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Pion-Pion Interaction in the Reaction $\bar{p}+p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$*

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Deviations from the expected two-pion effective-mass distributions have been found in the reaction $\bar{p}+p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0$. These occur in two regions, 300 Mev and 750 Mev, and could be explained by the $T=0$, pion-pion interaction reported by Abashian, Booth, and Crowe and the widely observed ρ meson ($T=1$, $J=1^-$ pion-pion resonance). The data in the 750-Mev region for $\pi^+\pi^-$ combinations ($Q=0$) show an apparent double-peak structure—one at 720 Mev ($\Gamma=20$ Mev), the other at 780 Mev ($\Gamma=60$ Mev). However, the $|Q|=1$ data ($\pi^+\pi^0$ and $\pi^-\pi^0$ combinations) are consistent with the previously observed single $T=1$, $J=1^-$ resonance. Our data give $M_\rho=767$ Mev ($\Gamma=110$ Mev). As these parameters also fit the $Q=0$ data, a detailed discussion of the statistical significance of the one-peak and two-peak hypotheses is given. A search has been made for possible neutral modes of decay of the heavy mesons, and no evidence has been found.

I. INTRODUCTION

THE π-π interaction, a subject of extensive theoretical[1] and experimental inquiry, has been evident in pion-nucleon,[2–4] proton-proton, electron-nucleon,[5] and proton-deuteron collisions,[6] and in $K^\pm$ decay.[7] Anderson *et al.*[3] established the $J=1$, pion-pion interaction as resonant at ≈750 Mev from the fact that the pion-pion cross section reached $12\pi\lambda\!\!\!^{-2}$. For the experimental structure of this resonance, their data, with that of others,[4] give a width of 140 Mev ($\Gamma/2=70$ Mev) and a central value of about 750 Mev.

Presented here are our results on the distributions of the invariant effective mass,

$$M_2=[(E_1+E_2)^2-(\mathbf{P}_1+\mathbf{P}_2)^2]^{\frac{1}{2}}, \qquad (1)$$

of pairs of pions produced in the reactions:

$$\bar{p}+p \rightarrow 2\pi^+ + 2\pi^- + \pi^0, \qquad (2)$$

$$\bar{p}+p \rightarrow 2\pi^+ + 2\pi^-, \qquad (3)$$

$$\bar{p}+p \rightarrow 2\pi^+ + 2\pi^- + n\pi^0, \quad (n=2,\ 3,\ \text{or}\ 4). \qquad (4)$$

Reactions (2–4) contain some unique features for the study of the π-π interaction: (a) absence of nucleons in the final state; (b) a variety of charge combinations, giving total charge states $Q=0$, 1, 2; and (c) a large amount of energy available for a two-pion system, covering the range 0.28 Bev $<M_2<$ 1.87 Bev.

II. EFFECTIVE-MASS RESOLUTION

For each value of M_2 given by Eq. (1), we calculate an uncertainty δM_2. From a sample of these uncertainties (selected for $M_2\approx 750$ Mev), we can form a resolution function, and measure its half-width at half-maximum, $\Gamma_{\text{resol}}/2$.[8] In kinematically fitted 5π annihilations we find: $\Gamma_{\text{resol}}/2=10\pm 2$ Mev for $Q=0$ ($\pi^+\pi^-$ pairs) and 20 Mev for $|Q|=1$ (involving one unmeasured π^0). The "6π" annihilations yield only unfitted pions; pairs of these with $Q=0$ yield $\Gamma_{\text{resol}}/2=15$ Mev.

We have checked our resolution estimates by measuring 90 $K_1{}^0$ decays produced by $\bar{p}+p \rightarrow K_1{}^0+K+n\pi$. We use the unfitted variables for the tracks from the $K_1{}^0$ decays and compare the M_2 distribution with our $\Gamma_{\text{resol}}/2=15$ Mev for unfitted tracks. From these $K_1{}^0$

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[8] For further details see B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters **7**, 178 (1961), and N. H. Xuong and G. R. Lynch, *ibid.* **7**, 327 (1961).

