Theory of a P-Wave π-Λ Resonance

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It is shown that a $P_{3/2}$ π-Λ resonance can conceivably occur as a consequence of the $P_{3/2}$ pion-nucleon resonance and the fact that the Λ hyperon can dissociate virtually into a $\bar{K}$ meson and a nucleon. Specific pion-hyperon interactions are not considered.

IN this note we present a calculation which suggests the existence of a P-wave π-Λ resonance at low energies *independent* of the nature of the Yukawa couplings of pions to Λ and Σ hyperons (provided that the latter do not exist with such large effective couplings as to invalidate the resonance mechanism discussed below). We consider this result significant in the light of the various conflicting evidences concerning the validity or invalidity of the original conjectures[1,2] on the nature of the relative Σ-Λ parity. The odd-parity conjecture[2] has some experimental support from photoproduction experiments,[3] from experiments on the nature of the 400-Mev/c $\bar{K}$-N anomaly,[4] and *perhaps* from associated production experiments.[5] On the other hand, the apparent wealth of pion-hyperon resonant states, as well as certain interpretations[6] of the nature of the hyperon-nucleon interaction, suggest even parity. It has been suggested[7,8] that, barring a direct experimental determination of the relative Σ-Λ parity, an important clue to this parity would be contained in an experimental determination of the spin of the Y_1^*, the π-Λ resonance at 1385 Mev.[9] Odd relative Σ-Λ parity and the bound-state model of the sigma hyperon[10] would

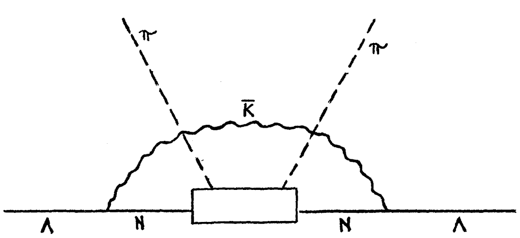

FIG. 1. General Feynman graph for π-Λ scattering according to the mechanism discussed in the text.

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[2] S. Barshay, Phys. Rev. Letters **1**, 97 (1958).
[3] F. Turkot, *Proceedings of 1960 International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 369.
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[6] R. H. Dalitz (to be published).
[7] S. Barshay and H. Pendleton, Phys. Rev. Letters **6**, 421 (1961).
[8] D. Amati, B. Vitale, and A. Stanghellini, Phys. Rev. Letters **5**, 524 (1960).
[9] M. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wozcicki, Phys. Rev. Letters **5**, 520 (1960).
[10] S. Barshay and M. Schwartz, Phys. Rev. Letters **4**, 618 (1960).

imply (in a reasonable potential model) an $S_{1/2}$ π-Λ scattering phase shift starting at π at zero energy and rotating down through $\pi/2$. Such a behavior would not, in general, appear as a narrow resonance. However, in the vicinity of the $\bar{K}$-N threshold, the reaction of this (closed) channel upon the π-Λ channel can induce a narrow resonant structure in the $S_{1/2}$ phase shift which includes both "potential" and reactive effects, as first demonstrated by Dalitz and Tuan.[11] An effective-range analysis of low-energy $\bar{K}$-N interactions[12] can accommodate such a situation, but this is but one of many accommodations. The existence of some such state would be useful in a dynamical theory of the pionic decay of Σ hyperons along the lines of the bound-state model.[13] On the other hand, Amati *et al.*[8] have demonstrated that a γ_5 coupling (even relative Σ-Λ parity) of pions to Λ and Σ hyperons can, in a nonrelativistic model with neglect of K-meson and pion-nucleon effects, lead to a $P_{3/2}$ π-Λ resonance.

We consider here a model in which (a) a Λ can dissociate into a $\bar{K}$ meson and a nucleon, and (b) a P-wave pion interacts in a resonant manner with a nucleon as described by a Chew-Low effective-range formulation.[14] We demonstrate that this mechanism can conceivably lead to a narrow low-energy P-wave π-Λ resonance in the state with total angular momentum 3/2, *independent* of the relative Σ-Λ parity.

