

Electron-Proton Scattering at Low Momentum Transfers

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We have measured the electron-proton scattering cross section at 248.9 Mev, 104.81°; 209.6 Mev, 149.75°; and 139.3 Mev, 104.19°. We find the following values: $F_1=0.767\pm 0.025$, $F_2=0.707\pm 0.028$, and $F_1/F_2=1.085\pm 0.025$ at $-q^2=2.98 \text{ f}^{-2}$. $F=0.902\pm 0.011$ at $-q^2=1.05 \text{ f}^{-2}$. The last result agrees with previous measurements. The others are new contributions.

INTRODUCTION

ELECTRON-PROTON scattering in so far as the Born approximation is valid, can be shown to depend upon two form factors, $F_1(q^2)$ and $F_2(q^2)$, which are functions of the 4-momentum transfer q alone.¹ This follows from simple invariance properties. It becomes an experimental goal to determine these form factors precisely over as large a range of momentum transfer as possible. The early Stanford measurements^{2,3} were consistent not only with this invariance property but also with the special assumption $F_1(q)=F_2(q)$. With the high-energy accelerators now available, these measurements have recently been extended to higher momentum transfers.⁴⁻⁸ A clear separation of F_1 and F_2 has been found with $F_1/F_2\approx 4$ at $-q^2=25 \text{ f}^{-2}$.

These experiments, and companion experiments with neutrons, have stimulated theoretical interpretation of the form factors. Thus Bergia *et al.*⁹ have compared the form factors with those of a (subtracted) dispersion theory with π - π resonances in $T=0$ and $T=1$ states. Bergia predicts that F_1 should be appreciably different from F_2 even at $-q^2=3 \text{ f}^{-2}$. These theoretical speculations were the stimulus for this work, which measures F_1/F_2 at $-q^2=2.98 \text{ f}^{-2}$ and measures absolutely F_1 at $q^2=-2.98$ and 1.05 f^{-2} and F_2 at $q^2=-2.98 \text{ f}^{-2}$ to a higher precision than heretofore. In particular we measured at: 248.9 Mev, 104.81° (nominally 250 Mev, 105°); 209.6 Mev, 149.75° (nominally 210 Mev, 150°); and 139.3 Mev, 104.19° (nominally 140 Mev, 104°). The first two give the same value of q^2 but contain

different mixtures of the form factors F_1 and F_2 ; at 105°, F_1 predominates and at 150° F_2 predominates.

METHOD

The principle of the method is shown in Fig. 1. The electron beam of the linear accelerator of the Faculty of Sciences of the University of Paris is passed through a hydrocarbon target; the intensity of the transmitted electrons is measured. The scattered electrons at an angle θ pass through an aperture of known area, which defines the solid angle, and then are bent by a spectrometer magnet to reach a Čerenkov counter. The aperture and the spectrometer are so adjusted that all electrons passing through the aperture enter the counter, provided that they have not lost more than 2.15% of their energy by radiation. Under these circumstances the calculation of the cross section becomes very simple, if the efficiency of the counter is known.

This method contrasts with that used in references 2, 3, 5, 6, and 7 in which the spectrometer was adjusted to give a moderate resolution, and the counting rate was studied as a function of spectrometer field. A peak was obtained at the energy for elastic electron-proton scattering, with a tail of lower energy electrons corresponding to those collisions where radiation has occurred. The cross section is obtained by integrating under the peak and dividing by the width of the spectrometer energy defining slit and dispersion constant. We will call this method the "peak area" method.

With our method such a curve of counting rate versus field gives a flat top. This has been used before in reference 4 and in this laboratory for the scattering of electrons from oxygen nuclei.¹⁰ It has, we believe, several

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¹ E.g., S. D. Drell and F. Zachariasen, *Electromagnetic Structure of Nucleons* (Oxford University Press, New York, 1961).

² E. E. Chambers and R. Hofstadter, *Phys. Rev.* **103**, 1454 (1956).

³ R. Hofstadter, *Ann. Rev. Nuclear Sci.* **7**, 231 (1957).

⁴ L. Hand, *Phys. Rev. Letters* **5**, 168 (1960).

⁵ F. Bumiller, M. Croissiaux, E. Dally, and R. Hofstadter, *Phys. Rev.* **124**, 1623 (1961).

