High-Energy Neutrino Reactions without Production of Intermediate Bosons

T. D. LEE AND C. N. YANG
Institute for Advanced Study, Princeton, New Jersey

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General forms of cross sections for the neutrino and antineutrino reactions at high energy are discussed. Consequences of the point structure of lepton currents are investigated. Particular efforts are made to separate out the results that are implied by different assumptions concerning weak interactions such as time-reversal invariance, conserved vector-current hypothesis, and $|\Delta\mathbf{I}|=1$ rule.

I. INTRODUCTION

IN the present theory of weak interactions it is assumed that all weak reactions which contain leptons and other strongly interacting particles can be described by an effective Lagrangian of the form

$$-\mathcal{L}_{\text{eff}}=\sum_{\lambda=1}^{4}\left[J_\lambda(x)j_\lambda(x)+J_\lambda{}^\star(x)j_\lambda{}^\star(x)\right], \tag{1}$$

where

$$j_\lambda(x)=-i[\psi_l{}^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_\nu], \tag{2}$$

$$j_\lambda{}^\star(x)=-i[\psi_\nu{}^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_l], \tag{3}$$

l stands for either e or μ, ψ_ν and ψ_l are the field operators for ν and l^-, respectively. J_λ and $J_\lambda{}^\star$ are current operators that act *only* on the strongly interacting particles. Because of the Hermiticity of $\mathcal{L}_{\text{eff}}$, we have

$$J_\lambda{}^\star=\eta_\lambda J_\lambda{}^\dagger,\quad (\dagger=\text{Hermitian conjugate}), \tag{4}$$

where

$$\eta_\lambda=+1\quad\text{for}\quad\lambda=1,\,2,\,3$$

and

$$\eta_\lambda=-1\quad\text{for}\quad\lambda=4. \tag{5}$$

The nature of J_λ and $J_\lambda{}^\star$ are known so far only in a few cases. In the relatively high-energy range (momentum transfer$\sim$a few hundred Mev) some limited information of the matrix elements of these current operators has been obtained from π decay and the leptonic modes of K decay and hyperon decay. The most extensively studied case is β decay in which only momentum transfer of the order of a few Mev is involved. In such a low-energy limit, the matrix elements of J_λ and $J_\lambda{}^\star$ are given by

$$\langle p|J_\lambda|n\rangle=(i/\sqrt{2})u_p{}^\dagger\gamma_4\gamma_\lambda(G_V-G_A\gamma_5)u_n \tag{6}$$

and

$$\langle n|J_\lambda{}^\star|p\rangle=(i/\sqrt{2})u_n{}^\dagger\gamma_4\gamma_\lambda(G_V{}^\star-G_A{}^\star\gamma_5)u_p, \tag{7}$$

where the symbol $\star$ on a c number means complex conjugation, u_n and u_p are the spinor solutions of the free Dirac equation with the same four momenta as the physical neutron and proton; G_V and G_A are the Fermi and Gamow-Teller coupling constants. Because of the presence of strong interactions and the possibility that the weak interactions may be transmitted through an intermediate Boson,[1] it is expected that in the high-energy range special forms like (6) and (7) do not hold and, in general, the behavior of the heavy-particle currents would be quite complicated.

On the other hand, one expects the lepton currents $j_\lambda(x)$ and $j_\lambda{}^\star(x)$, which interact only at a single space-time point in (1), to have a wider range of applicability.[2] The recent possibility of doing high-energy neutrino experiments[3] makes it, perhaps, feasible to establish the validity of this particular form of leptonic currents to the Bev range. It has already been pointed out[4,5] that the assumption of such a point structure of the lepton current introduces strong restrictions on the general forms of the cross sections for all neutrino and antineutrino reactions. In particular, the rates for reactions with different neutrino momentum $\mathbf{k}_\nu$ and different final lepton momentum $\mathbf{k}_l$ are mutually related, provided that in these reactions the energy transfer $E-m$ and the magnitude of momentum transfer P between the leptons and the strongly interacting particles are the same. (The only complicated and unknown part of the Lagrangian is the current J. For two reactions with the same $E-m$ and P, the matrix elements of J are related. Hence, the rates of these two reactions are related.) Some of these relationships have been stated in references 4 and 5. The purpose of this paper is partly to supply the mathematical details of these relationships and partly to give a more systematic discussion of the various theoretical implications of the high-energy neutrino experiments. Efforts are made to separate and dissociate the consequences that are implied by different assumptions concerning weak interactions such as time reversal invariance, the possible existence of a $|\Delta\mathbf{I}|=1$ rule, the conserved vector current hypothesis, etc. For the sake of clarity, the results are stated in the forms of several theorems.

In view of the present technological difficulties in performing the high-energy neutrino or antineutrino

[1] If the weak interactions are transmitted through an intermediate boson W, we choose the heavy particle current operators J_λ and $J_\lambda{}'$ to include the propagator of W.

[2] At the moderately high energy, the validity of the point structure hypothesis for the lepton currents can be tested by analyzing the leptonic decay modes of K decay. See A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957).

[3] M. Schwartz, Phys. Rev. Letters **4**, 306 (1960); B. Pontecorvo, Soviet Phys.—JETP, **37**, 1751 (1959). For further references see *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960). We are informed that such experiments are in progress at the Brookhaven National Laboratory and at CERN.

[4] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960).

[5] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

experiments, it seems also useful to assume (at least for the immediate future) the validity of the point structure of lepton currents, time-reversal invariance, and the conserved vector-current hypothesis, and to utilize the experimental results to measure the unknown matrix elements of J_λ such as the axial-vector form factor. Specific discussions on this aspect are given in Sec. III 4.

Throughout this paper we only consider the weak interaction to the lowest order.

II. NOTATIONS

In the subsequent sections, we will consider reactions of the general type

$$\nu(\text{or } \bar{\nu})+\text{nucleon (or nucleus)} \rightarrow l^{\mp}+\cdots,$$

but without the production of any intermediate bosons. The $l^{\mp}$ can be either $e^{\mp}$ or $\mu^{\mp}$ depending on the nature of the incoming ν (or $\bar{\nu}$). The following basic notations will be used throughout the paper: (All momenta and energies are in the laboratory system.)

$$\hbar=c=1;$$

$\mathbf{k}_\nu$, $\mathbf{k}$ = momenta of ν (or $\bar{\nu}$) and $l^{\mp}$, respectively;

$$\mathbf{P}=(\mathbf{k}_\nu-\mathbf{k}_l);$$

$$k_\nu,\ k_l,\ P=|\mathbf{k}_\nu|,\ |\mathbf{k}_l|,\ |\mathbf{P}|,\text{ respectively;}$$

z and x axes are parallel to $\mathbf{P}$ and $(\mathbf{k}_\nu\times\mathbf{P})$, respectively;

$$\theta=\text{angle between } \mathbf{k}_\nu \text{ and } \mathbf{k}_l;$$
$$\phi=\text{angle between } \mathbf{k}_\nu \text{ and } \mathbf{P};$$

m_l, m = masses of l and target nucleon (or target nucleus), respectively;

$$E_l=(m_l^2+k_l^2)^{\frac{1}{2}};$$
$$E=(k_\nu+m-E_l);$$
$$v_l=\text{velocity of } l^{\mp}=(k_l/E_l);$$
$$M=(E^2-P^2)^{\frac{1}{2}};$$

$q^2=(\text{4-momentum transfer})^2=P^2-(E-m)^2$;

$\mathcal{J}$ = spin of the target nucleus (which is always unpolarized).