TABLE I. Selection criteria.

Event type	χ^2 for 4π hypothesis	χ^2 for 5π hypothesis	$p_{\bar{p},\text{c.m.}}$ (Mev/c)	Other[a]	Number	$\Gamma_{\text{resol}}/2(M_2)$ (Mev)
5π	>15	<6.5	>643	···	1074	10 or 20
4π	<15	>6.5	>643	···	101	10
"6π"[b]	>15	>6.5	>643	M_u and $E_u>0$	1199	15
4π or 5π (ambiguous)	<15	<6.5	>643	···	47	···
background	>15	>6.5	<643	or M_u or $E_u<0$	332	···
Total processed					2753	

[a] Here M_u is the missing mass, E_u the missing energy.
[b] 6π events are actually about 60% 6π, 30% 7π, 10% 8π.

decays we find an M_2 peak with $\Gamma/2=13$ Mev centered at 499 ± 3 Mev ($0.2\%\pm0.6\%$ too high), which seems consistent with our δM estimates.

III. IDENTIFICATION OF REACTIONS

A total of 2700 four-prong events, produced by 1.61-Bev/c antiprotons [667-Mev/c center-of-mass (c.m.) momentum] in the 72-in. hydrogen bubble chamber, have been measured and processed through our system.[8a] Of these, 1074 five-pion (5π) events and 101 four-pion (4π) events were identified by using our kinematics program KICK. Since "six-pion" (6π) events contain two or more neutrals, fits cannot be obtained. We have selected a sample of 1199 events of reaction type (4) according to the following criteria: (a) Both missing mass and missing energy must be positive (this reduces the chance for wrong mass interpretation or mismeasurements) and (b) the momentum of the beam track must be >1.55 Bev/c. This minimizes the number of four-prong events produced by the pion contamination in our beam. (The average momentum of the pion contamination was about 1.50 Bev/c.) The χ^2 and other selection criteria as well as the numbers of events for each of the reactions are listed in Table I. The events that did not fit any of the above categories are the background, composed of annihilations into K mesons (uncharged as well as charged), four-prongs produced by pions, ambiguous events, and mismeasurements. The 2700 events were processed mainly by a computer. In order to select a pure sample of 4π's out of dominantly 5π and 6π events, the criterion for the 4π events had to be made rather strict; hence a large fraction of the real 4π's were probably thrown in the 5π sample. Some fraction of the "6π" events may be poor measurements of 4π and 5π events.

TABLE II. Numbers of pion pairs.

Charge state	Pion pairs involved	Number of pion pairs, fitted events only[a]	(Total, fitted plus unfitted events)[b]
$Q=0$	$\pi^+\pi^-$	1175×4	2374×4
$\lvert Q\rvert=1$	$\pi^+\pi^0$, $\pi^-\pi^0$	1074×4	1074×4
$\lvert Q\rvert=2$	$\pi^+\pi^+$, $\pi^-\pi^-$	1175×2	2374×2

[a] $4\pi+5\pi$ events.
[b] $4\pi+5\pi+$"6π" events.

The formation of a state of total charge $|Q|=1$ requires a neutral pion. Consequently, 4π and "6π" annihilations could not be used for this charge state. From the 2374 4π, 5π, and "6π" events, we were able to form the numbers of pion pairs shown in Table II.