⁶ R. R. Wilson, K. Berkelman, J. M. Cassels, and D. N. Olsen, *Nature* **188**, 94 (1960).

⁷ D. N. Olson, H. F. Schopper, and R. R. Wilson, *Phys. Rev. Letters* **6**, 286 (1961).

⁸ R. M. Littauer, H. F. Schopper, and R. R. Wilson, *Phys. Rev. Letters* **7**, 141 (1961).

⁹ S. Bergia, A. Stanghellini, S. Fubini, and C. Villi., *Phys. Rev. Letters* **6**, 367 (1961).

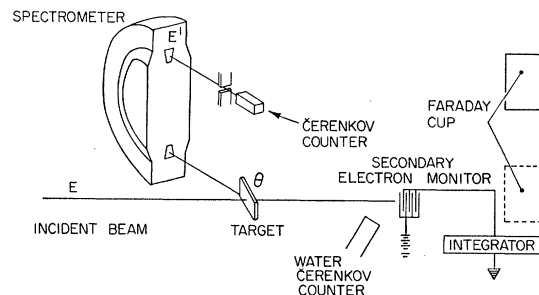


FIG. 1. Experimental arrangement.

¹⁰ F. Lacoste and G. R. Bishop, *Nuclear Phys.* **26**, 511 (1961).

TABLE I. Corrections and errors.

	140 Mev	250 Mev	F_1/F_2
Monitoring, scattering out, secondaries, etc.	0.996 ± 0.006	0.998 ± 0.005	1.004 ± 0.001
Target thickness angle, carbon subtraction	1.000 ± 0.006	1.000 ± 0.005	1.000 ± 0.002
Aperture Penetration, slit scattering, fringing field	0.997 ± 0.003	0.997 ± 0.003	1.000 ± 0.000
Angle measurement	1.000 ± 0.002	1.000 ± 0.0015	1.000 ± 0.002
Counter efficiency	1.010 ± 0.003	1.007 ± 0.003	1.000 ± 0.003
Profile check	1.002 ± 0.002	1.002 ± 0.002	1.000 ± 0.002
Count rate corrections	1.000 ± 0.001	1.000 ± 0.001	1.000 ± 0.001
Energy calibration			
Relative [$\delta E/E = \pm 0.15\%$]			1.000 ± 0.009
Absolute [$\delta E/E = \pm 0.5\%$]	1.000 ± 0.012	1.000 ± 0.016	1.000 ± 0.007
Radiative corrections			
Real bremsstrahlung	1.033 ± 0.001	1.028 ± 0.001	1.000 ± 0.000
Schwinger effect	1.158 ± 0.003	1.172 ± 0.004	0.994 ± 0.002
Statistics	1.000 ± 0.014	1.000 ± 0.007	1.000 ± 0.021
Over-all	1.202 ± 0.021	1.210 ± 0.021	0.999 ± 0.024

advantages when one is studying a well-known process. Firstly, the slit width and dispersion constant do not have to be known. Secondly, electrons, after passing through the aperture, cannot strike the side of the vacuum chamber or the energy-defining slit if they come from the process under study. Small-angle scattering from the vacuum chamber and the spectrometer slits do not, therefore, contribute to the counting rate. Thirdly, small machine-energy or spectrometer-stabilization changes do not affect the "flat top" method. If, however, the machine energy changes 0.1% during the scan of a 1% wide peak, an error in peak area is found of 10%. For the flat-top method, machine-energy changes of 0.1% produce 0.3% change in cross section, which with our parameters is approximately cancelled by an opposite correction for the radiative tail. A considerable improvement in accuracy is therefore expected.

Various corrections are necessary to the simple calculation of the form factors from the Rosenbluth formula. These are itemized in Table I and discussed in the sections below.

MONITORING

The basic monitoring device is a Faraday cup. This was constructed following earlier designs,^{11,12} 99.9% of the shower produced by an electron is stopped in the cup. Low-energy secondaries from the entrance foil are inhibited by a permanent magnet.

Unfortunately, it was not possible to bias the cup to check on the collection of these secondaries as is usual in checks on Faraday cups. However, in a cup of similar design at a similar electron energy¹² it has been found that such secondaries are less than 0.2% of the incident beam. The cup here is designed a little more conservatively so we take 0.2% as the possible error in monitoring.

