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Diffusion of Slow Electrons in Gases

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The diffusion of slow electrons in hydrogen, nitrogen, carbon dioxide, methane, ethylene, and cyclopropane in uniform electric fields has been investigated for ratios of electric field to pressure from 0.2 to 5.0 (v/cm)/mm Hg. Such measurements lead to a determination of the ratio of electron drift velocity to diffusion coefficient. By assuming a distribution in velocity of the electrons in the swarm, the Townsend energy factor k_T and the mean electron velocity can be computed as a function of E/P, where E is the electric field and P is the gas pressure. Where the electron drift velocity is also known, the mean free path at unit pressure, the average energy loss per collision, and the gas kinetic cross section can be calculated. The results are presented in tabular form.

I. INTRODUCTION

INVESTIGATION of the lateral spreading by diffusion of a steam of electrons moving through a gas under the action of a uniform electric field can lead to a determination of the ratio of the electron drift velocity to the diffusion coefficient. For an assumed energy distribution function of the electrons in the swarm, the average agitational energy of the electrons and the mean velocity of agitation can be inferred. If the electron-drift velocity is known, values of the mean free path at unit pressure, the mean proportion of energy lost by the electron per collision, and the gas kinetic cross section can be calculated. The results of early investigations of this type have been summarized by Healey and Reed[1]; more recently, Huxley and Zaazou,[2] and Crompton and Sutton[3] have investigated the diffusion of electrons in H_2, N_2, and air. While these swarm experiments lead only to average values of the various parameters and while certain of the assumptions required for the interpretation of the data are more arbitrary than realistic, they still offer the most convenient means of investigation in the ranges of low electron energies (less than a few electron volts). In the present experiments the diffusion of electrons in N_2, H_2, CO_2, CH_4, C_2H_4, and C_3H_6 has been investigated for a range of $E/P=0.2$ to 5.0 (v/cm)/mm Hg, where E is the electric field strength and P is the gas pressure.

II. SUMMARY OF THEORY

The distribution of electrons in a steam moving in a steady state of agitational motion under the action of a uniform electric field E along the z axis is described by

$$\nabla^2 n = (w/K)\partial n/\partial z, \qquad (1)$$

where n is the electron density, w is the electron drift velocity along the direction of the field and K is the coefficient of diffusion of the electrons. In writing Eq. (1) it is assumed that the electron density n is sufficiently low that the mutual repulsion of the electrons may be neglected and that an equilibrium condition exists such that w/K is constant throughout the gas. This equation has been solved by Huxley and Bennett[4] for the arrangement shown in Fig. 1. Electrons already in a steady state of motion under the action of the uniform electric field E, enter the diffusion chamber of depth h through a small hole in the upper plate. After drifting through the gas of the chamber they are collected by the receiving electrode system consisting of a central disk of radius b and a surrounding annular electrode of outer radius c, the two being separated by a narrow gap. A guard ring surrounds these electrodes and all three elements are maintained at ground potential. If i_b is the electron current to the central disk and i_c is the current to the annular electrode of outer radius c, the ratio R of the currents is $i_b/(i_b+i_c)$ is given by

$$R=\frac{i_b}{i_b+i_c}=\frac{\{1-(h/d)\exp[-(w/2K)(d-h)]\}}{\{1-(h/e)\exp[-(w/2K)(e-h)]\}}, \qquad (2)$$

where $d^2=h^2+b^2$ and $e^2=h^2+c^2$. This solution is obtained under the boundary condition of zero concentration at $z=0$ (the upper electrode), except at the origin, and

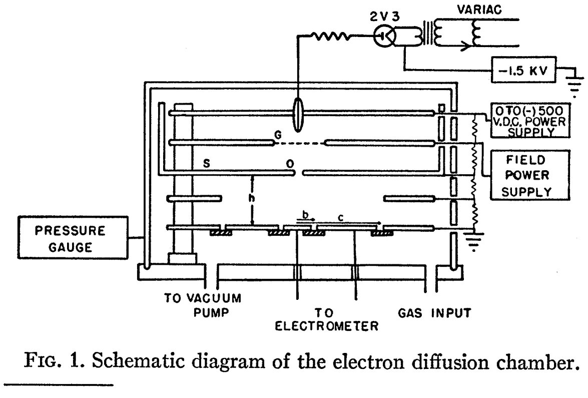

FIG. 1. Schematic diagram of the electron diffusion chamber.