It is found useful to introduce the following three functions of k_ν, k_l, and P:

$$x=(k_\nu+k_l+P)^{-1}(k_\nu+k_l-P);$$
$$y=(k_\nu-k_l+P)^{-1}(-k_\nu+k_l+P);$$

and

$$\Delta=(4\pi k_\nu q^2)^{-1}k_l[-(k_\nu-k_l)^2+P^2][(k_\nu+k_l)^2-P^2].$$

III. NEUTRINO REACTIONS WITHOUT MESON PRODUCTION AND $v_l=1$

1. Cross Sections

We consider first the simple reactions,

$$\nu+n\rightarrow p+l^- \tag{8}$$

and

$$\bar{\nu}+p\rightarrow n+l^+, \tag{9}$$

in which the velocity v_l of the lepton can be regarded approximately as 1. In this special case, we have

$$E=(m^2+P^2)^{\frac{1}{2}}, \tag{10}$$

$$(k_\nu-k_l)=(E-m); \tag{11}$$

$$q^2=2m(E-m), \tag{12}$$

or

$$q^2=2k_\nu k_l(1-\cos\theta).$$

The differential cross sections for reactions (8) and (9) depend only on two independent variables which may be chosen to be k_ν and q^2. The following theorem shows that if the Lagrangian has the "point structure" property for the lepton current as given by (1)–(3), then independently of the detailed form of J_λ, the differential cross sections for both reactions (8) and (9) (after summation over the spin directions of the initial and final nucleons) can be expressed in terms of three real functions which depend only on *one* variable q^2.

Theorem 1. If $v_l=1$, the differential cross sections for (8) and (9) can be expressed in the form:

$$d\sigma_\nu=(8\pi k_\nu^2)^{-1}[(k_\nu+k_l)^2-P^2]\times[xa_++x^{-1}a_-+b]d(q^2) \tag{13}$$

and

$$d\sigma_{\bar{\nu}}=(8\pi k_\nu^2)^{-1}[(k_\nu+k_l)^2-P^2]\times[xa_-+x^{-1}a_++b]d(q^2), \tag{14}$$

where

$$x=(k_\nu+k_l+P)^{-1}(k_\nu+k_l-P), \tag{15}$$

and $a_\pm$, b are real positive functions (called structure functions) of q^2 only.

Proof. To prove (13) we note first that in the special case $v_l=1$, there is only a single spin state for the lepton. Furthermore, the corresponding matrix elements $\langle j_\lambda\rangle$ satisfy

$$(k_\nu-k_l)_\lambda\langle j_\lambda\rangle=0, \tag{16}$$

which means

$$\langle j_4\rangle=i(E-m)^{-1}P\langle j_z\rangle. \tag{17}$$

Using (2) and the coordinate system chosen in Sec. II, the other spatial components of $\langle\mathbf{j}\rangle$ are found to be

$$\langle j_x,j_y,j_z\rangle=-2[i\sin\tfrac{1}{2}\theta,\ \sin(\phi+\tfrac{1}{2}\theta),\ \cos(\phi+\tfrac{1}{2}\theta)]. \tag{18}$$

Next, we analyze the initial and final nucleons in terms of definite spin states s along the $\mathbf{P}$ direction (in the laboratory system):

$$s=\uparrow(\text{i.e., } \tfrac{1}{2}) \quad\text{or}\quad \downarrow(\text{i.e., } -\tfrac{1}{2}).$$

For the no-spin-flip case $n_\uparrow\rightarrow p_\uparrow$ or $n_\downarrow\rightarrow p_\downarrow$, the matrix elements of J_x and J_y are zero. By using (17), we observe that only the matrix elements of $[(E-m)J_z+iPJ_4]$ contributes to the cross section. For the spin-flip cases $n_\downarrow\rightarrow p_\uparrow$ and $n_\uparrow\rightarrow p_\downarrow$, the only matrix elements that contribute are that of (J_x+iJ_y) and (J_x-iJ_y), respectively. Summing over all possible nucleon spin states, we obtain expression (13) for reaction (8). The functions $a_\pm$ and b are related to the matrix elements of J_λ by

$$a_+=(E+m)^{-1}(2E)|\langle p_\uparrow,\mathbf{P}|J_x|n_\downarrow,0\rangle|^2,$$
$$a_-=(E+m)^{-1}(2E)|\langle p_\downarrow,\mathbf{P}|J_x|n_\uparrow,0\rangle|^2,$$

and

$$b=(mP^2)^{-1}E\sum_{s=\uparrow,\downarrow}|\langle p_s,\mathbf{P}|(E-m)J_z+iPJ_4|n_s,0\rangle|^2,\quad(19)$$

where the states $|n_s,0\rangle$ and $|p_s,\mathbf{P}\rangle$ refer to that of a nucleon with spin s and momenta 0 and $\mathbf{P}$, respectively. These are all functions of q^2 only since P and E depend only on q^2.

To prove (14), we notice that the matrix elements of $j_\lambda{}^\star$ are related to that of j_λ by the operation of CR_x (where C=charge conjugation and R_x=reflection with respect to the y-z plane), which changes ν, l^- (left-handed) to $\bar{\nu}$, l^+ (right-handed) but leaves their linear momenta unchanged. Therefore,

$$\langle j_x{}^\star\rangle=+\langle j_x\rangle$$

and

$$\langle j_\lambda{}^\star\rangle=-\langle j_\lambda\rangle\quad\text{for}\quad\lambda\neq x.\quad(20)$$

Applying our previous discussions to reaction (9), the following expression for the differential cross section $d\sigma_{\bar{\nu}}$ can be readily derived:

$$d\sigma_{\bar{\nu}}=(8\pi k_\nu{}^2)^{-1}[(k_\nu+k_l)^2-P^2]\times[xa_-'+x^{-1}a_+'+b']d(q^2),\quad(21)$$

where

$$a_+'=(E+m)^{-1}(2E)|\langle p_\uparrow,\mathbf{P}|J_x{}^\star|p_\downarrow,0\rangle|^2,$$
$$a_-'=(E+m)^{-1}(2E)|\langle n_\downarrow,\mathbf{P}|J_x{}^\star|p_\uparrow,0\rangle|^2,$$

and

$$b'=(mP^2)^{-1}E\sum_{s=\uparrow,\downarrow}|\langle n_s,\mathbf{P}|(E-m)J_z{}^\star+iPJ_4{}^\star|p_s,0\rangle|^2.\quad(22)$$

It is important to observe that because of (20) the relative rates for the two spin-flip cases $\downarrow\to\uparrow$ and $\uparrow\to\downarrow$ are now given, respectively, by $x^{-1}a_+'$ and xa_-'; while for ν reactions they are xa_+ and $x^{-1}a_-$, respectively.

We then use the Hermiticity condition, Eq. (4), which relates, e.g.,

$$\langle n_\uparrow,\mathbf{P}|J_x{}^\star|p_\downarrow,0\rangle=\langle p_\downarrow,0|J_x|n_\uparrow,\mathbf{P}\rangle^\star,\quad(23)$$

where the $\star$ on the right-hand side means simply the complex conjugation. A 180° rotation around the x axis changes the matrix element $|\langle p_\downarrow,0|J_x|n_\uparrow,\mathbf{P}\rangle|^2$ into $|\langle p_\uparrow,0|J_x|n_\downarrow,-\mathbf{P}\rangle|^2$, which can, in turn, be shown to be identical with $|\langle p_\uparrow,\mathbf{P}|J_x|n_\downarrow,0\rangle|^2$ by a subsequent simple Lorentz transformation that leaves x and y axes unchanged but transforms $|n_s,-\mathbf{P}\rangle$ and $|p_s,0\rangle$ into $|n_s,0\rangle$ and $|p_s,\mathbf{P}\rangle$, respectively. Therefore, we obtain

$$a_+=a_+'.\quad(24)$$

In an identical way, it can be shown that

$$a_-=a_-',$$

and

$$b=b'.\quad(25)$$

Theorem 1 is, then, proved.

Remarks.