When the Faraday cup is in position, neutrons cause too large a background in the Čerenkov counter. Therefore the Faraday cup has been used for frequent cali-

brations of a secondary emission monitor.¹³ These have been made without a target in place. Small corrections (0.3%) are necessary for losses by scattering in the exit foil of the machine vacuum pipe and the secondary emitter foils.

The monitor efficiency varies with energy; in particular it is about 1% less efficient at 210 Mev than at 250 Mev. There were also fluctuations of up to 1% during the course of this experiment, and a change of 0.5% with a change of a factor of 20 in beam current. These observations agree with those of Bumiller.¹⁴ Accordingly calibrations were made frequently, before, after, and during each batch of runs, if possible without stopping the machine, and at the same beam intensity as was used in the experiment.

When experimental runs were made, the target scattered the beam to cover an area about 3 times as large as that when the target was not in place. It is thus possible that inhomogeneities of the monitor could vitiate our results. This has been checked by two methods. First, the beam was displaced across the monitor two centimeters, independent of the direction in which this was done, the apparent monitor efficiency *increased* by 2%. It is believed that at least half of this is due to an increase in the scattering out of the Faraday cup. An occasional monitor calibration was made with the target in place with an apparent *increase* of 1% in the monitor efficiency, due also to an increase in the scattering out.

Accordingly we assign an error of 0.5% to the monitoring in addition to a correction of (0.2 ± 0.1)% for scattering out.

The charge collected was measured by a slide-back voltmeter and a standard condenser. The condenser has a polyethylene dielectric which shows no stored charge effects up to 0.1%. Its capacity was measured on a General Radio Bridge at 1000 cps to an accuracy of 0.2%. The voltage was measured on a digital voltmeter to 4 significant figures and calibrated by a standard cell. A small correction (1/A) is necessary for the finite amplification of the slide-back amplifier. Small drifts were observed, but these were of random sign.

TARGET

A polyethylene target was oscillated up and down. This served two purposes; first, it prevented the thermal and radiative decomposition of the target and second, the target thickness was an average over a reasonably large area. The target was placed in a helium atmosphere. This reduced the scatterings and background as compared with air, and increased the cooling of the target as compared with vacuum. There were no problems with secondary processes, such as back scattering from vacuum-chamber walls of forward-scattered electrons.

¹¹ K. L. Brown and G. W. Tautfest, Rev. Sci. Instr. **27**, 696 (1956).

¹² G. W. Tautfest and W. K. H. Panofsky, Phys. Rev. **105**, 1356 (1957).

¹³ G. W. Tautfest and R. H. Fechter, Rev. Sci. Instr. **26**, 229 (1955).

¹⁴ F. Bumiller, Berkeley Conference on Instrumentation for High Energy Physics, 1960.

The carbon background, about $\frac{1}{4}$ of the total, was measured from a carbon target of almost equal number of carbon atoms. Corrections for the different radiative effects, energy loss, and average scattering energy of the carbon target compared with the polyethylene are less than 0.1% of the hydrogen counts and were neglected. An error of 0.2% is assigned to the subtraction.

The target was placed at an angle of $37.8 \pm 0.2^\circ$ to the incident beam. The angle was chosen to equalize the losses for the two scattering angles (105° and 150°). An error of 0.5% is assigned to thickness and angle measurements.

Counts with no target in place were less than 0.5% of the whole, and were corrected by the carbon subtraction.

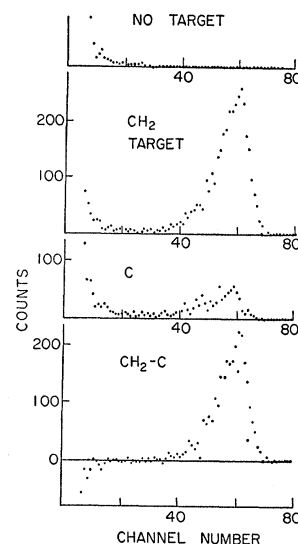
DEFINING APERTURE

The defining aperture was of tungsten 1-cm thick in the direction of the beam. Errors can arise in three ways. First, electrons losing less than 2.15% of their energy (the acceptance of the spectrometer) in passing through the slit edges, could still be counted. This penetration occurs over 1 mm of the slit and could increase the area by 0.1%. Secondly, electrons striking the face can scatter out again and pass through the spectrometer. On another occasion it would be better to reduce the thickness of the tungsten to 2 mm to avoid this effect. The solid angle was increased by an estimated $(0.2 \pm 0.2)\%$ by this. Thirdly the fringing field at the edges of the spectrometer increases the solid angle slightly by $(0.3 \pm 0.1)\%$.