IV. RESULTS

We have studied the M_2 distributions in 1175 fitted 4π and 5π reactions. Figures 1(a), (b), and (c) display the results for $|Q|=0$, 1, and 2, respectively. The error flags are $N^{\frac{1}{2}}$, where N is the number of pion pairs per 10-Mev interval. To gain slightly in statistical accuracy, we have added [in Figs. 2(a) and (c)] the 1199 unfitted "6π" events of type (4) to the $Q=0$ and $|Q|=2$ distributions of fitted data from Fig. 1. The histogram interval was chosen to be 20 Mev in this instance. In

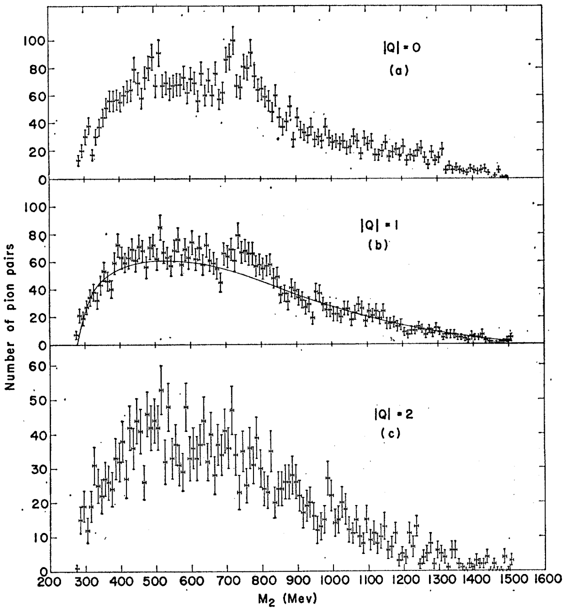

FIG. 1. Effective mass distributions of fitted data, plotted in 10-Mev intervals. (a) $Q=0$, 1074 $5\pi+101$ 4π events. (b) $|Q|=1$, 1074 5π events. The solid curve is Lorentz-invariant phase space, arbitrarily normalized. (c) $|Q|=2$, 1074 $5\pi+101$ 4π events.

[8a] The $\bar{p}$ path length in this experiment is such that a reaction with a cross section of 1 mb, yields 150 events. Thus, 2700 events represent a cross section of ≈20 mb.

Fig. 2(b) the same data as in Fig. 1(b) are plotted in 20-Mev intervals. Possible deviations from the smooth distributions are observed in two regions of M_2, which we now discuss.

A. The 750-Mev Region

1. *Effective-Mass Distributions*

The $Q=0$ data [$\pi^+\pi^-$ pairs of Figs. 1(a) and 2(a)] deviate somewhat from the expected $T=1$, $J=1$ resonant state previously reported. At first inspection there appears to be evidence for a double peak in this region.

The $|Q|=1$ data [$\pi^+\pi^0$ and $\pi^-\pi^0$ pairs of Fig. 1(b)] appear consistent with the expected resonance. The $|Q|=2$ state [$\pi^+\pi^+$ and $\pi^-\pi^-$ pairs of Figs. 1(c) and 2(b)] does not show any significant deviations from a smooth distribution. In fact, the χ^2 analysis of the data of Fig. 2(c) for the entire $|Q|=2$ M_2 spectrum shows that they fit a smooth curve with a 90% probability level. This is an indication that the correlation between pairs of the same event are unimportant and that the errors, $N^{\frac{1}{2}}$, are realistic.