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[1] R. H. Healey and J. W. Reed, *The Behavior of Slow Electrons in Gases* (Amalgamated Wireless Ltd., Sidney, Australia, 1941).

[2] L. G. H. Huxley and A. A. Zaazou, Proc. Roy. Soc. (London) **A196**, 402 (1949).

[3] R. W. Crompton and D. J. Sutton, Proc. Roy. Soc. (London) **A215**, 467 (1952).

[4] L. G. H. Huxley and F. W. Bennett, Phil. Mag. **30**, 396 (1940).

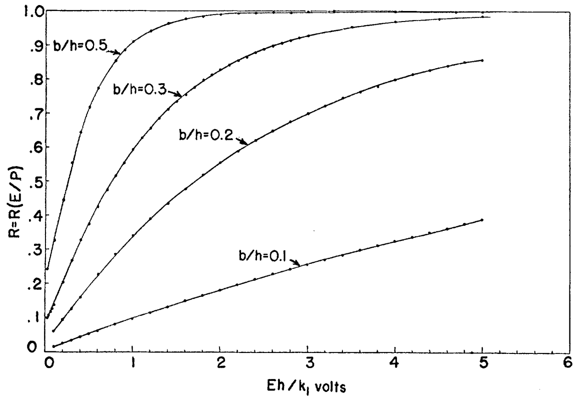

FIG. 2. Curves showing the electron current ratio R as a function of Eh/k_1 for several values of b/h.

over the entire electrode system at $z=h$. The electron source is taken to be a singlet source at the origin and the dimensions of the chamber are such that second- and higher-order terms in the series solution may be neglected.

Although the quantity determined from the experimental measurement of the current ratio is w/K, it is more common to write the value of R in terms of parameter k_1 related to the Townsend energy factor,

$$k_T=\frac{\text{average agitation energy of the electrons}}{\text{average energy of the gas molecules}},$$

by the relation,

$$k_1=Ak_T, \qquad (3)$$

where A is a constant depending on the energy distribution function of the electrons in the swarm. The average electron energy factor k_1 may be obtained from w/K as follows:

$$w/K=NeE/R_0Tk_1=38.92E/k_1, \qquad (4)$$

where N is Avogadro's number, e is the electronic charge, R_0 is the gas constant, E is the electric field in volts/cm, and $T=(273+25)$°K. This assumes that the drift velocity is small compared with the mean velocity of agitation of the electron, that the molecules may be considered as fixed elastic scattering centers with all directions of motion of the electrons equally probable after collision, and that the electron mean free path does not vary significantly with the electron agitation velocity. Equation (2) can be written

$$R=\frac{1-(1+(b/h)^2)^{-\frac{1}{2}}\exp\{-19.46(Eh/k_1)[(1+(b/h)^2)^{\frac{1}{2}}-1]\}}{1-(1+(c/h)^2)^{-\frac{1}{2}}\exp\{-19.46(Eh/k_1)[(1+(c/h)^2)^{\frac{1}{2}}-1]\}}. \qquad (5)$$

Values of R for a constant ratio of $c/h=1.5$ and for several values of the ratio b/h are plotted in Fig. 2.

While the distribution of velocities in the electron stream is not known, the two distributions most commonly considered are the classical Maxwellian distribution and the Druyvestyn distribution[5] which can be obtained from the solution of the Boltzmann transport equation for constant collision cross sections. Relations, derived by Huxley and Zaazou,[2] for the calculation of the Townsend energy factor, the root-mean-square velocity, the mean free path at unit pressure, the average energy loss per collision, and the gas kinetic cross section for these two velocity distributions are summarized in Table I.