(i) By using (6), we obtain at $q^2=0$,

$$a_+(q^2=0)=a_-(q^2=0)=\tfrac{1}{2}|G_A|^2,\quad b(q^2=0)=|G_V|^2.\quad(26)$$

(ii) The validity of theorem 1 depends only on the special form, Eqs. (1)–(3), of the effective Lagrangian and is independent of any assumptions about J_λ such as time reversal invariance, $|\Delta\mathbf{I}|=1$ rule, etc.

(iii) It is useful to express J_λ as a sum of a vector part V_λ and and axial-vector part A_λ,

$$J_\lambda=V_\lambda+A_\lambda.\quad(27)$$

Under a reflection with respect to the (y-z) plane, a_+ [given in (19)] becomes

$$a_+=(E+m)^{-1}(2E)|\langle p_\downarrow,\mathbf{P}|V_x-A_x|n_\uparrow,0\rangle|^2.$$

Therefore, the difference between a_+ and a_- (consequently, also the difference between $d\sigma_\nu$ and $d\sigma_{\bar{\nu}}$) depends only on the interference term between V_λ and A_λ.

2. An Alternative Form

It is also possible to derive theorem 1 by working directly with the general covariant forms of the matrix elements of J_λ and $J_\lambda{}^\star$:

$$\langle p|J_\lambda|n\rangle=(i/\sqrt{2})u_p{}^\dagger\gamma_4[\gamma_\lambda(g_V+g_A\gamma_5)+i(n_\lambda+p_\lambda)(f_V+f_A\gamma_5)+i(n_\lambda-p_\lambda)(h_V+h_A\gamma_5)]u_n\quad(28)$$

and

$$\langle n|J_\lambda{}^\star|p\rangle=(i/\sqrt{2})u_n{}^\dagger\gamma_4[\gamma_\lambda(g_V{}^\star+g_A{}^\star\gamma_5)+i(p_\lambda+n_\lambda)(f_V{}^\star-f_A{}^\star\gamma_5)+i(p_\lambda-n_\lambda)(-h_V{}^\star+h_A{}^\star\gamma_5)]u_p,\quad(29)$$

where n_λ and p_λ are, respectively, the four-momenta of the states n and p; g_V, g_A, $\cdots h_A$ are complex functions which depend only on q^2. The Fermi and Gamow-Teller constants are related to these functions at $q^2=0$:

$$g_V(q^2=0)-2mf_V(q^2=0)=G_V\cong10^{-5}m^{-2}$$

and

$$g_A(q^2=0)=-G_A\cong1.2G_V.\quad(30)$$

If $v_l=1$, then because of (16), h_V and h_A do not contribute to either reaction (8) or reaction (9). The relevant matrix elements depend only on *four* (complex) functions g_V, g_A, f_V, and f_A. In terms of these four functions, the previously obtained three (real) structure functions $a_\pm$, b become

$$a_\pm=\tfrac{1}{2}(E+m)^{-2}|g_A(E+m)\mp g_VP|^2$$

and

$$b=(E+m)^{-1}(2m)[|g_V-f_V(E+m)|^2+|f_AP|^2]. \quad (31)$$

Remarks.

(i) If time reversal invariance holds, then g_V, g_A, f_V, f_A are all real functions.

(ii) It has been proposed[5] that for those weak reactions that conserve the strangeness quantum number S among the strongly interacting particles (i.e., $\Delta S=0$), the corresponding change of isotopic spin obeys the $|\Delta\mathbf{I}|=1$ rule[6] which implies that the heavy-particle currents J_λ and $J_\lambda^\star$ are components of a single spin vector[7]:

$$J_\lambda^\star=-[\exp(i\pi I_y)]J_\lambda[\exp(-i\pi I_y)], \quad (32)$$

where $\exp(i\pi I_y)$ is the 180° rotation along the y axis in the isotopic spin space.

If the $|\Delta\mathbf{I}|=1$ rule holds, then g_V, g_A, f_V are real but f_A is pure imaginary.

(iii) If both time reversal invariance and $|\Delta\mathbf{I}|=1$ rule hold, then

$$f_A=0, \quad (33)$$

and g_V, g_A, f_V are real. In this case, measurements on cross sections $d\sigma_\nu$ and $d\sigma_{\bar\nu}$ are sufficient to determine g_V, g_A, and f_V:

$$g_A=(1/\sqrt{2})[(a_-)^{\frac{1}{2}}+(a_+)^{\frac{1}{2}}], \quad (34)$$

$$(E+m)^{-1}Pg_V=(1/\sqrt{2})[(a_-)^{\frac{1}{2}}-(a_+)^{\frac{1}{2}}] \quad (35)$$

and

$$g_V-(E+m)f_V=[\tfrac{1}{2}m^{-1}b(E+m)]^{\frac{1}{2}}. \quad (36)$$

(iv) If either time reversal invariance or $|\Delta\mathbf{I}|=1$ rule holds but not both, then (34) and (35) are still correct. Measurements on $d\sigma_\nu$ and $d\sigma_{\bar\nu}$ determines only g_V and g_A. To obtain f_V and f_A, polarization measurements become important. (See Sec. III 5.)

[6] The $|\Delta\mathbf{I}|=1$ rule follows if the schizon scheme of reference 5 is correct. However, the validity of $|\Delta\mathbf{I}|=1$ rule has a much wider basis and should be discussed independently of the existence of intermediate bosons. For example, $|\Delta\mathbf{I}|=1$ rule also follows if one assumes $J_\lambda=(iG_V/\sqrt{2})\psi_p^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_n$, where ψ_n and ψ_p are the (unrenormalized) operators for the nucleons.

[7] To be specific, the $|\Delta\mathbf{I}|=1$ rule is defined as follows: For the case $\Delta S=0$, the heavy particle current operators J_λ and $-J_\lambda^\star$ transform under an isotopic spin rotation like the $|I=1, I_z=1\rangle$ and $|I=1, I_z=-1\rangle$ components of an $I=1$ multiplet, where we use the notation of A. Edmond, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). Notice that in this definition not only must J_λ and $J_{-\lambda}^\star$ each individually transform according to $I=1$, but they must belong to the *same* multiplet. In this sense the $|\Delta\mathbf{I}|=1$ rule is stronger than the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule.

3. Simple Consequences of Theorem 1

In this section, we list some simple consequences of theorem 1.

(i) In the forward direction, $\theta=0$ (consequently, $q^2=0$). For arbitrary k_ν,

$$d\sigma_\nu=d\sigma_{\bar\nu}=(2\pi)^{-1}[|G_V|^2+|G_A|^2]d(q^2). \quad (37)$$

(ii) At arbitrary angle θ and arbitrary k_ν,

$$\left|\frac{d\sigma_\nu-d\sigma_{\bar\nu}}{d\sigma_\nu+d\sigma_{\bar\nu}}\right|\leq[(k_\nu+k_l)^2+P^2]^{-1}[2P(k_\nu+k_l)]. \quad (38)$$

(iii) In the low-energy limit $k_\nu\to0$, the maximum value of $q^2\sim4k_\nu^2\to0$. The cross sections $d\sigma_\nu$ and $d\sigma_{\bar\nu}$ become (to the lowest order in k_ν)

$$d\sigma_\nu=d\sigma_{\bar\nu}=(2\pi)^{-1}k_\nu^2[|G_V|^2(1+\cos\theta)+|G_A|^2(3-\cos\theta)]d(\cos\theta). \quad (39)$$

(iv) In the very high energy region,

$$\lim_{k_\nu\to\infty}d\sigma_\nu=\lim_{k_\nu\to\infty}d\sigma_{\bar\nu}=(2\pi)^{-1}(a_++a_-+b)dq^2. \quad (40)$$

(v) If the integral $\int_0^\infty(a_++a_-+b)dq^2$ exists, then in the very high energy region,

$$\lim_{k_\nu\to\infty}\int d\sigma_\nu=\lim_{k_\nu\to\infty}\int d\sigma_{\bar\nu}=(2\pi)^{-1}\int_0^\infty(a_++a_-+b)dq^2. \quad (41)$$