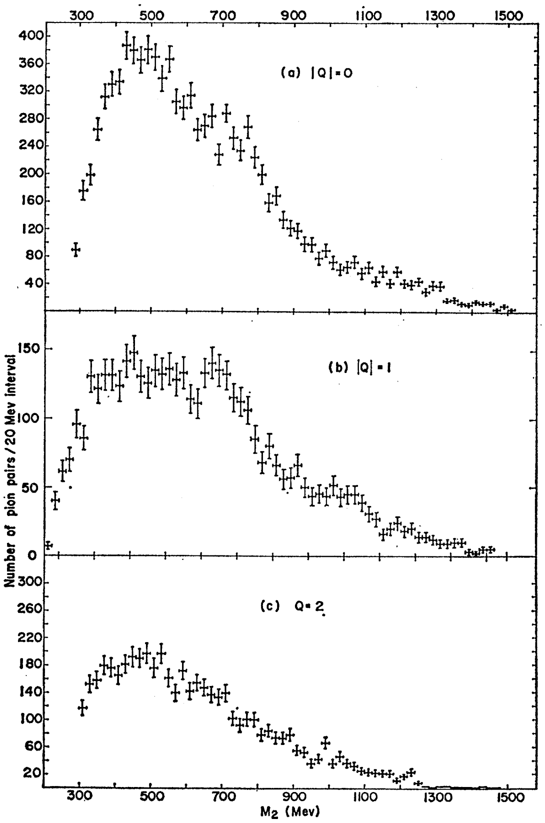

FIG. 2. Effective mass distributions of fitted plus unfitted data, plotted in 20-Mev intervals. (a) $Q=0$: 1074 5π+101 4π+1199 "6π" events. (b) $|Q|=1$: 1074 5π events (the same data as in Fig. 1(b), but replotted in 20-Mev intervals). (c) $|Q|=2$: 1074 5π+101 4π+1199 "6π" events.

To understand the unexpected result in the $Q=0$ state, we subject the following hypotheses to χ^2 tests.

2. *Hypothesis 1 (Single Resonance)*

In this hypothesis we assume that only one $T=1$, $J=1$ resonance exists and that the anomalous behavior of the $Q=0$ state is caused by a statistical fluctuation.

a. χ^2 tests. After making a background subtraction, we have fitted a $J=1$ resonance curve of the type

$$\frac{dn}{dM_2}\propto\frac{M_\rho^2\Gamma_\rho(M_2)\Gamma}{(M_2^2-M_\rho^2)^2+M_\rho^2\Gamma_\rho^2(M_2)} \tag{5}$$

to both charge states, where

$$\Gamma_\rho(M_2)\equiv\Gamma\left(\frac{M_\rho}{M_2}\right)\left(\frac{M_2^2-(2M_\pi)^2}{M_\rho^2-(2M_\pi)^2}\right)^{\frac{3}{2}}, \tag{6}$$

M_ρ is the mass of the ρ meson, and Γ its "width."

The results of least-squares fits to various combinations of the data are presented in Table III. The upper, middle, and lower portions of Fig. 3(a) display the fit to the total sample ($4\pi+5\pi+$"6π") for $|Q|=0$, 1, and 0+1, respectively.

Uncertainties in the parameters of the fits are difficult to estimate because of the serious background subtraction. However, comparison of the independent $Q=0$ and $|Q|=1$ samples shows that M_ρ fluctuates by a few Mev, and Γ by 10 to 20 Mev. The value of χ^2 and its associated probability depends on this background and upon the number and size of bins used. We could make the $Q=0$ fits appear more probable by using all the data instead of the 30 bins near the ρ peak, but this seems misleading since previous experiments had already focused attention on this region.[3,4] We could make the $Q=0$ fit appear even less probable by grouping the data in 20-Mev bins, but instead we prefer to make the analysis discussed below [Sec. IV. 4(b)].

Table III displays the fraction of annihilations in which ρ mesons are produced. When fitted to the neutral M_2 spectrum of 1175×4 pion pairs, the integral of Eq. (5) contains 496 pairs, representing about 425 distinct events (because of statistical chance plus physical correlations, some events have two pairs under the ρ peak). Thus 425/1175 ($\approx$36%) of these annihilations make neutral ρ mesons. When fitted to the $Q=1$ spectrum (1074×4 pairs), the integral contains 274 pairs ($\approx$250 distinct events) corresponding to $\rho^\pm$ production with a probability of 25%. Accordingly, a given 5π annihilation produces a ρ with a probability of >50% ($\approx$36% for ρ^0 and $\approx$25% for $\rho^\pm$). [The statistical model predicts ρ^0: $(\rho^++\rho^-)$ of about 0.7:1.0.][9]

b. Search for decay properties. In spite of the large background, we have attempted to observe the decay properties of the ρ meson. Since the ρ is a two-pion $T=1$ resonance, it must have odd J. If the ρ is polarized

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[9] A. Hussain (private communication).

