

Polarization and Differential Cross Section for Neutron Scattering from Silicon*

R. O. LANE, A. J. ELWYN, AND A. LANGSDORF, JR.
Argonne National Laboratory, Argonne, Illinois

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The polarization of neutrons scattered from Si and the unpolarized differential cross section for the process are measured at 5 angles and at neutron energies from 0.2 to 0.7 Mev. The differential cross sections in the present work, together with previously measured cross sections, are analyzed in terms of the properties of the resonance levels seen in Si^{29} ($\text{Si}^{28}+n$) by means of the R -function formalism of Lane and Thomas. The levels in Si^{29} at neutron energies of 0.536 Mev and 0.571 Mev are assigned to be D_1 and P_1 , respectively. The phase shifts obtained from this analysis are used to predict the polarization in the scattering from Si^{28} . The predicted values agree well with the measured polarization in scattering from natural Si. However, the rapid variation of the polarization with energy limits the usefulness of Si as an analyzer near the resonance energies. At neutron energies below 0.4 Mev and above 0.65 Mev the polarization is practically zero. The agreement between calculated and measured polarizations gives added confirmation of previous measurements of the polarization of the neutrons in the $\text{Li}^7(p,n)\text{Be}^7$ reaction, which has been used as the neutron source in the present study.

INTRODUCTION

IN studies of the polarization of neutrons produced in nuclear reactions initiated (for example) by charged particles, the usual method is to measure the left-right asymmetry in the "second" scattering of these neutrons from various materials. The scatterers then serve as polarization analyzers. For neutron energies below 1.0 Mev, such measurements of polarization are hampered by the lack of suitable materials for use as analyzers. The zero-spin nuclei O^{16} and Mg^{24} and the nonzero-spin nucleus Li^7 have proved useful in this application,¹⁻⁵ each being effective, however, only at certain energies and each having its own difficulties in use. It is the purpose of the present work to determine the extent to which another material, natural Si, can be used as an analyzer in the neutron energy region from 0.2 to 0.7 Mev.

In measurements of left-right asymmetries as a function of angle, one also obtains the differential cross section for the scattering of unpolarized neutrons. (This is obtained as the sum of the "left" and "right" intensity measurements.) When these data for Si are analyzed by means of the R -function formalism⁶ in terms of the properties of the resonance levels seen in neutron total cross section measurements, the polarization can be calculated. Comparisons between measured and predicted values of the polarization serve to indicate the degree to which Si is useful as a polarization analyzer and to test the general formalism in this situation.

EXPERIMENT

The experiment consisted in measuring the left-right asymmetries in the scattering of partially polarized neutrons from samples of natural Si. The experimental arrangement has been described previously,^{4,7} and will only be mentioned briefly here.

Partially polarized neutrons were obtained from the $\text{Li}^7(p,n)\text{Be}^7$ reaction at an angle of 51° (lab) with respect to the direction of the incident protons. The proton beam was supplied by the Argonne 5-Mev electrostatic accelerator. The polarized neutrons passed through the transverse field of an electromagnet⁸ before striking the silicon scatterer. The apparatus was arranged so that the reaction and scattering events were coplanar. Left-right asymmetry measurements at given scattering angles were obtained by measuring the intensity of scattered neutrons first with the magnet off and then with the magnet on at a value of magnetic field sufficient to precess the spins of the neutrons by 180° . The magnet was contained inside a shield surrounding the source and formed an integral part of the collimator through which the neutrons emerged from the source shield.

The scatterers were normal elemental silicon powder held in thin-walled containers in the form of rectangular slabs. These containers had 0.005 in. steel walls and were 20 in. high \times 10 in. wide with nominal thicknesses of $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ in. The samples contained 0.0096, 0.0226, and 0.0492 atoms/b, respectively, along the path of the incident beam. Background runs were taken with an identical but empty container in place of the scatterer. The scatterer was placed at the center of a circular track around which 5 detectors could be positioned. Each detector consisted of a shielded tank at the center of which was placed a bank of ten BF_3 counters in an oil moderator. The neutron beam was

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¹ H. R. Striebel, S. E. Darden, and W. Haeberli, *Nuclear Phys.* **6**, 188 (1958).