III. APPARATUS AND METHOD

The diffusion chamber, shown schematically in Fig. 1, was constructed of brass and all interior surfaces were gold plated. The gaskets and insulator lead-throughs for the electrical connections were made of Teflon and the supporting posts for the electrodes and guard rings were made of fluorothene. Oil-diffusion pumps were used, and the system was checked with a helium-leak detector; when clean and tight it showed an out-gas rate of only 0.3 μ/hr. The depth h of the chamber was 3 cm and the radius c of the outer collector was 4.5 cm to the center of a 0.010-in. air gap. Central collecting electrodes of radii 0.3, 0.6, 0.9, and 1.5 cm to the center of a 0.005-in. air gap were used. Gas pressures were read with mechanical gauges calibrated against an oil manometer. The electric fields were supplied by well-regulated power supplies and the applied potentials were measured with a differential voltmeter calibrated against a standard cell.

Electron currents of the order of 2×10^{-12} amp to the collecting electrodes were measured by both the rate-of-drift method using a conventional vacuum tube electrometer and potentiometer and by a feedback electrometer having a grid current of about 3×10^{-15} amp. The electron source, located in the top of the chamber, was either a hot filament or a thin wire mounted in a quartz tube and supplied with a high voltage through a current-limiting circuit after the method of Huxley and Zaazou. In both methods, the currents were quite steady and the current ratios reproducible to 1%.

Commercial tank gases were used and efforts were made to eliminate electronegative contaminants since electron attachment to form negative ions of greatly different mobilities in the stream would lead to spurious results, The stated purity of the gases used was as follows: H_2—99.8%· N_2—99.99%, CO_2—99.956%, CH_4—99.0%, C_2H_4—99.5%, and C_3H_6—99.5%. The CO_2, CH_4, C_2H_4, and C_3H_6 were further purified by dis-

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[5] M. J. Druyvestyn, Physica 10, 69 (1930).

TABLE I. Quantities dependent on the distribution of velocities in the electron swarm. [The numerical factors are computed for w in cm/sec, E/P in (volts/cm)/mm Hg, and $T=298°\text{K}$.]

	Velocity distribution Maxwell	 Druyvestyn
Townsend energy factor, k_T	$k_T=k_1$	$k_T=0.875k_1$
Root-mean-square velocity, $\langle u^2\rangle^{\frac{1}{2}}$	$\langle u^2\rangle^{\frac{1}{2}}=1.16\times10^7k_1^{\frac{1}{2}}$	$\langle u^2\rangle^{\frac{1}{2}}=1.09\times10^7k_1^{\frac{1}{2}}$
Mean electron velocity, $\bar{u}$	$\bar{u}=1.07\times10^7k_1^{\frac{1}{2}}$	$\bar{u}=1.04\times10^7k_1^{\frac{1}{2}}$
Mean free path at unit pressure, L	$L=7.20\times10^{-9}wk_1^{\frac{1}{2}}/(E/P)$	$L=7.47\times10^{-9}wk_1^{\frac{1}{2}}/(E/P)$
Average energy loss per collision, η	$\eta=1.74\times10^{-14}w^2/k_1$	$\eta=2.14\times10^{-14}w^2/k_1$
Gas kinetic cross section, σ	$\sigma=4.26\times10^{-9}(E/P)/wk_1^{\frac{1}{2}}$	$\sigma=4.14\times10^{-9}(E/P)/wk_1^{\frac{1}{2}}$

TABLE II. Summary of results for hydrogen.

E/P $\left(\frac{\text{volts/cm}}{\text{mm Hg}}\right)$	w/K ($\text{cm}^{-1}\,\text{mm}^{-1}$)	Maxwellian $k_T=k_1$	Druyvestyn $k_T=0.875k_1$	$\bar{u}\times10^{-7}$ (cm/sec)	$w\times10^{-5}$ (cm/sec)	$L\times10^2$ (cm)	$\eta\times10^3$	$\sigma\times10^{16}$ (cm^2)
0.2	2.31	3.37	2.95	1.91	4.8	3.29	1.46	9.39
0.4	3.09	5.04	4.41	2.33	6.6	2.77	1.85	11.2
0.6	3.46	6.75	5.91	2.70	8.0	2.59	2.03	11.9
0.8	3.74	8.34	7.30	3.00	9.0	2.43	2.08	12.7
1.0	3.97	9.82	8.59	3.26	9.9	2.32	2.14	13.3
1.2	4.20	11.1	9.74	3.47	10.7	2.22	2.20	13.9
1.4	4.40	12.4	10.8	3.66	11.7	2.20	2.37	14.0
1.6	4.63	13.5	11.8	3.82	12.6	2.16	2.52	14.3
1.8	4.85	14.5	12.6	3.95	13.6	2.15	2.74	14.4
2.0	5.05	15.4	13.5	4.08	14.3	2.10	2.84	14.7
2.5	5.48	17.8	15.5	4.38	16.1	2.03	3.12	15.2
3.0	5.92	19.7	17.3	4.62	18.0	1.99	3.51	15.5
3.5	6.23	21.9	19.1	4.86	19.6	1.96	3.76	15.8
4.0	6.65	23.4	20.5	5.03	21.3	1.93	4.15	16.0
4.5	7.00	25.0	22.0	5.20	23.0	1.91	4.52	16.2
5.0	7.24	26.9	23.6	5.40	24.5	1.90	4.77	16.3