TABLE III. Single-peak fits to several combinations of data.

Sample	Q	Events in whole spectrum	M_ρ (Mev)	Γ (Mev)	Integral under Eq. (5) (events)	χ^2	Probability $(\chi^2,\langle\chi^2\rangle_{\text{av}}=26)^{\text{a}}$ (%)
Fitted	0	1175[b]	763	126	496	34.8	12
Total	0	2106	768	112	440	35.6	10
Fitted	1	1074	765	98	274	19.6	80
Total	0+1	3180[b]	767	110	737	30.4	25

[a] The average value of χ^2 is the number of bins (30) minus the number of parameters of the resonance curve (3) minus the number of parameters of the background (n_b). If the background were determined entirely by the nonresonant $|Q|=2$ data, n_b would be zero; if it were unconstrained and approximated by $a+b(EM_2)$, n_b would be two. Since we took $|Q|=2$ data partly into account, we call n_b one.

[b] These fits were made when only 931 of the 1199 "6π" events had been processed.

or aligned during production, then its breakup will be of the form $1+a_2\cos^2\theta$ (odd powers of $\cos\theta$ are forbidden; higher even terms could be present for $J\geqslant 3$). For small a_2, the statistical uncertainty δa_2 is $(45/4N)^{\frac{1}{2}}$, where N is the number of events. After increasing δa_2 to account for the fact that in the region of the ρ peak the signal-to-background ratio is about 1:2, we find $\delta a_2\approx 0.25$. Within this uncertainty, we find that the "decay" pion momentum in the ρ rest frame is not correlated with either (a) the ρ direction of flight $\mathbf{p}_\rho$, (b) the beam direction $\mathbf{p}_{\text{beam}}$, or (c) the production normal $\mathbf{n}=\mathbf{p}_{\text{beam}}\times\mathbf{p}_\rho$. The ρ production distribution is also isotropic.

c. Search for correlations. We find no evidence that ρ mesons tend to be produced in pairs or in association with ω mesons, or that there is momentum correlation between either of the two pions that form the ρ and other pions produced in the annihilation.

3. *Hypothesis 2 (Double Resonance)*

In this hypothesis we assume that there is a double peak in both $Q=0$ and $|Q|=1$ states. We will refer to the lower-mass peak as ρ_1 and to the higher one as ρ_2. These data are best fit by two curves that have the shape of our resolution function.

The total ($4\pi+5\pi+$"6π") $Q=0$ data plotted in 10-Mev intervals, are displayed in the upper portion of Fig. 3(b). The middle and lower portions display $|Q|=1$ and our combined total data, respectively.

a. χ^2 tests. The best fits of two resonance functions with the form of Eq. (5) are displayed in Table IV. Although the $Q=1$ data do not suggest a double peak, two different fits to them are displayed. The first fit is not constrained in any way to agree with the $Q=0$ fits, and is not very consistent with them. The second fit is forced to have values equal to the best fit for the over-all data, and still has a low value of χ^2.

We conclude that the data satisfy very well the hypothesis of two peaks at 720 and 780 Mev, with $\Gamma_1=30$ Mev and $\Gamma_2=60$ Mev. It must be noted, however, that while high χ^2 values discriminate *against* wrong hypotheses, low χ^2 cannot be used as evidence *for* a hypothesis.

Finally we note that even if the ρ_1 and ρ_2 peaks are real, their isotopic spin cannot be determined. One or both of them are at least $T=1$.

b. Search for correlations. In a vain attempt to find a difference between the events in the ρ_1 and ρ_2 regions, the production and "decay" correlations discussed earlier for all ρ events were examined separately for ρ_1 and ρ_2, but again no correlations were seen.