² S. E. Darden, T. R. Donaghue, and C. A. Kelsey, *Nuclear Phys.* **22**, 439 (1960).

³ S. M. Austin, S. E. Darden, A. Okazaki, and F. Wilhelmi, *Nuclear Phys.* **22**, 451 (1960).

⁴ A. J. Elwyn and R. O. Lane, *Nuclear Phys.* (to be published).

⁵ W. John and F. V. Martin, *Phys. Rev.* **124**, 830 (1961).

⁶ A. M. Lane and R. G. Thomas, *Revs. Modern Phys.* **30**, 257 (1958).

⁷ R. O. Lane, A. Langsdorf, Jr., J. E. Monahan, and A. J. Elwyn, *Ann. Phys.* **12**, 135 (1961).

⁸ A. Langsdorf, Jr., A. J. Elwyn, and R. O. Lane (to be published).

TABLE I. Polarization of neutrons from the reaction $\text{Li}^7(p,n)\text{Be}^7$ at 51° as obtained from reference 4.

E_{lab} (Mev)	$P_1(51^\circ)$	E_{lab} (Mev)	$P_1(51^\circ)$
0.400	0.41±0.06	0.560	0.24±0.05
0.450	0.20±0.05	0.565	0.25±0.06
0.500	0.22±0.05	0.571	0.25±0.06
0.525	0.27±0.04	0.581	0.25±0.06
0.530	0.23±0.05	0.600	0.25±0.05
0.534	0.24±0.05	0.625	0.26±0.05
0.538	0.23±0.05	0.650	0.26±0.05
0.543	0.24±0.05	0.700	0.27±0.05
0.549	0.24±0.05		

monitored by two more BF_3 counters in moderator. They sampled the flux passing through the source collimator.

Neutrons were produced by proton bombardment of evaporated metallic Li targets which were about 40 kev thick (to protons with energy of about 1.9 Mev) for the measurements in the energy regions far from resonances. Targets 8–15 kev thick were used at energies near resonances. Thickness was measured by the rise-curve method at threshold. After passing through the source collimator, the neutrons were incident on the scatterers at an angle of 45° with respect to the normal to the plane of the slab. For this experiment the 5 detectors were set at laboratory scattering angles of 20° , 52° , 85° , 117° , and 150° , except at energies of 0.4, 0.45, and 0.50 Mev for which the measurements were made at the laboratory angles of 20° , 55° , 90° , 125° , and 159° .

RESULTS AND ANALYSIS

From the measured left-right asymmetry ratios $L(\theta)/R(\theta)$, determined from the scattering with the magnet first off and then on, the product,

$$P_1(51^\circ)P_2(\theta) = \frac{1 - L(\theta)/R(\theta)}{1 + L(\theta)/R(\theta)}, \quad (1)$$

TABLE II. Polarization results for silicon at a scattering angle of $\theta_{\text{c.m.}} = 21^\circ$.

E_{lab} (Mev)	$P_1(51^\circ)P_2(21^\circ)$	$P_2(21^\circ)$
0.400	0.00 ±0.02	0.01±0.05
0.450	0.00 ±0.03	0.00±0.14
0.500	0.02 ±0.02	0.08±0.10
0.525	0.00 ±0.02	0.01±0.06
0.530	0.04 ±0.02	0.18±0.08
0.534	0.04 ±0.02	0.17±0.07
0.538	0.01 ±0.02	0.04±0.06
0.543	0.02 ±0.02	0.08±0.06
0.549	0.01 ±0.02	0.05±0.08
0.560	0.07 ±0.01	0.28±0.07
0.565	0.045±0.01	0.18±0.06
0.571	0.00 ±0.01	0.02±0.03
0.581	0.00 ±0.01	0.01±0.03
0.600	-0.02 ±0.01	-0.07±0.04
0.625	-0.01 ±0.01	-0.03±0.04
0.650	-0.03 ±0.01	-0.10±0.05
0.700	0.01 ±0.01	0.02±0.05