The general Feynman graph under consideration is shown in Fig. 1. With pseudovector coupling between K meson, Λ, and nucleon, and with neglect of baryon recoil in the intermediate states, we can write the transition operator in the spin space of the Λ hyperon, as follows:

$$M(\mathbf{k},\mathbf{k}',\omega')=\frac{\frac{4}{3}f_K^2}{m_K^2}\int\frac{d\mathbf{l}\,\boldsymbol{\sigma}\cdot\mathbf{l}\,T_k(\mathbf{k}',l)\,\boldsymbol{\sigma}\cdot\mathbf{l}}{(2\pi)^3(2E)(E-\delta)^2},\qquad(1)$$

where f_K is the rationalized effective pseudovector coupling for $\Lambda\rightarrow N+\bar{K}$, m_K is the K-meson mass; $\boldsymbol{\sigma}$ is the Pauli spin operator; $E=(l^2+m_K^2)^{1/2}$; δ is the lambda-nucleon mass difference; and $T_k(\mathbf{k}',l)$ is a P-wave pion-nucleon scattering operator which takes a pion of momentum $\mathbf{k}$ into a pion of momentum $\mathbf{k}'$. We have considered only isotopic spin 3/2 for the pion-

[11] R. H. Dalitz and S. F. Tuan, Ann. Phys. **10**, 307 (1960).
[12] M. Ross and G. Shaw, Phys. Rev. (to be published).
[13] S. Barshay, Ann. Phys. (to be published).
[14] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

nucleon system and this is the origin of the factor of 4/3 when one notes that the Λ can dissociate into both K^--p and $\bar{K}^0$-n systems. We now consider only that part of the $T_{\mathbf{k}}(\mathbf{k}',l)$ which corresponds to the pion-nucleon system in a $P_3/_2$ state. We write

$$T_{\mathbf{k}}(\mathbf{k}',l)=T_k(k',l)P_{3/2}(\hat{k},\hat{k}'), \tag{2}$$

where the $P_{3/2}$ projection operator is defined in terms of the unit vectors $\hat{k}$ and $\hat{k}'$ by

$$P_{3/2}(\hat{k},\hat{k}')=2\hat{k}\cdot\hat{k}'-i\boldsymbol{\sigma}\cdot\hat{k}'\times\hat{k}. \tag{3}$$

Inserting (2) into (1) and moving the $\boldsymbol{\sigma}\cdot\mathbf{l}$ on the left through $T_{\mathbf{k}}(\mathbf{k}',l)$, using the commutation properties of the components of $\boldsymbol{\sigma}$, we get

$$M(\mathbf{k},\mathbf{k}',\omega')=\frac{4(f_K^2/4\pi)}{3\pi m_K^2}\int\frac{l^4dl\, T_k(k',l)}{E(E-\delta)^2}\times\left(\frac{10}{9}P_{\frac{3}{2}}(\hat{k},\hat{k}')-\frac{2}{9}P_{\frac{1}{2}}(\hat{k},\hat{k}')\right), \tag{4}$$

where the $P_{1/2}$ projection operator is defined by

$$P_{1/2}(\hat{k},\hat{k}')=\hat{k}\cdot\hat{k}'+i\boldsymbol{\sigma}\cdot\hat{k}'\times\hat{k}. \tag{5}$$

We now write down the equation satisfied by $T_k(k',l)$ in the *one-pion approximation.*

$$T_k(k',l)=-B_k(k',l)-\sum_q\left(\frac{\langle T_{\mathbf{k}'}{}^{\dagger}(\mathbf{q})T_{\mathbf{k}}(\mathbf{q})\rangle}{-\omega'+\omega_q+E-\delta-i\eta}+\frac{\langle T_{\mathbf{k}}{}^{\dagger}(\mathbf{q})T_{\mathbf{k}'}(\mathbf{q})\rangle}{\omega'+\omega_q+E-\delta}\right), \tag{6}$$

where

$$B_k(k',l)=\frac{2kk'}{3(\omega\omega')^{\frac{1}{2}}}\frac{1}{\omega'+E-\delta}\frac{f^2}{\mu^2}.$$

In these equations $\omega'=(k'^2+\mu^2)^{\frac{1}{2}}$, $\omega=(k^2+\mu^2)^{\frac{1}{2}}$, $\omega_q=(q^2+\mu^2)^{\frac{1}{2}}$ with μ the pion mass; the sum on q denotes a sum over isotopic indices as well as an integral over the three-momentum $\mathbf{q}$; the symbols $\langle\cdots\rangle$ denote that the function which is the coefficient of the isotopic spin 3/2, angular momentum 3/2 projection operator after the sum over q is performed is to be taken; f is the rationalized renormalized pion-nucleon pseudovector coupling constant; and finally the dagger denotes Hermitian adjoint. The validity of Eq. (6) can most readily be demonstrated by expanding the right-hand side by iteration of the Born approximation, $B_k(k',l)$, and then inserting this series into Eq. (4) and identifying the result term by term with the perturbation theory series (with exclusion of renormalization graphs) for all Feynman graphs contained in the one-pion approximation to the general graph of Fig. 1.