4. *Further Discussion*

a. Search for neutral decay modes. We have searched for a neutral decay mode of the ρ meson, ω meson, or

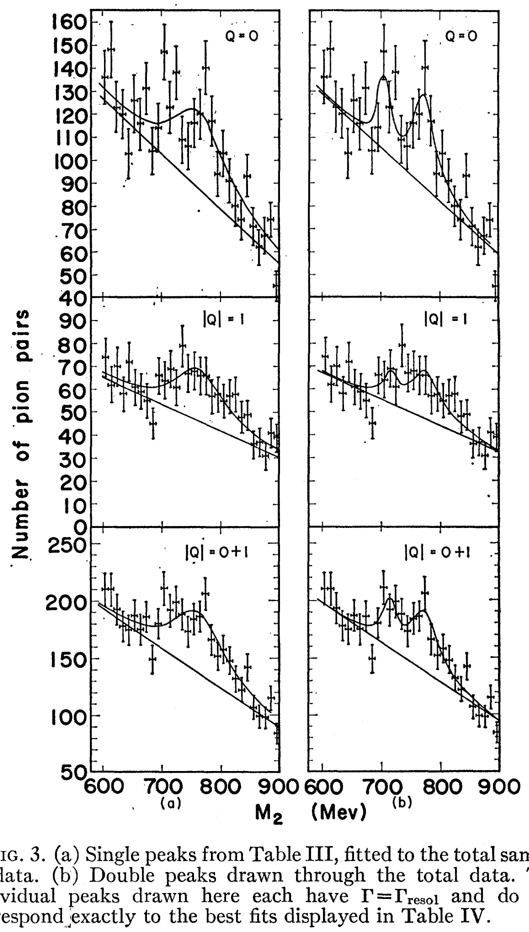

FIG. 3. (a) Single peaks from Table III, fitted to the total sample of data. (b) Double peaks drawn through the total data. The individual peaks drawn here each have $\Gamma=\Gamma_{\text{resol}}$ and do not correspond exactly to the best fits displayed in Table IV.

TABLE IV. Double-peak fits to several combinations of data.

Sample	Q	Events in whole spectrum	$M_{\rho 1}:M_{\rho 2}$ (Mev)	$\Gamma_1:\Gamma_2$ (Mev)	Integral 1 +integral 2 (events)	χ^2	Probability $(\chi^2,\langle\chi^2\rangle_{av}=23)$[b] (%)
Fitted	0	1175	720 :778	20[a]:70	108+220	23.5	40
Total	0	2106[d]	709 :777	25[a]:44	107+236	25.2	30
Fitted	1	1074	749 :817	54 :40[a]	165+65	17.2	80
Fitted[c]	1	1073	717[c]:778[c]	40[a]:59[c]	56+158	22.4	50
Total	0+1	3180[d]	717 :778	30[a]:59	163+419	24.3	40

[a] A slightly better fit can be obtained with a smaller value of Γ, but it is unreasonable to allow Γ to be less than Γ_{resol}.
[b] $\langle\chi^2\rangle_{av}$ is the number of bins (30) minus the parameters of two resonance curves (three each) minus the number of parameters n_b in the background (n_b is again taken to be 1).
[c] This fit is constrained to agree with the fit to the total data given in the next row.
[d] These fits were made when only 931 of the 1199 "6π" events had been processed.

any other particle by studying the missing-mass (MM) spectrum of the "6π" data as displayed in Fig. 4. Our experimental resolution Γ(MM) is about 100 Mev, and we estimate that we would surely notice a peak if it contained >75 events. Seeing no peaks, we can set some upper limits on branching ratios.