It is important to be sure that all electrons passing through the aperture pass through the spectrometer and reach the counter. This was checked in two ways. First, a fluorescent screen was placed in the counter position. The vertical direction of the incident electron beam was changed by a magnet placed at the target position and the spectrometer magnet was moved from side to side. The entrance face of the spectrometer could thus be explored with the electron beam for different spectrometer currents. The aperture was chosen to be within the acceptance region. We note that the solid angle (0.0038 sr) is less than the maximum allowed by the spectrometer vacuum chamber (0.008 sr). This gives a margin of safety to avoid spectrometer aberrations and scattering on the walls of the vacuum chamber.

Secondly, the aperture was tested by scattering of electrons from carbon at 180 Mev and 70° . The profile of counting rate versus spectrometer current was plotted using the full aperture and also using the top 6% and the bottom 11% of the aperture. The profiles had the same shape and were normalized to the full area by the ratios of the areas to an accuracy of 0.5% of the total. We conclude that the spectrometer accepts all electrons that pass through the aperture, even in the extreme edge. In addition the slit penetration and scattering corrections are verified to 0.5%.

FIG. 2. Pulse-height distribution from the Čerenkov counter.



ANGLE MEASUREMENT

The beam direction was measured by two fluorescent screens 180 cm apart viewed by television cameras. The direction agreed with the nominal direction to $\pm 0.015^\circ$. The center of rotation of the spectrometer was found by a plumb line and theodolite to 0.03 cm. The target was placed in this position to 0.02 cm. The over-all angular precision is believed correct to $\pm 0.03^\circ$, and any deviation from perfect alignment tends to occur in the same direction for both the angles chosen.

Corrections to the measurement of angles arise from an error (0.15°) in the zero of the spectrometer scale; and scattering out of the horizontal plane due to the finite vertical extent of the defining aperture. Random variations of angle were obtained during the runs by beam direction changes. These were checked every half hour by a fluorescent screen, and averaged $\pm 0.05^\circ$ for each run.

COUNTER EFFICIENCY

The Čerenkov counter was a $10 \times 7 \times 10$ cm piece of Lucite fastened to a photomultiplier type 56 AVP. The pulses passed through a linear gate¹⁵ and then to pulse height analyzer. The pulse height spectrum of pulses from hydrogen scattering gave a peak with a small tail at low pulse heights. At the same energy the pulses from inelastic carbon scattering gave larger tails, because some of the electrons could scatter from, or penetrate the energy defining slit. The spectrum after subtraction of the carbon counts was analyzed to give the efficiency of the Čerenkov counter. These spectra are shown in Fig. 2.

In such a Čerenkov counter, the process giving a pulse less than the full amount are, radiation, scattering out, and bad light collection. None of these processes is

¹⁵ F. Lacoste, Laboratoire de l'Accélérateur Linéaire, Orsay Rapport LHE 7 (unpublished).

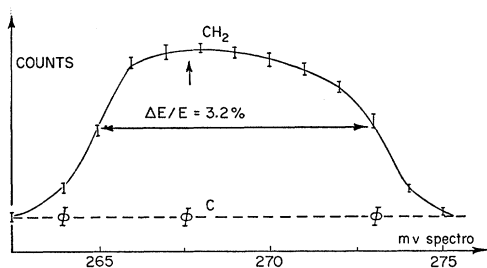


FIG. 3. Counting rate as a function of spectrometer setting. The arrow indicates the point where the statistical value was taken.

expected to give zero pulse height; accordingly we estimate the efficiency of the counter by extrapolating the pulse-height distribution to zero pulse height under the noise counts. Two extrapolation curves seem reasonable. In one, the tail is extrapolated linearly to zero pulse height, and in the other it is extrapolated at constant count-rate-per-pulse-height interval. The area under the whole curve is then taken to be 100% and the error taken from these two extremes.

It is interesting to note that if we were measuring the cross section by the peak area method, with our experimental arrangement, we would have a pulse-height distribution similar to that with the carbon scattering and a larger uncertainty in the counter efficiency.