Consider the coefficient of $P_{3/2}(\hat{k},\hat{k}')$ in Eq. (4) and define a new function $h(\omega')$ as follows:

$$\frac{40(f_K^2/4\pi)}{27\pi m_K^2}\int\frac{l^4dl\, T_k(k',l)}{E(E-\delta)^2}\cong\frac{-4\pi kk'}{2(\omega\omega')^{\frac{1}{2}}}h(\omega'), \tag{7}$$
$$h(\omega')=e^{i\delta(k')}\sin\delta(k')/(k')^3,$$

where $\delta(k')$ is the π-Λ scattering phase shift in the $P_{3/2}$ state. The function $h(\omega')$ has cuts from $\omega'=-m_K+\delta$ to $-\infty$ and from $m_K+\mu-\infty$ to s; the function $h(\omega')$ goes like $1/\omega'$ as $\omega'\to\infty$ and further it can be shown that crossing symmetry $-M(\mathbf{k}',\mathbf{k},-\omega')=M(\mathbf{k},\mathbf{k}',\omega')$, implies $h(-\omega')\cong\frac{1}{3}h(\omega')$. We can therefore write

$$h(\omega'+i\eta)=\frac{1}{\pi}\int_{m_K-\delta}^{m_K+2\mu-\delta}\frac{G(x)dx}{x+\omega'}+\frac{1}{\pi}\int_{m_K+2\mu-\delta}^{\infty}\frac{G(x)dx}{x+\omega'}+\frac{1}{\pi}\int_{m_K+\mu-\delta}^{\infty}\frac{F(x)dx}{x-\omega'-i\eta}, \tag{8}$$

where $G(x)$ and $F(x)$ are suitable weight functions given in terms of the discontinuities of $h(\omega')$ across the left and right cuts, respectively. We now approximate the first term on the right of Eq. (8) as follows:

$$\frac{1}{\pi}\int_{m_K-\delta}^{m_K+2\mu-\delta}\frac{G(x)dx}{x+\omega'}\cong\frac{c}{x+M}$$

with

$$c=\frac{1}{\pi}\int_{m_K-\delta}^{m_K+2\mu-\delta}G(x)dx, \tag{9}$$

and

$$M\cong m_K-\delta+\mu.$$

We have replaced this small piece of the cut by a pole at its midpoint. In this approximation the quantity c can be estimated in terms of the residue of the Born approximation for $h(\omega'+i\eta)$ at the pole. We obtain

$$c\cong\left(\frac{f_K^2}{4\pi}\right)\left(\frac{f^2}{4\pi\mu^2}\right)\left(\frac{40}{27}\right)\left(\frac{4}{3}\right)\frac{1}{m_K^2}\int_0^{l_m}\frac{l^4dl}{E(E-\delta)^2}, \tag{10}$$

where $l_m\cong(18\mu^2)^{\frac{1}{2}}$. It is most important to note at this point that c contains *two* small multiplicative factors, $(f^2/4\pi)\sim0.08$ and $(f_K^2/4\pi)\sim0.35$ (for an effective pseudoscalar coupling between K meson, Λ, and nucleon of $(g^2/4\pi)\sim5$). A very approximate estimate of the integral in (10) gives

$$c\sim2.3(f_K^2/4\pi)(f^2/4\pi\mu^2). \tag{11}$$

At this point we may proceed to follow precisely the development of the effective-range formulation of P-wave pion-nucleon scattering.[14,15] Define a function

$$g(\omega'+i\eta)=g(z)=\frac{c}{(z+M)h(z)}. \tag{12}$$

[15] One simply constructs a unitary amplitude (in the elastic scattering approximation) from what is essentially the Born approximation, Eq. (7).

This function satisfies the following equation:

$$g(z)=1+\frac{(z+M)}{c\pi}\int_{\mu}^{\infty}dx\left(\frac{\rho_1(x)}{x-z}+\frac{\rho_2(x)}{x+z}\right), \quad (13)$$

with

$$\rho_1(x)=-c^2q^3/(M+x)^2,$$

where $x^2=q^2+\mu^2$, and

$$\rho_2(x)\cong\frac{-c^2q^3}{(M+x)^2}\left|\frac{g(-x-i\eta)}{g(x+i\eta)}\right|^2.$$