was calculated for each value of θ . Here $P_1(51^\circ)$ is the polarization of the neutrons emitted from the $\text{Li}^7(p,n)\text{Be}^7$ reaction at 51° , and $P_2(\theta)$ is the polarization that would arise if an unpolarized beam of neutrons incident on the Si analyzer were scattered through an angle θ . The choice of algebraic sign in Eq. (1) is consistent with the Basel convention⁹ for the positive direction of the polarization vector and is based on the assumption that, as viewed from above, the angle 51° is measured in a clockwise sense from the direction of incidence of the proton beam.

A number of corrections to the experimentally observed $P_1(51^\circ)P_2(\theta)$ products were considered. (i) In order to analyze the results in terms of energy levels in $\text{Si}^{28}+n$, the effects of the minor isotopes in the natural Si scatterer (analyzer) were considered. However, since the minor isotopes made up only 7.8% of the natural material [Si^{29} (4.7%) and Si^{30} (3.1%)] it was felt that these effects were for the most part

TABLE III. Polarization results for silicon at a scattering angle of $\theta_{\text{c.m.}} = 53^\circ$.

E_{lab} (Mev)	$P_1(51^\circ)P_2(53^\circ)$	$P_2(53^\circ)$
0.400	0.02 ±0.02 ^a	0.04±0.04 ^a
0.450	0.02 ±0.02 ^a	0.08±0.08 ^a
0.500	0.01 ±0.02 ^a	0.05±0.08 ^a
0.525	0.03 ±0.01	0.14±0.07
0.530	0.057±0.016	0.24±0.08
0.534	0.083±0.016	0.35±0.10
0.538	0.082±0.016	0.34±0.10
0.543	0.065±0.016	0.27±0.09
0.549	0.089±0.017	0.37±0.10
0.560	0.139±0.012	0.57±0.13
0.565	0.110±0.010	0.45±0.10
0.571	0.046±0.008	0.18±0.05
0.581	0.00 ±0.01	0.01±0.03
0.600	-0.02 ±0.01	-0.09±0.05
0.625	-0.02 ±0.01	-0.10±0.05
0.650	-0.02 ±0.01	-0.06±0.05
0.700	-0.01 ±0.01	-0.06±0.05

^a Measurement actually made at $\theta_{\text{c.m.}} = 56^\circ$.

negligible. (ii) The moderated BF_3 counters used in this study could not distinguish between the elastically scattered neutrons and any lower energy groups that might be present. However, the first excited state in Si^{28} is at 1.78 Mev so that inelastic scattering does not occur in the energy region studied in this experiment. Also, a second group of neutrons from the $\text{Li}^7(p,n)\text{Be}^7$ reaction, leading to a level in Be^7 at 0.43 Mev, does not have a significant intensity relative to the ground state group at the energies used.¹⁰ Therefore no correction for such effects was necessary. (iii) Effects of the finite size of the scatterer and detectors, which might lead to spurious asymmetries in the left-right

⁹ *Proceedings of the International Symposium on Polarization Phenomena of Nucleons*, Helv. Phys. Acta, Supplement VI (1961), p. 436.

¹⁰ A. Smith (private communication); P. R. Bevington, W. W. Rolland, and H. W. Lewis, Phys. Rev. **121**, 871 (1961).

asymmetry measurements,¹¹ are avoided entirely by the use of a magnetic field to rotate the spins of the incident neutrons. Furthermore, the observation of the ratios $L(\theta)/R(\theta)$ by the rotation of the spins removes uncertainties caused by differing efficiencies of two detectors or by differing environmental conditions for a single detector alternated between left and right. (iv) The scatterer subtended an angle of approximately 4° at the source, and the detectors subtended approximately 4° – 5° at the scatterer. The averaging effect present in the observed asymmetries due to this angular resolution was indeed negligible in this case, so no correction was made for such effects. (v) Possible depolarization effects connected with use of a magnetic field have also been considered⁸ but are negligible in the present situation.