ENERGY SPREAD AND PROFILE CHECK

In order to obtain a flat-top profile the energy acceptance slit of the spectrometer must be appreciably larger than the energy spread of the scattered electrons. Contributions to this energy spread are listed in Table II.

The over-all slit width was 3.25% of the scattered electron energy. The measured profiles varied between 3.1 and 3.4% corresponding to changes of machine energy of $\pm 0.15\%$ while measuring the profile. A typical profile is shown in Fig. 3. The top of the profile is not, of course, completely flat, because of changes in radiative corrections and kinematics. The point of operation for the data accumulation runs is shown by the arrow. This is 1.1% from the midpoint of the profile on the low-field side, and according to Table II should give a margin of safety.

In the run procedure, the edges of the profiles were checked every few hours and after any major machine change. The estimated accuracy of setting to the chosen point is $\pm 0.03\%$.

From the profile we can also estimate the quantity $\Delta E/E$ for the radiative corrections. This becomes $(2.15 \pm 0.1)\%$.

COUNTING RATE CORRECTIONS

No more than one pulse from the Čerenkov counter can be counted per beam pulse (and because of the stretcher circuit the largest pulse will always be chosen).

The counting rate was then limited to 2 per second to limit the correction for loss of counts to 2% for our 50-cps accelerator.

If the beam remains steady, this correction is easy to apply, if not, uncertainty can occur. The uncertainty was minimized by using an auxiliary Čerenkov counter measuring general room background. The pulses from this counter were integrated with a 20 μ sec time constant and displayed on a second pulse-height analyzer. This gave the distribution of beam intensity. Counting-rate corrections were made in accordance with these distributions. On no occasion did the correction differ by more than 0.5% from the simple correction, and we estimate an accuracy of 0.1%. It is clear that on another occasion the count rate could be increased.

ENERGY CALIBRATION

Although the 250-Mev deviation system of the accelerator has itself been calibrated to an estimated 0.25%, it is not possible to use this calibration, as the direction of the beam as it leaves the accelerator is not sufficiently well known and can change. Indeed, energy changes of 0.8% have been observed for the same nominal beam energy. The energy calibration is therefore made by measuring the energy of the scattered electrons and adding 0.38 Mev (for the $-q^2=3$ runs) for average ionization loss in the target and windows.

The spectrometer current chosen for data accumulation is $\frac{1}{3}$ of the way along the profile from the low-field end (Fig. 3). The scattered electrons pass $\frac{1}{3}$ of the way along the energy defining slit from the top. This point is then the point at which we wish to know the energy calibration of the spectrometer. This height is the height used in other experiments in this laboratory^{10,15} to within 1 mm.

Two calibrations at the 120-Mev/c region have been made, one with α particles, and one with a floating wire, to a nominal accuracy and reproducibility of 0.1% and 0.2%, respectively.¹⁶ They disagree by 0.7%. We take the absolute value given by the former and assign an error of 0.5%.

TABLE II. Contributions to energy spread (%).

	140 Mev	210 Mev	250 Mev
Ionization loss	0.10	0.70	0.59
Beam energy width	0.30	0.30	0.30
Bad focussing of spectrometer	0.05	0.05	0.05
Beam height	0.15	0.15	0.15
Scattering by foils	0.05	0.05	0.05
Finite angular acceptance	0.77	0.28	0.66
Radiative tail	0.10	0.10	0.10
Total added linearly	1.52	1.63	1.90
Maximum distance $\frac{1}{3}$ down point to peak	0.76	0.82	0.95
Nominal position	1.1	1.1	1.1

¹⁶ The α -particle measurements were performed by G. Bishop and B. Milman, the floating wire measurements by P. Bounin and G. Proca.

To extend the calibration to the higher energies desired we can use two methods. First we may assume the floating wire method is correct for *relative* values, and secondly we can measure the magnetic field. The magnetic-field measurement was made one azimuth only by a Hall-effect probe calibrated by proton resonance.¹⁷ A small correction must then be made to allow for the fact that the proportion of fringing field changes as the magnet saturates. These two methods disagree for the ratio between the 186- and 148-Mev calibrations (which ratio is of the most vital importance in the ratio F_1/F_2). The disagreement is 1.0020. We take the mean and assign an error to this ratio of ± 0.0015 .

These are significant sources of error in this experiment and the largest source for the absolute cross sections. One of us (P.L.) hopes to reduce these in the future. The sensitivity of the cross section to changes in scattered electron energy is calculated from the point cross sections and the known values of F^2 versus q^2 from references 2 and 3.