From Eqs. (7) and (12) we have

$$\mathrm{Re}g(\omega')=\frac{c(k')^3\cot\delta(k')}{\omega'+M}. \quad (14)$$

Using Eqs. (13) and (14) with approximation of the "crossed" term, we have

$$\frac{c(k')^3\cot\delta(k')}{\omega'+M}\cong 1-\frac{(M+\omega')}{\pi}P\int_{\mu}^{x_m}2cq^3dx/(x+M)^2(x-\omega')\cong 1-(M+\omega')r, \quad (15)$$

where P denotes the principal value, x_m is some cutoff, and r is an approximately energy-independent effective range given by[16]

$$\pi r\sim P\int_{\mu}^{x_m}2cq^3dx/x(x+M)^2. \quad (16)$$

This should be compared immediately with the (3,3) effective-range of Chew-Low theory[14]:

$$\pi r_3\sim P\int_{\mu}^{x_m}4\left(\frac{f^2}{4\pi\mu^2}\right)\frac{q^3dx}{x^3}\cong\left(\frac{1}{2\cdot 2\mu}\right)\pi. \quad (17)$$

Inserting (11) into (16) and comparing with (17), we find $r<r_3$ as a *direct consequence* of $(f_K^2/4\pi)<1$. Equation (15) thus indicates that a low-energy resonance could develop in the $P_{3/2}$ π-Λ state. Indeed, if we identify such a state with the Y_1^* at $\omega'\sim 270$ Mev, we obtain $r\sim 1/5.25\mu$ which is decidedly smaller than r_3. We may go even further and make a direct comparison of the width of a resonant state described by Eq. (15), Γ_Λ, to the width of the (3,3) pion-nucleon resonance, Γ_N. We obtain

$$\frac{\Gamma_\Lambda}{\Gamma_N}=\frac{cq_\Lambda^3}{\frac{4}{3}(f^2/4\pi\mu^2)q_N^3}, \quad (18)$$

where q_Λ and q_N are the center-of-mass momenta at resonance in the π-Λ and π-N systems, respectively. We see that, even for comparable q_Λ and q_N, the fact that $c/(f^2/4\pi)\propto(f_K^2/4\pi)<1$ gives immediately a *narrowing* of the π-Λ resonance. With a value of $r/r_3\sim c/(f^2/4\pi\mu^2)\sim 1/2.4$, Eq. (18) gives $\Gamma_\Lambda\sim 0.5\Gamma_N$. In this model there is a direct connection between the smaller P-wave effective-range for π-Λ scattering as compared to π-N scattering and the narrowing of the π-Λ resonance; both are connected to the smallness of the effective pseudovector coupling of K mesons to Λ's and nucleons. From Eq. (4) we see immediately that the effective range in the $P_{1/2}$ π-Λ state is negative and therefore the corresponding effective-range formula implies no resonance in this state.[17]

We would like to summarize. The entire derivation is crude, but Eq. (15) does suggest that a low-energy π-Λ resonance in the $P_{3/2}$ state can arise as a consequence of the well-known pion-nucleon resonance and the fact the Λ can dissociate virtually into a $\bar{K}$ meson and a nucleon. A modified effective-range formula [Eq. (15)] can be applied; the effective-range is smaller than in the pion-nucleon case and the width of the π-Λ resonance is narrower than the π-N resonance. Both of these facts follow simply from the fact that the effective pseudovector coupling of K mesons to Λ's and nucleons is likely to be less than unity (if the K were scalar relative to ΛN, the scalar coupling would also likely be less than unity and the calculation goes through with small modification). Direct pion-hyperon couplings have been neglected. Therefore the model is the logical opposite of that of Amati *et al.*,[8] in which a π-Λ $P_{3/2}$ resonance is produced as a consequence of special forms of Yukawa couplings of pions to Λ's and Σ's, with neglect of K-meson and pion-nucleon effects. It would seem that since a π-Λ $P_{3/2}$ resonance may be expected on the basis of Eq. (15), its existence may not be a clean indication of the validity of certain models of direct pion-hyperon couplings. The other experimental data supposedly bearing on the nature of these couplings is that concerning the hyperon-nucleon interaction.[6] It is not clear that the theory of these processes can distinguish between direct coupling mechanisms and strong 2π-Λ interactions arising from the general graph of Fig. 1. We would say that a *direct* experimental determination of the relative Σ-Λ parity must still be viewed as crucial in pointing the way to fruitful theories of pion-hyperon interactions.

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[16] We have estimated the contribution to r from the cut in $T_{\mathbf{k}}(\mathbf{k}',l)$ arising from the (3,3) pion-nucleon isobar. Since this represents an additional attraction, r tends to increase. This increase is less than $\sim$10% because of the relatively narrow width of the isobar.

[17] One can show that inclusion of all of the Born terms in $T_{\mathbf{k}}(\mathbf{k}',l)$ does not alter the conclusion as to the absence of a $P_{\frac{1}{2}}$ resonance.