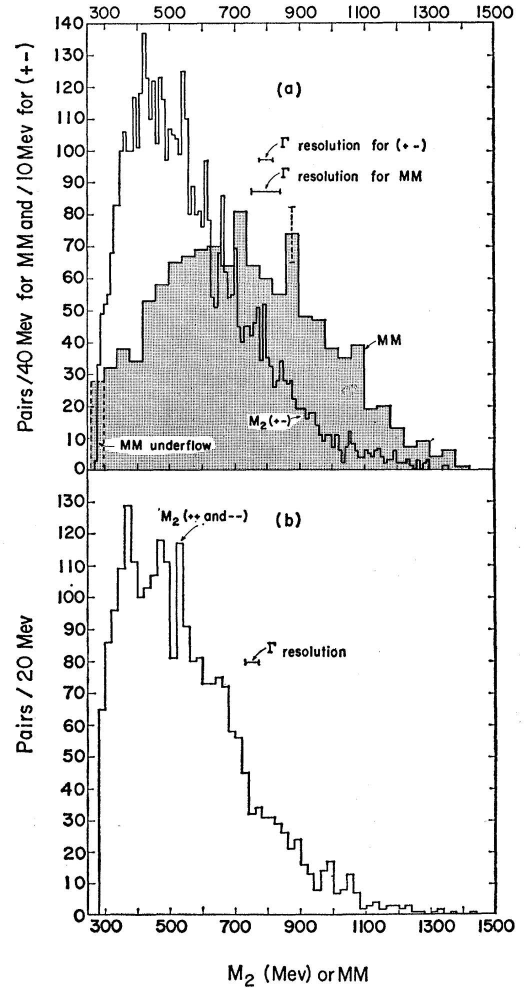

FIG. 4. The M_2 distributions for the 1199 "6π" events. (a) The two $Q=0$ groupings, i.e., one missing-mass, MM, (shaded) and four $(+-)$ combinations per event. (b) The $|Q|=2$ combinations for each event. At 750 Mev, the missing mass has a typical resolution $\Gamma_{resol}(\text{MM})=90$ Mev; the charged pairs have $\Gamma_{resol}(+-)=\Gamma_{resol}(++)=30$ Mev. At any other value of MM, the formula is $\Gamma_{resol}(\text{MM})=45$ Mev [(750 Mev/MM], and similarly for $M_2(+-)$ and $M_2(++)$.

First we consider the ω meson. The same experiment that yielded the four-prong events discussed in this paper yielded also 78±18 six-prong annihilations determined to be $\bar{p}+p\rightarrow 2\pi^++2\pi^-+(\omega\rightarrow\pi^++\pi^-+\pi^0)$.[8] If the branching ratio for $(\omega\rightarrow 3\pi^0+\pi^0\gamma)/(\omega\rightarrow\pi^+\pi^-\pi^0)$ were as high as unity, we would have been able to observe a peak in the missing-mass spectrum. An upper limit for this branching ratio is ≈ 0.5. (We did not expect to see this peak, since the ω is thought to have isotopic spin zero, and hence its strong decay into $3\pi^0$ is forbidden.)

Next we consider the ρ, even though it cannot decay into $2\pi^0$ if it really has $J=1$. The same six-prong events that yielded 78 ω mesons, yielded 110±20 ρ mesons; so again a branching ratio $(\rho\rightarrow\text{neutrals})/(\rho\rightarrow\pi^++\pi^-)$ cannot be as high as unity.

Finally there is the η^0, which has a mass of 550 Mev[10] and decays into $\pi^++\pi^-+\pi^0$ or into neutrals with a branching ratio of about 1:3.[11] We see no evidence for either of these modes, or for the mode $\eta\rightarrow\pi^++\pi^-+\gamma$, or any charged η.

