# Influence of a Resonant Absorber whose Thickness Varies Linearly with Time on the Spectral Shape of an Incident Lorentzian Wave Packet*

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The influence of a resonant absorber whose thickness varies linearly with time on the spectral shape of an incident Lorentzian wave packet is discussed, using classical dispersion theory. The resonant line is broadened and, for high enough velocities with which the thickness of the absorber changes, the line is split into two bands with a minimum at the resonant frequency $\omega_0$.

## INTRODUCTION

THE transmission of a Lorentzian wave packet of $\gamma$ rays through a resonant absorber of given thickness has recently been studied theoretically by Lynch *et al.*[1] in connection with Mössbauer experiments on the time dependence of resonantly filtered $\gamma$ rays. It is of some interest to consider the shape of a Lorentzian wave packet (corresponding to the emission of recoil-free $\gamma$ rays from an ideal source) after transmission through an ideal resonant absorber whose thickness varies with time. Each Fourier component $a(\omega)e^{i\omega t}$ of the wave packet whose time dependence is given by

$$a(t)=\int_{-\infty}^{\infty} a(\omega)e^{i\omega t}d\omega$$

is then subjected to a phase shift

$$\varphi=-(\omega/c)(n-1)d,$$

when passing through an absorber of thickness $d$ and refractive index $n$. If $n$ is independent of $\omega$ (the case of a nonresonant absorber), then the whole packet receives a constant phase shift and there is no change in shape. If the thickness $d$ varies linearly with time, $d=vt$, then the phase shift produces a frequency shift

$$\Delta\omega=-(\omega/c)(n-1)v.$$

This situation has recently been examined experimentally by Grodzins and Phillips[2] using the Mössbauer effect. The observed frequency shifts, produced by nonresonant absorbers whose thickness varies linearly with time, were used to determine the refractive indices, arising from the interaction of $\gamma$ rays with the atomic electrons. If the absorber used, however, absorbs resonantly as a consequence of the Mössbauer effect, the situation is more complicated, since the part of the refractive index produced by the nuclear interaction is strongly dependent on $\omega$ in the resonant region and the shape of the transmitted wave may be changed considerably. In this note, this problem has been examined using classical dispersion theory.

## THEORY

The original incident wave packet, assumed to be Lorentzian, will have the form

$$a(t)=e^{-(\lambda/2)t}e^{i\omega_0 t},$$

where $1/\lambda$ is the mean lifetime of the nuclear state and $\lambda$ is the width of the Fourier spectrum of the packet at half maximum. The Fourier spectrum is given by:

$$a(t)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{e^{i\omega t}}{\omega-\omega_0-i\lambda/2}d\omega.$$

Then, as in classical dispersion theory and as assumed by Lynch *et al.*, the refractive index is given by

$$n^2=1+\frac{r}{\omega_0^2-\omega^2+i\lambda\omega},$$

and to a good approximation

$$n-1=\frac{r/2}{\omega_0^2-\omega^2+i\lambda\omega}.$$

Now, if we assume, again, that $d=vt$, then every component $a(\omega)e^{i\omega t}$ is shifted in frequency by

$$\Delta\omega=-(\omega/c)\,\mathrm{Re}(n-1)v.$$

For $\omega<\omega_0$, $\mathrm{Re}(n-1)$ is positive and for $\omega>\omega_0$, $\mathrm{Re}(n-1)$ is negative (Fig. 1), so we must expect that the shape of

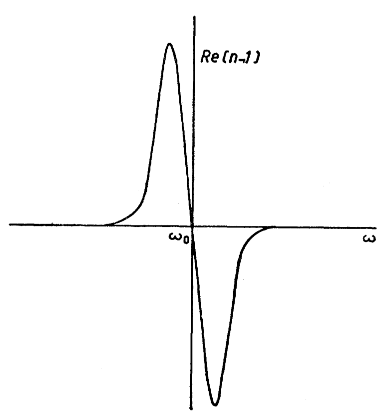


FIG. 1. Dependence of the real part of the refractive index on frequency near resonance.

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[1] F. J. Lynch, R. E. Holland, and M. Hamermesh, Phys. Rev. **120**, 513 (1960).
[2] L. Grodzins and E. A. Phillips, Phys. Rev. **124**, 774 (1961).

the transmitted wave packet will be widened. A frequency-dependent attenuation of the wave will also occur, and this will influence the shape to a lesser extent, being determined by the variation of $\mathrm{Im}(n-1)$ near $\omega_0$.