4. Measurement of Form Factors

To measure the form factors it seems desirable, at least for the present, to assume time reversal invariance and the $|\Delta\mathbf{I}|=1$ rule so as not to have too many unknown functions. Under these assumptions, $f_A=0$ as shown in III 2, and substitution of (31) into (13) and (14) gives

$$\begin{aligned}d\sigma_{\nu(\bar\nu)}=(4\pi)^{-1}(dq^2)\{&(2k_\nu^2)^{-1}q^2(g_A^2-g_V^2)+(g_A\pm g_V)^2\\&+[1-(2mk_\nu)^{-1}q^2]^2(g_A\mp g_V)^2\\&+[2-(2mk_\nu^2)^{-1}q^2(m+2k_\nu)]\\&\times[(4m^2+q^2)f_V^2-4mf_Vg_V]\}, \quad (42)\end{aligned}$$

where the upper and lower signs refer to ν-reaction (8) and $\bar\nu$-reaction (9), respectively. For sometime to come, because of lack of detailed experience data, these equations would undoubtedly be still insufficiently restrictive to determine the three form factors g_V, f_V, and g_A. It is therefore desirable to introduce further assumptions.

It has been proposed by Feynman and Gell-Mann[8] that the vector part of J_λ satisfies a conservation law and that it is proportional to the corresponding isotopic vector part of the electromagnetic current. Let $F_Q(q^2)$ and $F_M(q^2)$ be, respectively, the isotopic

[8] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

vector part s of the charge and magnetic form factors for the nucleon. We have[9]

$$g_V = G_V[F_Q + (\mu_p - \mu_n)F_M]$$

and

$$f_V = G_V(2m)^{-1}(\mu_p - \mu_n)F_M, \tag{43}$$

where F_Q and F_M are normalized to unity at $q^2=0$; $\mu_p \cong 1.79$ and $\mu_n \cong -1.90$ are, respectively, the anomalous part of the magnetic moments of proton and neutron in units of the nuclear Bohr magneton. The form factors F_Q and F_M have been measured[10] extensively by the electron scattering experiments.

Using the proportional current hypothesis the only unknown form factor in (42) is g_A. It can be measured by measuring either $d\sigma_\nu$ or $d\sigma_{\bar\nu}$. in particular, the difference,

$$(d\sigma_\nu - d\sigma_{\bar\nu}) = (\pi m k_\nu)^{-1} q^2[1-(4mk_\nu)^{-1}q^2] g_V g_A d(q^2), \tag{44}$$

gives a sensitive determination of g_A. [See (97) and (99) for the modification of above equations if $v_l \neq 1$.]

Consider an experiment with a neutrino (or antineutrino) beam whose spectrum is $I(k_\nu)dk_\nu$. Let $N(q^2)dq^2$ be the number of events with a (four-momentum transfer)^2 between q^2 and $q^2+d(q^2)$. It is useful to define the moments I_n of the incoming spectrum:

$$I_n = I_n(q^2) \equiv \int (4\pi)^{-1}(k_\nu)^{-n} I(k_\nu) dk_\nu,$$

where the integration extends from $[m_l + (2m)^{-1}q^2]$ to ∞ and $n=0, 1, 2$. The number of events for reactions (8) and (9) at a given q^2 can be obtained from (42) by integrating over the incoming spectrum:

$$\begin{aligned}[N(q^2)]_{\nu(\bar\nu)} = {} & \tfrac{1}{2}I_2 q^2(g_A{}^2 - g_V{}^2) + I_0(g_A \pm g_V)^2 \\ & + [I_0 - I_1(q^2/m) + \tfrac{1}{4}I_2(q^2/m)^2](g_A \mp g_V)^2 \\ & + [2I_0 - I_1(q^2/m) - \tfrac{1}{2}I_2 q^2] \\ & \qquad \times [(4m^2+q^2)f_V{}^2 - 4mf_Vg_V], \end{aligned} \tag{45}$$

where the upper and lower signs are for ν and $\bar\nu$ reactions, respectively. By using (43) and (45), g_A can be directly deduced from the experimental results.

5. Longitudinal Polarization

In this section we discuss briefly the longitudinal polarization of the final nucleon for reactions (8) and (9), assuming that the initial nucleon is unpolarized. By using exactly the same arguments as that in Sec. III 1 it can be shown readily that the average helicities $\langle s\rangle$ of the final p and the final n at a given four-momentum transfer q^2 are given, respectively, by

$$2\langle s\rangle_p = (xa_+ + x^{-1}a_- + b)^{-1}(xa_+ - x^{-1}a_- + d)$$

and

$$2\langle s\rangle_n = (xa_- + x^{-1}a_+ + b)^{-1}(x^{-1}a_+ - xa_- - d), \tag{46}$$

where $\langle s\rangle$ is defined to be the z-component spin (z axis parallel to $\mathbf{P}$) averaged over $s=\frac{1}{2}$ and $-\frac{1}{2}$; a_+, a_-, b are given by (19) and d is given by

$$\begin{aligned} d = (mP^2)^{-1}E[\,&|\langle p_\uparrow, \mathbf{P}|(E-m)J_z + iPJ_4|n_\uparrow, 0\rangle|^2 \\ &- |\langle p_\downarrow, \mathbf{P}|(E-m)J_z + iPJ_4|n_\downarrow, 0\rangle|^2].\end{aligned}$$

It is clear that d is a function of q^2 only. In terms of g_V, f_V, etc., d can be written as

$$d = (E+m)^{-1}(2mP)\{[-g_V + f_V(E+m)]^\star f_A + \text{c.c.}\}.$$

Remarks.

(i) In the forward direction $\theta=0$,

$$\langle s\rangle_p = \langle s\rangle_n = 0.$$

(ii) In the low-energy limit ($q^2 \to 0$),

$$\langle s\rangle_p = \langle s\rangle_n = 0.$$

(iii) In the very high energy limit ($k_\nu \to \infty$),

$$\langle s\rangle_p = -\langle s\rangle_n = (a_+ + a_- + b)^{-1}(a_+ - a_- + d).$$

(iv) If $|\Delta\mathbf{I}|=1$ rule holds (independently of whether time reversal invariance is true or not),

$$d=0.$$

IV. NEUTRINO REACTIONS WITH MESON PRODUCTIONS AND $v_l = 1$

1. Cross Sections

In Sec. IV, we consider general reactions of the type

$$\nu + N \to \Gamma + l^- \tag{47}$$

and

$$\bar\nu + N' \to \Gamma' + l^+, \tag{48}$$

but retaining the approximation $v_l = 1$, where N (or N') represents an unpolarized nucleus and Γ (or Γ') is an arbitrary complex consisting of any number of strongly interacting particles. The strangeness quantum number of Γ (or Γ') may be non-zero. For simplicity, we shall consider in this paper only those cases in which Γ (or Γ') is a *noncoherent* mixture of states with different helicities, where the helicity of a particle or a complex of particles is defined to be the quantum number of its total angular momentum component along the direction of its total momentum.

In this more general case the momentum $\mathbf{P}$ and the energy E of the complex Γ (or Γ') are independent variables. It is convenient to call M, defined in Sec. II,

[9] If the weak interactions are transmitted through an intermediate Boson W than (43) should be changed into

$$g_V = G_V[F_Q + (\mu_p - \mu_n)F_M][1 + m_W{}^{-2}q^2]$$

and

$$f_V = G_V\left(\frac{\mu_p - \mu_n}{2m}\right)F_M[1 + m_W{}^{-2}q^2],$$

where m_W is the mass of W.