TABLE III. w/K data in nitrogen, carbon dioxide, methane, ethylene, and cyclopropane.

E/P $\left(\frac{\text{volts/cm}}{\text{mm Hg}}\right)$	w/K ($\text{cm}^{-1}\,\text{mm}^{-1}$) Nitrogen	 Carbon dioxide	 Methane	 Ethylene	 Cyclopropane
0.2	1.01	...	3.17	...	2.75
0.4	1.16	...	4.13	5.77	5.37
0.6	1.28	...	4.65	8.08	7.28
0.8	1.42	13.6	4.66	9.67	8.82
1.0	1.60	16.6	4.53	11.0	9.74
1.2	1.78	19.3	4.49	11.7	10.7
1.4	1.95	22.4	4.34	12.5	11.5
1.6	2.13	24.7	4.18	12.7	12.2
1.8	2.31	26.6	4.02	13.0	12.7
2.0	2.48	28.5	3.92	13.5	13.1
2.5	2.85	31.3	3.71	13.4	14.1
3.0	3.30	32.8	3.46	12.8	14.5
3.5	3.61	31.6	3.40	12.6	14.9
4.0	4.04	29.2	3.24	12.1	15.3
4.5	4.38	25.3	3.27	11.7	15.4
5.0	4.76	21.7	3.19	11.5	15.4

tillation from traps cooled to liquid nitrogen temperature. The hydrogen was purified by passage through a commerical unit (Deoxo Purifier, Baker and Co., East Newark, New Jersey) to catalytically combine the oxygen with hydrogen to form water, then through a $Mg(Cl—O_4)_2$-filled drying tube and metal traps cooled with liquid nitrogen. The nitrogen was passed through traps cooled to liquid nitrogen temperatures. Other experiments[6] in this laboratory had previously shown that these procedures were adequate to remove all measurable traces of electronegative contaminants from the gases listed above.

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[6] G. S. Hurst and T. E. Bortner, Phys. Rev. **114**, 116 (1959).

IV. RESULTS

The results are summarized in Tables II and III. Experimentally determined values of w/K and sample calculations of related parameters in hydrogen are given in Table II while w/K data for the other gases are summarized in Table III. The Townsend energy factor has been calculated for assumed velocity distributions of both the Maxwellian and Druyvestyn types. The mean electron velocity $\bar{u}$, mean free path at unit pressure L, average energy loss per collision η, and gas kinetic cross section σ are calculated for the Druyvestyn distribution only. These parameters for gases other than hydrogen may be obtained using the relations in Table I. The values of drift velocity w used in the calculation of L, η, and σ are the data of Nielsen and Bradbury.[7] The results for H_2 and N_2 are in good agreement with those of Crompton and Sutton.[3] Values of the Townsend energy factor k_T for CO_2 are slightly larger than those obtained by Bailey and Rudd[8] and by Skinker,[9] with corresponding changes in the other parameters. At the lower values of E/P, the values of k_T are larger in C_2H_4 than those obtained by Bannon and Brose.[10] CH_4 and C_3H_6 had not been investigated previously.

ACKNOWLEDGMENTS

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[9] M. F. Skinker, Phil. Mag. **44**, 994 (1922).

[10] J. Bannon and H. L. Brose, Phil. Mag. **6**, 817 (1928).