In Fig. 4 the missing-mass distribution is compared with the $Q=0$, M_2 distribution of the "6π" data. The difference between these two distributions, we believe, can be attributed to the presence of approximately (30 to 50%) seven-pion reactions ($3\pi^0$) plus ~10% eight-pions ($4\pi^0$) in the "6π" sample.

b. Monte Carlo test of statistical significance. The χ^2 test is unsatisfactory for testing the above hypotheses because it does not take into account the correlations between adjacent points. To obtain a better understanding of how unlikely it is for the data to look as much like two peaks as our data when in fact the resonance has no structure, we used a computer to generate 72 Monte Carlo histograms of the M_2 distribution for 3022 pion pairs in 10-Mev intervals from 620 to 920 Mev. The generation was made according to the

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[10] A. Pevsner, R. Kraemer, M. Nussbaum, C. Richardson, P. Schlein, R. Strand, T. Toohig, M. Block, A. Engler, R. Cessaroli, and C. Meltzer, Phys. Rev. Letters 7, 421 (1961).

[11] P. Bastien, J. P. Berge, O. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters 8, 114 (1962).

fitted single-resonance curve [Fig. 3(a)]. These histograms were studied independently by 20 experimental physicists who were asked which of them looked most suggestive of a double peak. Unknown to them the real data were also included in the sample. The actual data were the second most frequently chosen histogram. Six of the physicists picked it as the most suggestive of a double peak. Although such a test is an extremely subjective one, we can conclude from it that there is about a 1 to 3% chance of obtaining data near 750 Mev in any M_2 distribution which are as suggestive of a double peak as are our data when only a single resonance exists.

c. Upper limit for the decay $\omega \rightarrow \pi^+ + \pi^-$. In our 5π data there are about 100 ω mesons[10] which decay by their normal mode $\omega \rightarrow \pi^+ + \pi^- + \pi^0$. If they decay into the G-forbidden mode $\pi^+ + \pi^-$ with a branching fraction f, then we would see in our 4π fits $100f$ events with a value of M_2 near $m_\omega = 787$ Mev (near the upper half of the ρ peak). Among our 101 4π events, we see fewer than 20 ρ's and no evidence for extra events near 787 Mev. We conclude that the branching fraction f is less than about 10%.

5. Conclusions

Whether one accepts the double-peak hypothesis ($\approx 50\%$ probable) when the expected hypothesis (1) is $\approx 10\%$ probable is a matter of individual judgment. We feel that such a judgment cannot be made on the basis of our data alone, and conclude that further data are needed to settle this question. Indeed, they are needed to establish the character of ρ_1 and ρ_2 if the double peak is real.

B. The 300-Mev Region

In the 300-Mev region of the neutral M_2 spectrum we observe a discontinuity in the experimental points, suggestive of the low-energy pion-pion interaction found by Abashian, Booth, and Crowe (ABC).[6] This is seen in Fig. 5. The discontinuity of the rise of the points occurs at 320 Mev, which is near the central value of the 300-Mev ABC peak. The absence of similar peaks in charge states 1 and 2 is also consistent with their assignment $T=0$ to this interaction. On the other hand, our data do not exclude the presence of such an effect in all these states. Without detailed knowledge of the phase-space "background" in that region, we do not know how to treat the data.

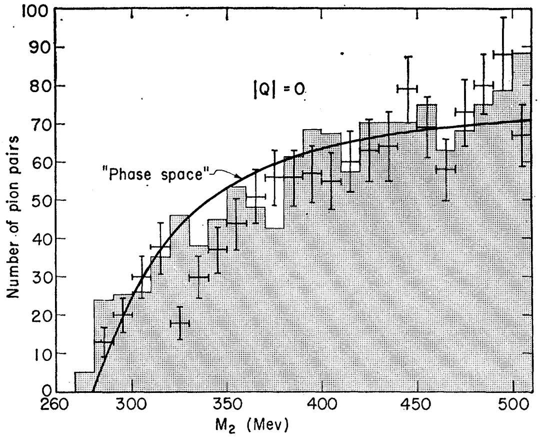

FIG. 5. Evidence for a discontinuity in the $Q=0$ M_2 distribution at 320 Mev. The shaded area is the spectrum $(|Q|=1)+(|Q|=2)$ for the fitted data.

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