[10] See, e.g., the review article by R. Hofstadter, S. Bumiller and M. R. Yearian, Rev. Mod. Phys. **30**, 482 (1958). For some of the more recent measurements, see D. N. Olson, H. F. Schopper and R. R. Wilson, Phys. Rev. Letters **6**, 286 (1961); R. Hofstadter, C. de Vries and R. Herman, Phys. Rev. Letters **6**, 290 (1961); R. Hofstadter and R. Herman, Phys. Rev. Letters **6**, 293 (1961).

the effective mass of Γ (or Γ'). Apart from the intrinsic characteristics of Γ and Γ', such as the nature and the number of different particles involved, their isotopic spin, etc., there are *three independent variables*, say, k_ν, k_l, and θ. (Or, we may choose k_ν, q^2, and M as the three independent variables.)

If the "point structure" of the lepton current holds, then for any arbitrary given Γ or Γ' the cross sections $d\sigma_\nu(\Gamma)$, or $d\sigma_{\bar\nu}(\Gamma')$, can be expressed as sums of three terms each of which has an unknown dependence only on *two* variables which may be chosen to be q^2 and M.

Theorem 2. If $v_l=1$, the differential cross sections for (47) and (48) can be written as[4]

$$d\sigma_\nu(\Gamma)=(4\pi k_\nu)^{-1}k_l[(k_\nu+k_l)^2-P^2]\times[xA_+ + x^{-1}A_- + B]dk_l d(\cos\theta) \quad (49)$$

and

$$d\sigma_{\bar\nu}(\Gamma')=(4\pi k_\nu)^{-1}k_l[(k_\nu+k_l)^2-P^2]\times[xA_-' + x^{-1}A_+' + B']dk_l d(\cos\theta) \quad (50)$$

where x is defined in (15), $A_\pm$, B (or $A_\pm'$, B') are positive real functions that depend on only q^2, M, and other intrinsic characteristics of the complex Γ (or Γ').

Proof. We follow the proof of theorem 1 and resolve Γ (or Γ') and N (or N') into states with definite components of angular momentum along **P** direction. These angular momentum components are denoted by s_Γ and s_N respectively. For any given s_Γ, reactions (47) and (48) are then analyzed into noncoherent transitions in which $s_N=s_\Gamma\pm1$ and $s_N=s_\Gamma$. For these transitions only the matrix elements $\langle s_\Gamma|J_x\mp iJ_y|s_N=s_\Gamma\pm1\rangle$ and $\langle s_\Gamma|(E-m)J_z+iPJ_4|s_N=s_\Gamma\rangle$ contribute.

To obtain the density of final states one first considers a single definite state of Γ (or Γ') with an effective mass M. The Jacobian for the transformation from the variables M, q^2 to k_l and $\cos\theta$ is easily computed from

$$k_\nu-k_l=E-m,\quad q^2=-M^2-m^2+2mE,$$

and

$$q^2=2k_\nu k_l(1-\cos\theta).$$

One obtains

$$2k_\nu k_l m dk_l d\cos\theta=Md(q^2)dM. \quad (51)$$

One introduces the density of states $\rho(M)$ so that $\rho(M)dM$ = number of states Γ (or Γ') having an effective mass between M and $M+dM$. Theorem 2 can then be readily proved by summing over different spin configurations and using the explicit form of j_λ and $j_\lambda^\star$ given by (17), (18), and (20). The functions $A_\pm$, B and $A_\pm'$, B' in (49) and (50) are given by

$$A_\pm=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M)\times|\langle s_\Gamma|J_x|s_N=s_\Gamma\mp1\rangle|^2,$$

$$B=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2E)\rho(M)\times|\langle s_\Gamma|(E-m)J_z+iPJ_4|s_N=s_\Gamma\rangle|^2, \quad (52)$$

$$A_\pm'=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M)\times|\langle s_\Gamma|J_x^\star|s_N=s_\Gamma\mp1\rangle|^2,$$

and

$$B'=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2E)\rho(M)\times|\langle s_\Gamma|(E-m)J_z^\star+iPJ_4^\star|s_N=s_\Gamma\rangle|^2, \quad (53)$$

where the sum $\sum$ extends over the appropriate mixture s_Γ for the final state. For example, if Γ is polarized with a definite helicity then there is only one term in the sum. It is important to note that these structure functions occur linearly in the cross sections. Thus, theorem 2 can be applied to any sum of final states Γ or Γ'.

Remarks.

(i) It is easy to see that if Γ (or Γ') consists of only a single nucleon, then $\rho(M)=\delta(M-m)$ and (52), (53) reduce to (19) and (22), respectively.

(ii) It has been discussed before[4] that the validity of theorem 2 can be tested by performing a capture experiment with a neutrino (or antineutrino) beam of a known spectrum $I(k_\nu)dk_\nu$ and measuring for each event the values of x, P and E. If $N(x,P,E)dxdPdE$ is the number of events, then theorem 2 implies that

$$\int I^{-1}k_\nu^2 NdPdE=(a_1+a_2x+a_3x^2)(1-x)^{-4},$$

where the integration extends over all events with fixed x, and a_1, a_2, a_3 are numerical constants.

2. $|\Delta\mathbf{I}|=1$ Rule

That the ν-reaction (47) is completely unrelated to the $\bar\nu$-reaction (48) in the case of $\Delta S\neq0$ is obvious. If $\Delta S=0$, the rate for a neutrino reaction with meson production can be related to that of antineutrino reaction if the $|\Delta\mathbf{I}|=1$ rule holds. This is to be contrasted with theorem 1 which relates reactions (8) and (9) through Hermiticity (independent of $\Delta\mathbf{I}=1$ rule).

Theorem 3. If $|\Delta\mathbf{I}|=1$ rule holds, then in (49) and (50)

$$A_\pm=A_\pm' \quad\text{and}\quad B=B', \quad (54)$$

provided that N' and Γ' are, respectively, the isotopic symmetrical states of N and Γ; i.e.,

$$|N'\rangle=\exp(i\pi I_y)|N\rangle$$

and

$$|\Gamma'\rangle=\exp(i\pi I_y)|\Gamma\rangle. \quad (55)$$

Theorem 3 follows directly by applying (32) to (52) and (53). The validity of theorems 2 and 3 is independent of time reversal invariance. The following equalities and inequalities are some immediate consequences of the $|\Delta\mathbf{I}|=1$ rule and theorems 2 and 3:

(i) At $\theta=0$, $q^2=0$, hence, $A_\pm=0$. Therefore,

$$d\sigma_\nu(\Gamma)=d\sigma_{\bar\nu}(\Gamma'). \quad (56)$$

(ii) At arbitrary θ, k_l, and k_ν we have

$$\left|\frac{d\sigma_\nu(\Gamma)-d\sigma_{\bar\nu}(\Gamma')}{d\sigma_\nu(\Gamma)+d\sigma_{\bar\nu}(\Gamma')}\right| \leq [P^2+(k_\nu+k_l)^2]^{-1}[2P(k_\nu+k_l)]. \quad (57)$$

(iii) In the limit in which $k_\nu\to\infty$, but q^2, M (and also other characteristics of Γ and Γ') are kept fixed:

$$\lim_{k_\nu\to\infty} d\sigma_\nu(\Gamma)=\lim_{k_\nu\to\infty} d\sigma_{\bar\nu}(\Gamma') =(2\pi m)^{-1}[A_+ + A_- + B]MdMd(q^2). \quad (58)$$

V. GENERAL CASE ($v_l\neq 1$)

1. Cross Sections

All the previous theorems can be easily generalized without the approximation $v_l=1$. If $v_l\neq 1$, the helicity of l becomes no longer definite. Let $d\sigma_\nu(\Gamma,l_L^-)$ and $d\sigma_\nu(\Gamma,l_R^-)$ be the differential cross sections for reaction (47) in which the helicity of l^- is $-\frac{1}{2}$ and $+\frac{1}{2}$, respectively. Similarly, let $d\sigma_{\bar\nu}(\Gamma,l_R^+)$ and $d\sigma_{\bar\nu}(\Gamma,l_L^+)$ be that for reaction (48) in which the helicity of l^+ is $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. We now state the generalization of theorem 2:

Theorem 4.

$$d\sigma_\nu(\Gamma,l_L^-)=dk_ld(\cos\theta)\tfrac{1}{2}(1+v_l)\Delta \times[xA_+ + x^{-1}A_- + yB_+ + y^{-1}B_- + C] \quad (59)$$

$$d\sigma_\nu(\Gamma,l_R^-)=dk_ld(\cos\theta)\tfrac{1}{2}(1-v_l)\Delta \times[xB_+ + x^{-1}B_- + yA_+ + y^{-1}A_- - C] \quad (60)$$

$$d\sigma_{\bar\nu}(\Gamma',l_R^+)=dk_ld(\cos\theta)\tfrac{1}{2}(1+v_l)\Delta \times[xA_-' + x^{-1}A_+' + yB_+' + y^{-1}B_-' + C'] \quad (61)$$

and

$$d\sigma_{\bar\nu}(\Gamma',l_L^+)=dk_ld(\cos\theta)\tfrac{1}{2}(1-v_l)\Delta \times[xB_+' + x^{-1}B_-' + yA_-' + y^{-1}A_+' - C'], \quad (62)$$

where x is defined in (15),

$$y=(k_\nu-k_l+P)^{-1}(-k_\nu+k_l+P) \quad (63)$$

and

$$\Delta=(4\pi k_\nu q^2)^{-1}k_l[-(k_\nu-k_l)^2+P^2] \times[(k_\nu+k_l)^2-P^2], \quad (64)$$

where the functions A, B, C, A', B', C' are functions only of q^2 and M.

Proof. If $v_l\neq 1$, then instead of (17) and (18) the matrix elements of j_λ for final lepton states l_L^- and l_R^- are given by

$$\langle l_L^-|j_\lambda|\nu\rangle=[2(1+v_l)]^{\frac{1}{2}}[-i\sin\tfrac{1}{2}\theta, -\sin(\phi+\tfrac{1}{2}\theta), -\cos(\phi+\tfrac{1}{2}\theta), -i\cos\tfrac{1}{2}\theta] \quad (65)$$

and

$$\langle l_R^-|j_\lambda|\nu\rangle=[2(1-v_l)]^{\frac{1}{2}}[\cos\tfrac{1}{2}\theta, -i\cos(\phi+\tfrac{1}{2}\theta), i\sin(\phi+\tfrac{1}{2}\theta), -\sin\tfrac{1}{2}\theta]. \quad (66)$$

Similar to (20) the matrix element of $j_\lambda^\star$ is related to j_λ by

$$\begin{aligned}\langle l_R^+|j_x^\star|\bar\nu\rangle&=+\langle l_L^-|j_x|\nu\rangle,\\ \langle l_L^+|j_x^\star|\bar\nu\rangle&=+\langle l_R^-|j_x|\nu\rangle,\\ \langle l_R^+|j_\lambda^\star|\bar\nu\rangle&=-\langle l_L^-|j_\lambda|\nu\rangle \quad \text{for } \lambda\neq x,\end{aligned} \quad (67)$$

and

$$\langle l_L^+|j_\lambda^\star|\bar\nu\rangle=-\langle l_R^-|j_\lambda|\nu\rangle \quad \text{for } \lambda\neq x.$$

The validity of (59)–(62) can now be readily established by using the proof of theorem 2. Similar to (52) and (53) the functions $A_\pm$, $B_\pm$, etc. are related to J_λ and $J_\lambda^\star$ by

$$\begin{aligned}A_\pm&=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M) \times|\langle s_\Gamma|J_x|s_N=s_\Gamma\mp1\rangle|^2,\\ B_\pm&=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M) \times|\langle s_\Gamma|\tfrac{1}{2}(J_z\mp iJ_4)|s_N=s_\Gamma\rangle|^2,\\ C&=-\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M) \times[\langle s_\Gamma|\tfrac{1}{2}(J_z+iJ_4)|s_N=s_\Gamma\rangle^\star \times\langle s_\Gamma|\tfrac{1}{2}(J_z-iJ_4)|s_N=s_\Gamma\rangle+\text{c.c.}],\end{aligned} \quad (68)$$

and

$$\begin{aligned}A_\pm'&=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M) \times|\langle s_\Gamma|J_x^\star|s_N=s_\Gamma\mp1\rangle|^2,\\ B_\pm'&=\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M) \times|\langle s_\Gamma|\tfrac{1}{2}(J_z^\star\mp iJ_4^\star)|s_N=s_\Gamma\rangle|^2,\\ C'&=-\sum[MP^2(2\mathcal{J}+1)]^{-1}(2Eq^2)\rho(M) \times[\langle s_\Gamma|\tfrac{1}{2}(J_z^\star+iJ_4^\star)|s_N=s_\Gamma\rangle^\star \times\langle s_\Gamma|\tfrac{1}{2}(J_z^\star-iJ_4^\star)|s_N=s_\Gamma\rangle+\text{c.c.}],\end{aligned} \quad (69)$$

where the sum $\sum$ extends over the appropriate mixture of s_Γ for the final states Γ or Γ'.

Remarks.

(i) It is clear that

$$A_\pm\geqq 0,\quad B_\pm\geqq 0,$$

and

$$B_+B_-\geqq\tfrac{1}{4}C^2. \quad (70)$$

Identical expressions also hold for $A_\pm'$, $B_\pm'$, and C'.

(ii) If $v_l=1$, then $d\sigma_\nu(\Gamma,l_R^-)=d\sigma_{\bar\nu}(\Gamma',l_L^+)=0$. Furthermore, by using

$$\begin{aligned}q^2&=[P^2-(k_\nu-k_l)^2],\\ y^{\pm1}&=(q^2)^{-1}[P\mp(E-m)]^2,\\ q^2&=-M^2-m^2+2mE,\end{aligned} \quad (71)$$

Theorem 4 reduces to theorem 2, and

$$(yB_+ + y^{-1}B_- + C)=B$$

and

$$(yB_+' + y^{-1}B_-' + C')=B'. \quad (72)$$

2. $|\Delta \mathbf{I}|=1$ Rule

Similar to theorem 3, we have

Theorem 5. If the $|\Delta \mathbf{I}|=1$ rule holds then

$$A_{\pm}=A_{\pm}',$$
$$B_{\pm}=B_{\pm}',$$

and

$$C=C', \tag{73}$$

provided that the states N' and Γ' are, respectively, the isotopic spin symmetric states of N and Γ.

The following equalities and inequalities are some further immediate consequences of the $|\Delta \mathbf{I}|=1$ rule:

(i) At $\theta=0$, $y=0$, $\Delta=0$, but $\Delta y^{-1}\neq 0$. Therefore,

$$d\sigma_\nu(\Gamma,l_L^-)=d\sigma_{\bar\nu}(\Gamma',l_R^+). \tag{74}$$

However, the cross section $d\sigma_\nu(\Gamma,l_R^-)$ may be quite different from $d\sigma_{\bar\nu}(\Gamma',l_L^+)$.