RADIATIVE CORRECTIONS

There are two forms of radiative correction. One corresponds to bremsstrahlung by the target and vacuum chamber windows. In all, a total of 0.0073 radiation length of material is in the path of the incident or scattered beams before the spectrometer. If we assume that the number of photons radiated having an energy between k and $k+dk$, is dk/k , a fraction of the electrons equal to $(0.0073) \log(E/\Delta E)$ loses more than a fraction ΔE of their energy. A small (0.05%) correction is applicable because of the different energies of incident and scattered electrons. The correction becomes 1.024 ± 0.001 for $\Delta E/E = 0.0210$, where the error comes from our approximation to the bremsstrahlung formula.

More important is the radiation during scattering (the so-called Schwinger effect). We take here the calculations of Tsai¹⁸ which include radiation by the recoil proton. Our computational precision is adequate, and we have included the error due to our uncertainty of $\Delta E/E$ [(2.10 \pm 0.05)%]. This uncertainty in $\Delta E/E$ is the same for all energies if the spectrometer dispersion remains constant. However, as the spectrometer saturates, it is known that the field index $n = (r/B)dB/dr$ decreases, altering the dispersion. This has been estimated and the correction made, (0.2 \pm 0.1)%. Because of the spread of incident energies $\Delta E/E$ changes from electron to electron. The *average* is the value 2.10% and the spread about the average gives 1% increase in the correction or an extra 0.20% of correction to the hydrogen counts.

RESULTS

Three targets were used for the 250- and 210-Mev cross sections giving altogether about 100 000 counts.

¹⁷ We are grateful to M. Ezrati, C.E.A. Saclay, for performing this calibration.

¹⁸ Yung-Su Tsai, Phys. Rev. **122**, 1898 (1961).

TABLE III. Tabulated experimental results.

		Weighted mean
$\sigma_{250}/\sigma_{210}$	$\begin{cases} 2.300(1 \pm 0.023) \\ 2.290(1 \pm 0.019) \\ 2.315(1 \pm 0.015) \end{cases}$	2.304(1 \pm 0.010)
σ_{250}		$5.399 \times 10^{-32} \text{ cm}^2$
σ_{140}		$1.998 \times 10^{-31} \text{ cm}^2$
$\sigma_{250}/\sigma_{210}$ corrected	2.303	
σ_{250} corrected	$6.533 \times 10^{-32} \text{ cm}^2$	
σ_{140} corrected	$2.389 \times 10^{-31} \text{ cm}^2$	
σ_{250} point	$1.173 \times 10^{-31} \text{ cm}^2$	
σ_{140} point	$2.935 \times 10^{-31} \text{ cm}^2$	
$\sigma_{250}/\sigma_{210}$ point	2.222	
$F^2(q^2 = -2.98, \theta = 105^\circ)$	0.557 ± 0.012	
$F^2(q^2 = -1.05, \theta = 104^\circ)$	0.814 ± 0.018	
$F_1/F_2(q^2 = -2.98)$	1.085 ± 0.025	
$F_1(q^2 = -2.98)$	0.767 ± 0.025	
$F_2(q^2 = -2.98)$	0.707 ± 0.028	

For each target and incident energy, five to ten CH₂ runs were made and the counts were analyzed with a χ^2 test. The over-all χ^2 was 58 with 42 degrees of freedom compared with 48 expected from counting statistics alone. Since the observed χ^2 includes the errors due to count rate corrections, monitor fluctuations, small energy changes, different angular setting, etc., the agreement is satisfactory.

The energy setting of the spectrometer changed by 0.3% (between extremes) during the 5 days of the run. The point cross sections were calculated for the energy at the weighted mean of the spectrometer settings. The accuracy of finding this mean is $\pm 0.03\%$.

Table III shows the ratios of the cross sections 250 to 210 Mev from the 3 targets, and the cross sections at 250- and 140-Mev points. Included also are the calculated "point" cross sections and the final values of F^2 , defined as the ratio to the point cross section.