The time dependence of the transmitted wave packet through a resonant absorber of constant thickness $d$ has already been calculated.[1] It is given by

$$a'(t)=e^{-(\lambda/2)t}e^{i\omega_0 t}J_0\left[\left(\frac{r}{c}d\right)^{\frac{1}{2}}t^{\frac{1}{2}}\right].$$

Now, if we put $d=vt$ (for the case of an absorber whose thickness varies linearly with time) in the above expression, we can find the spectral shape of the transmitted wave packet $a'(\omega)$, since

$$a'(\omega)=\int_0^\infty a'(t)e^{-i\omega t}dt.$$

In our case, this integral has the form

$$\int_0^\infty e^{-[\lambda/2+i(\omega-\omega_0)]t}J_0\left[\left(\frac{r}{c}v\right)^{\frac{1}{2}}t\right]dt,$$

which is a well-known integral of Lipschitz.[3] The result is

$$a'(\omega)=\{(r/c)v+[\lambda/2+i(\omega-\omega_0)]^2\}^{-\frac{1}{2}},$$

and

$$|a'(\omega)|^2=\{\lambda^2(\omega-\omega_0)^2+[(r/c)v+\lambda^2/4-(\omega-\omega_0)^2]^2\}^{-\frac{1}{2}}=[R(\omega)]^{-\frac{1}{2}}.$$

Let us see how the shape of the wave packet is changed when we change $v$. We shall look for the maxima of the intensity given by

$$(d/d\omega)R(\omega)=4(\omega-\omega_0)[(\omega-\omega_0)^2+\tfrac{1}{4}\lambda^2-(r/c)v]=0.$$

For $v\rightarrow 0$, we approach a Lorentz shape. For $v<v_c=c\lambda^2/4r$, there is only one maximum at $\omega=\omega_0$. When $v=v_c$, we still have only one maximum, at $\omega_0$, but it is a triple zero of $dR/d\omega$. Its height at $\omega_0$ is half the height of the incident Lorentz packet, and its width at half maximum is about twice the width of the latter.

When $v>v_c$, on the other hand, there is a minimum at $\omega_0$ and maxima at

$$\omega=\omega_0\mp[(r/c)v-\lambda^2/4]^{\frac{1}{2}}.$$

For example, if $v=5v_c$, the distance between the two maxima is $2\lambda$, and the minimum at $\omega_0$ is about 70% of the height at the maxima. The situation is described schematically in Fig. 2.

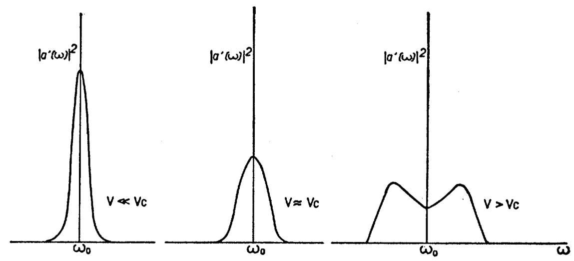


FIG. 2. Schematic behavior of the line shape of the transmitted wave packet for various velocities with which the thickness of the absorber is changed.

As pointed out in the introduction, in any practical case connected with Mössbauer experiments there is also an electronic contribution $k$ to the refractive index which is constant through the linewidth. For an absorber whose thickness varies linearly with time, this gives a frequency shift

$$\Delta\omega_0=-(\omega_0/c)kv.$$

In general, therefore, we should replace $\omega_0$ in our last equations by

$$\omega_0'=\omega_0-(\omega_0/c)kv.$$

In order to compare the effects of the two contributions to the refractive index, let us consider as an example the case of the 14-kev $\gamma$ line in $Fe^{57}$. From the experimental results of Lynch *et al.* (using their numerical result for the parameter $\beta=rd/c\lambda$), we obtain

$$r\approx 3\times 10^{22}\ \mathrm{sec}^{-2},$$

and thus

$$v_c=c\lambda^2/4r\approx 12.5\ \mathrm{cm/sec}.$$

Then, for the distance $D$ between the two maxima, we get

$$D=[4(r/c)v-\lambda^2]^{\frac{1}{2}}\approx(4v-50)^{\frac{1}{2}}\times 10^6\ \mathrm{sec}^{-1}.$$

The frequency shift due to the electronic contribution ($k\approx-10^{-6}$) to the refractive index is

$$\Delta\omega_0=-(\omega_0/c)kv\approx 10^3 v\ \mathrm{sec}^{-1}.$$

From these equations, we see that when $v$ is larger than $v_c$ the splitting $D$ which then occurs is much larger than the shift $\Delta\omega_0$.

**ACKNOWLEDGMENT**

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[3] H. Lamb, *Hydrodynamics* (Dover Publications, New York, 1945), 6th ed., p. 138.