(ii) For all θ and k_l,

$$\left|\frac{d\sigma_\nu(\Gamma,l_L^-)-d\sigma_{\bar\nu}(\Gamma',l_R^+)}{d\sigma_\nu(\Gamma,l_L^-)+d\sigma_{\bar\nu}(\Gamma',l_R^+)}\right| \leqq [P^2+(k_\nu+k_l)^2]^{-1}[2P(k_\nu+k_l)] \tag{75}$$

and

$$\left|\frac{d\sigma_\nu(\Gamma,l_R^-)-d\sigma_{\bar\nu}(\Gamma',l_L^+)}{d\sigma_\nu(\Gamma,l_R^-)+d\sigma_{\bar\nu}(\Gamma',l_L^+)}\right| \leqq [P^2+(k_\nu-k_l)^2]^{-1}[2P(k_\nu-k_l)]. \tag{76}$$

(iii) If we increase k_ν but keep q^2, M and the other characteristics of Γ and Γ' fixed, then we have

$$\lim_{k_\nu\to\infty} d\sigma_\nu(\Gamma,l_R^-)=\lim_{k_\nu\to\infty} d\sigma_{\bar\nu}(\Gamma',l_L^+)=0, \tag{77}$$

$$\lim_{k_\nu\to\infty} d\sigma_\nu(\Gamma,l_L^-)=\lim_{k_\nu\to\infty} d\sigma_{\bar\nu}(\Gamma',l_R^+)=d\sigma_\infty(\Gamma), \tag{78}$$

where

$$d\sigma_\infty(\Gamma)=(2\pi m)^{-1}\{[A_+ + A_- + C]+(q^2)^{-1} \times[P-(E-m)]^2B_+ + (q^2)^{-1}[P+(E-m)]^2B_-\} \times MdMd(q^2). \tag{79}$$

3. Γ (or Γ') = Single Spin $\frac{1}{2}$ Particle

Next, we consider the special case that in reaction (47) Γ is an arbitrary single spin $\frac{1}{2}$ particle (e.g., Γ may be Σ^+). Let $d\sigma_\nu(\Gamma_s,l'_s)$ be the differential cross section for the reaction

$$\nu+n\rightarrow \Gamma_s^+ + l_{s'}^-, \tag{80}$$

where s [or s']$=L$ or R depending on the helicity of Γ [or l] being $-\frac{1}{2}$ or $+\frac{1}{2}$. The corresponding structure functions for reaction (80) are denoted by $A_\pm(\Gamma_s)$, $B_\pm(\Gamma_s)$ and $C(\Gamma_s)$.

There are altogether four different reactions depending on s, $s'=L$ or R. In applying theorem 4 to these four cross sections several obvious simplifications occur:

(i) The density of states $\rho(M)$ is a δ function

$$\rho(M)=\delta(M-m_\Gamma), \tag{81}$$

where m_Γ is the mass of Γ. Therefore, among q^2 and M there is only one single variable. By using the transformation

$$dk_ld(\cos\theta)=(2mk_\nu k_l^2)^{-1}E_lMd(q^2)dM,$$

the integral over dM can be easily performed.

(ii) For the reaction that produces Γ_L^+ the function A_+ [defined in (68)] satisfies

$$A_+(\Gamma_L)=0; \tag{82}$$

for the reaction that produces Γ_R^+,

$$A_-(\Gamma_R)=0. \tag{83}$$

Therefore, in general for any given spin $\frac{1}{2}$ particle Γ these four cross sections $d\sigma_\nu(\Gamma_s,l_{s'})$ depend on eight functions $A_+(\Gamma_L)$, $A_-(\Gamma_R)$, $B_\pm(\Gamma_L)$, $B_\pm(\Gamma_R)$, $C(\Gamma_L)$, and $C(\Gamma_R)$, each of which, apart from the trivial factor $\delta(M-m_\Gamma)$, depends only on one variable.

(iii) If time-reversal invariance holds for the weak interactions, then we have

$$B_+(\Gamma_s)B_-(\Gamma_s)=\tfrac{1}{4}[C(\Gamma_s)]^2, \tag{84}$$

where $s=L$ or R. Therefore among these eight functions only six are independent.

(iv) If $v_l=1$, then we have for the neutrino reactions (independent of time reversal invariance),

$$d\sigma_\nu(\Gamma_s,l_R^-)=0,\quad (s=L,\ R)$$
$$d\sigma_\nu(\Gamma_L,l_L^-)=dk_ld(\cos\theta)\Delta[x^{-1}A_-(\Gamma_L)+B(\Gamma_L)],$$

and

$$d\sigma_\nu(\Gamma_R,l_L^-)=dk_\nu d(\cos\theta)\Delta[xA_+(\Gamma_R)+B(\Gamma_R)]. \tag{85}$$

The cross sections $d\sigma_\nu(\Gamma_s,l_{s'}^-)$ depend, in this case, only on four structure functions.

In an almost identical way all the above discussions can be applied to reactions induced by $\bar\nu$.

4. Γ (or Γ') = Single Nucleon

For completeness, we discuss again the special case that Γ (or Γ') is a single nucleon but without the approximation $v_l=1$. Similar to theorem 1, the reactions,

$$\nu+n\rightarrow p_s+l_{s'}^- \tag{86}$$

and

$$\bar\nu+p\rightarrow n_s+l_{s'}^+, \tag{87}$$

are related to each other through the hermiticity of $\mathfrak{L}_{\text{eff}}$. (86) and (87) represent altogether 8 processes depending on the helicities s, $s'=L$ or R. Theorem 4 can be directly applied to each of these reactions.

Eliminating the trivial factor of $\delta(M-m)$ in (59)–(62), we obtain the following theorem for the differential cross sections for (86) and (87):

Theorem 6.

$$d\sigma_\nu(p_s,l_L^-)=\tfrac{1}{2}(1+v_l)K[xa_+(s)+x^{-1}a_-(s)+yb_+(s)+y^{-1}b_-(s)+c(s)]d(q^2),$$

$$d\sigma_\nu(p_s,l_R^-)=\tfrac{1}{2}(1-v_l)K[xb_+(s)+x^{-1}b_-(s)+ya_+(s)+y^{-1}a_-(s)-c(s)]d(q^2),$$

$$d\sigma_{\bar\nu}(n_s,l_R^+)=\tfrac{1}{2}(1+v_l)K[xa_-'(s)+x^{-1}a_+'(s)+yb_+'(s)+y^{-1}b_-'(s)+c'(s)]d(q^2)$$

and

$$d\sigma_{\bar\nu}(n_s,l_L^+)=\tfrac{1}{2}(1+v_l)K[xb_+'(s)+x^{-1}b_-'(s)+ya_-'(s)+y^{-1}a_+'(s)-c'(s)]d(q^2), \quad (88)$$

where

$$K=(8\pi k_\nu^2 v_l q^2)^{-1}[P^2-(k_\nu-k_l)^2][(k_\nu+k_l)^2-P^2]. \quad (89)$$

$a_\pm(s)$, $a_\pm'(s)$, etc., can be expressed explicitly in terms of the six (complex) functions g_V, g_A, $\cdots$, h_A introduced in (28) and (29):

(i) Final nucleon state $=p_R$.

$$a_+(R)=\tfrac{1}{2}(E+m)^{-2}|g_VP-g_A(m+E)|^2,$$
$$a_-(R)=0,$$
$$b_\pm(R)=\tfrac{1}{4}(E+m)^{-1}(E\pm P)|g_V-f_V(m+E)\pm h_VP\mp g_A-f_AP\pm h_A(E-m)|^2, \quad (90)$$
$$c(R)=\tfrac{1}{4}(E+m)^{-1}m[g_V-f_V(m+E)-h_VP+g_A-f_AP-h_A(E-m)][g_V-f_V(m+E)+h_VP-g_A-f_AP+h_A(E-m)]^\star+\text{c.c.}$$

(ii) Final nucleon state $=p_L$.

$$a_+(L)=0$$
$$a_-(L)=\tfrac{1}{2}(E+m)^{-2}|g_VP+g_A(m+E)|^2,$$
$$b_\pm(L)=\tfrac{1}{4}(E+m)^{-1}(E\pm P)|g_V-f_V(m+E)\pm h_VP\pm g_A+f_AP\mp h_A(E-m)|^2,$$

and

$$c(L)=\tfrac{1}{4}(E+m)^{-1}m[g_V-f_V(m+E)-h_VP-g_A+f_AP+h_A(E-m)][g_V-f_V(m+E)+h_VP+g_A+f_AP-h_A(E-m)]^\star+\text{c.c.} \quad (91)$$

