c = 1.4\, E_{Th}$.

Finally, the extension of the absorption technique to energies below 10 kev reemphasizes the fact that no simple power-law for the range-energy relation can be strictly valid over the whole region from 0.6 to 1500 kev investigated so far. In agreement with the Bohr–Bethe theory, the exponent in the range-energy relation varies from as low a value as 1.1 near 1 kev to a maximum value of 1.7 near 100 kev, thus never attaining the value of 2 required by the classical Thomson–Whiddington law.

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[20] J. R. Young, Phys. Rev. **103**, 292 (1956).