DISCUSSION

The absolute values of F^2 fall on the curve used by Hofstadter^{2,3} to correlate his early results (see Fig. 4). The agreement is gratifying, since our measurements are absolute, whereas the others were relative except for one point of 6% precision. We can regard this agreement

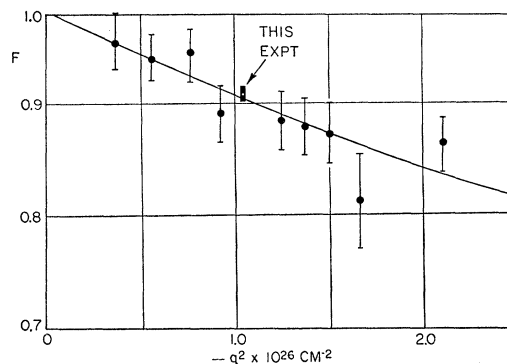


FIG. 4. $F(q^2)$ as a function of q^2 . The measurement reported here is indicated by a heavy bar.

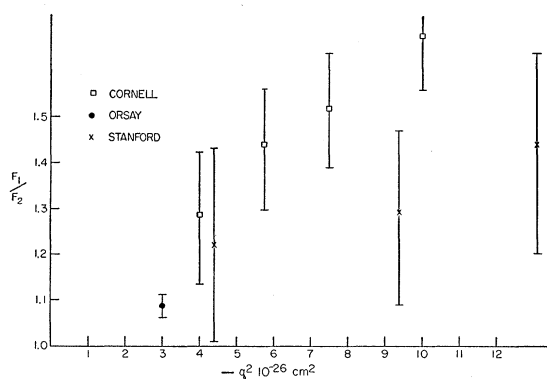


FIG. 5. The values of F_1/F_2 from this experiment compared with Cornell (reference 8) and Stanford (reference 5). The values have been derived direct from the cross sections quoted by these authors and errors derived on the same basis as errors in this experiment.

in two ways. First, the relative values of the two points at $-q^2 = 1.05 \text{ f}^{-2}$ and $-q^2 = 2.98 \text{ f}^{-2}$ immediately confirm the slope of the curve $F^2(q^2)$ vs q^2 to 5%. Secondly, if we assume the form factors are given by Hofstadter's results, the absolute value of the cross section confirms the radiative correction as calculated by Tsai.¹⁸ This confirmation is additional to that of Panofsky and Tautfest.

Our value of $F_1/F_2 = 1.085 \pm 0.025$ at $-q^2 = 2.98 \text{ f}^{-2}$ is in agreement with the prediction of Fubini⁸ which is 1.1.

In Fig. 5, we plot our value of F_1/F_2 and the values obtained, at different transfers by the Stanford and Cornell groups.

The value of $F^2(-q^2 = 1.05, \theta = 104^\circ)$ is essentially a measure of F_1 . If we suppose that, for that transfer $F_1 = F_2$, we deduce a value of the root mean square radius of the proton which is $a = 0.79 \pm 0.04 \text{ f}$.

Note added in proof. Since the above was written, it has become clear that it is preferable to express the scattering in terms of the electric and magnetic form factors G_E and G_M ,¹⁸ which [L. Hand, D. Miller, and Richard Wilson, Phys. Rev. Letters 8, 110 (1962)] are also the helicity amplitudes used in dispersion theory.⁹ G_M is given by the 150° scattering with little contribution from G_E . The error in magnetic field calibration does not enter appreciably into backward scattering. We can therefore quote a high precision for G_M .

$$G_E(q^2 = -2.98) = 0.725 \pm 0.022,$$

$$G_M(q^2 = -2.98) = 2.035 \pm 0.016,$$

$$G_M/(1+K) = 0.729 \pm 0.006,$$

$$G_E(q^2 = -1.05) = 0.884 \pm 0.009.$$

The mean square charge or magnetic radius of the proton becomes $0.84 \pm 0.04 \text{ f}$.

More detailed analysis will await further measurements that one of us (P.L.) intends to pursue.

ACKNOWLEDGMENTS

In an experiment of this sort all the experience of the laboratory is utilized and gratefully acknowledged. In addition we would like to thank in particular Victor Round for constructional help; Dr. Boris Milman for valuable set up and calibration aid; Boris Dudelzak for help in many nights of experiment, and the machine operating crew under Dr. Burnod for approximately 10^{19} electrons. We are grateful to Professor H. Halban for putting the facilities of the Laboratory at our disposal, and for his continued interest in this work. R. Wilson thanks the John Simon Guggenheim Foundation for a financial grant.