(iii) Final nucleon state $=n_s$ $(s=L,R)$. In this case the structure functions $a_+'(s)$, $a_-'(s)$, $\cdots c'(s)$ can be obtained from the corresponding functions $a_+(s)$, $a_-(s)$, $\cdots$, $c(s)$ by the substitution

$$g_V\to g_V^\star,\quad g_A\to g_A^\star,$$
$$f_V\to f_V^\star,\quad f_A\to -f_A^\star, \quad (92)$$
$$h_V\to -h_V^\star,\quad h_A\to h_A^\star.$$

As a result, we have the following theorem:

Theorem 7. Independent of either time reversal invariance or the $|\Delta\mathbf{I}|=1$ rule,

$$a_\pm'(s)=a_\pm(s),\quad (s=L,R)$$
$$(E\mp P)b_\pm'(L)=(E\pm P)b_\mp(R),$$
$$(E\mp P)b_\pm'(R)=(E\pm P)b_\mp(L),$$
$$c'(R)=c(L),$$

and

$$c'(L)=c(R). \quad (93)$$

Therefore, the eight processes (86) and (87) depend at most on eight independent real functions of q^2. By using (73) and (84), it follows that

(i) If time-reversal invariance holds, then among these eight only six are independent. We have

$$b_+(s)b_-(s)=\tfrac{1}{4}[c(s)]^2, \quad (94)$$

where $s=L$ or R.

(ii) If the $|\Delta\mathbf{I}|=1$ rule holds, then

$$(E\pm P)b_\mp(R)=(E\mp P)b_\pm(L)$$

and

$$c(R)=c(L). \quad (95)$$

Thus, among the eight real functions only five are independent.

(iii) If both time reversal invariance and $|\Delta\mathbf{I}|=1$ rule holds then by combining (94) and (95) there are only four independent real functions of q^2 appearing in the cross sections for the eight processes (86) and (87).

(iv) If time-reversal invariance holds then g_V, g_A, f_V, f_A, h_V, h_A must all be real. If $|\Delta\mathbf{I}|=1$ rule holds, then g_V, g_A, f_V, h_A are real but f_A and h_V are both pure imaginary. Therefore, if both time-reversal invariance and $|\Delta\mathbf{I}|=1$ rule holds, then g_V, g_A, f_V, h_A are real and

$$f_A=h_V=0. \quad (96)$$

The above properties (i)–(iii) can also be derived by directly working with the explicit forms (90)–(92).

(v) If time reversal invariance and $|\Delta\mathbf{I}|=1$ rule are both valid, then the differential cross sections for (86) and (87), after summing over the helicities of the final nucleons, become

$$d\sigma_\nu(l_L^-)=\tfrac{1}{2}(1+v_l)K[xa_++x^{-1}a_-+yb_++y^{-1}b_-+c]d(q^2),$$

$$d\sigma_\nu(l_R^-)=\tfrac{1}{2}(1-v_l)K[xb_++x^{-1}b_-+ya_++y^{-1}a_--c]d(q^2),$$

$$d\sigma_{\bar\nu}(l_R^+)=\tfrac{1}{2}(1+v_l)K[xa_-+x^{-1}a_++yb_++y^{-1}b_-+c]d(q^2),$$

and

$$d\sigma_{\bar\nu}(l_L^+)=\tfrac{1}{2}(1-v_l)K[xb_++x^{-1}b_-+ya_-+y^{-1}a_+-c]d(q^2), \quad (97)$$

where K is given by (89) and

$$a_{\pm}=\tfrac{1}{2}(E+m)^{-2}[g_V P\mp g_A(m+E)]^2,$$
$$b_{\pm}=\tfrac{1}{2}(E+m)^{-1}(E\pm P)\{[g_V-f_V(m+E)]^2+[g_A-h_A(E-m)]^2\}, \quad (98)$$
$$c=(E+m)^{-1}m\{[g_V-f_V(m+E)]^2-[g_A-h_A(E-m)]^2\}.$$

If we assume further that the conserved current hypothesis holds, then g_V and f_V are given explicitly by (43). The functions g_A and h_A can then be determined by using (97) and (98). In particular, the difference,

$$(d\sigma_\nu-d\sigma_{\bar\nu})\equiv\sum_{s=L,R}[d\sigma_\nu(l_s^-)-d\sigma_{\bar\nu}(l_s^+)],$$

gives a sensitive determination of g_A. We have, similar to (44) but without the $v_l=1$ approximation,

$$(d\sigma_\nu-d\sigma_{\bar\nu})=(4\pi m^2k_\nu^2)^{-1}q^2[4mk_\nu-q^2-m_l^2]\times g_V g_A d(q^2). \quad (99)$$

(vi) If $v_l=1$, then

$$K=(8\pi k_\nu^2)^{-1}[(k_\nu+k_l)^2-P^2],$$
$$y=(E+P)^{-1}m,$$

and

$$y^{-1}=(E-P)^{-1}m.$$

The $a_\pm$, b, d functions of Sec. III are related to the present ones by

$$a_+=a_+(R),$$
$$a_-=a_-(L),$$
$$b=\sum_{s=L,R}[yb_+(s)+y^{-1}b_-(s)+c(s)],$$

and

$$d=[yb_+(R)+y^{-1}b_-(R)+c(R)]-[yb_+(L)+y^{-1}b_-(L)+c(L)]. \quad (100)$$

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Self-Consistent Model for Nonleptonic Decays of Σ and Λ Hyperons*

VIRENDRA SINGH† AND BHALCHANDRA M. UDGAONKAR‡
Lawrence Radiation Laboratory, University of California, Berkeley, California
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A self-consistent calculation of pionic Σ and Λ decays has been carried out in the pole approximation of an S matrix approach in order to get information on (a) the angular momentum in which the decay $\Sigma^+\to n\pi^+$ takes place, (b) the relative (ΣΛ) parity, (c) the possible existence of other than global symmetric solutions. On the basis of existing experimental data, the model predicts that $\Sigma^+\to n\pi^+$ decay must occur in the s wave, and, somewhat less definitely, that (ΣΛ) parity is even.

INTRODUCTION

RECENTLY, Beall *et al.* have established that the helicities of the protons in the nonleptonic decays of Σ^+ and Λ^0 are opposite.[1] This result, while confirming an important prediction of the global-symmetry hypothesis, contradicts the predictions of several other models of hyperon decay. In particular, it disagrees with the bound-pion model of Barshay and Schwartz,[2] in which the Λ decay is taken as the primary decay, and thus invalidates one of the arguments used by Nambu and Sakurai in favor of odd Σ-Λ parity.[3] We have, therefore, considered a simple self-consistent model in which both these decays are treated as equally fundamental, with parameters to be determined by requirements of consistency. We have, then, tried to seek answers to the following questions: (1) Are there solutions other than the global-symmetric one that fit the experimental data? (2) Does odd Σ-Λ parity fit the data better, or vice versa? (3) Can one predict which of the two decays—$\Sigma^+\to n\pi^+$ or $\Sigma^-\to n\pi^-$—goes into s wave and which into p wave? With regard to the last question, it has been well known for some time, from the experimental data on the Σ triangle of Gell-Mann and Rosenfeld,[4,5] that one of these decays must go into s-wave and the other into p wave, but it has not been possible so far to say which goes to which.

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† On leave from the Tata Institute of Fundamental Research, Bombay, India.
‡ On leave from the Atomic Energy Establishment Trombay, Bombay, India.

[1] E. F. Beall, B. Cork, D. Keefe, P. G. Murphy, and W. A. Wenzel, Phys. Rev. Letters, **7**, 285 (1961).
[2] S. Barshay and M. Schwartz, Phys. Rev. Letters **4**, 618 (1960).
[3] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **6**, 377 (1961).
[4] M. Gell-Mann and A. H. Rosenfeld, in *Ann. Rev. Nuclear Sci.* **7**, 407 (1957).
[5] B. Cork, L. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. **120**, 100 (1960).