The experimentally observed P_1P_2 products were corrected for the effects of multiple scattering. This correction was based on the approximation that the

TABLE IV. Polarization results for silicon at a scattering angle of $\theta_{c.m.} = 87^\circ$.

E_{lab} (Mev)	$P_1(51^\circ)P_2(87^\circ)$	$P_2(87^\circ)$
0.400	0.01 ± 0.02^a	0.02 ± 0.04^a
0.450	0.02 ± 0.02^a	0.07 ± 0.07^a
0.500	0.02 ± 0.02^a	0.07 ± 0.08^a
0.525	0.08 ± 0.02	0.32 ± 0.09
0.530	0.06 ± 0.02	0.26 ± 0.08
0.534	0.05 ± 0.02	0.23 ± 0.08
0.538	0.08 ± 0.02	0.33 ± 0.10
0.543	0.06 ± 0.02	0.24 ± 0.08
0.549	0.10 ± 0.02	0.40 ± 0.11
0.560	0.17 ± 0.01	0.71 ± 0.15
0.565	0.16 ± 0.01	0.66 ± 0.14
0.571	0.13 ± 0.01	0.50 ± 0.11
0.581	0.02 ± 0.02	0.08 ± 0.06
0.600	-0.06 ± 0.02	-0.23 ± 0.08
0.625	-0.04 ± 0.02	-0.15 ± 0.07
0.650	-0.03 ± 0.02	-0.11 ± 0.06
0.700	0.00 ± 0.02	0.01 ± 0.06

^a Measurement actually made at $\theta_{c.m.} = 92^\circ$.

second and subsequent scatterings produced no net polarization and was calculated by a Monte Carlo technique¹² using the results of reference 7 as input. The approximation involved in this calculation ordinarily makes the values of P_1P_2 too large. However, in the present case in which the measured P_1P_2 products themselves are not large, it is thought to be a good approximation. Multiple-scattering corrections to polarization measurements have been discussed more fully in an earlier report.⁴

The corrected products $P_1(51^\circ)P_2(\theta)$ were divided by the values of $P_1(51^\circ)$ shown in Table I as determined from earlier measurements of Elwyn and Lane,⁴ which

¹¹ J. E. Monahan and A. J. Elwyn, Argonne National Laboratory Report ANL 6420, 1961 (unpublished); Nuclear Instr. (to be published); J. E. Evans, Atomic Energy Research Establishment Report AERE-R3347, 1960 (unpublished).

¹² R. O. Lane and W. F. Miller, Nuclear Instr. and Methods (to be published).

TABLE V. Polarization results for silicon at a scattering angle of $\theta_{c.m.} = 119^\circ$.

E_{lab} (Mev)	$P_1(51^\circ)P_2(119^\circ)$	$P_2(119^\circ)$
0.400	-0.01 ± 0.02^a	-0.03 ± 0.05^a
0.450	-0.00 ± 0.22^a	-0.01 ± 0.09^a
0.500	0.03 ± 0.02^a	0.11 ± 0.09^a
0.525	0.04 ± 0.02	0.15 ± 0.07
0.530	0.02 ± 0.02	0.08 ± 0.07
0.534	-0.01 ± 0.02	-0.02 ± 0.06
0.538	0.03 ± 0.02	0.04 ± 0.07
0.543	0.05 ± 0.02	0.21 ± 0.07
0.549	0.07 ± 0.01	0.28 ± 0.08
0.560	0.11 ± 0.01	0.46 ± 0.11
0.565	0.12 ± 0.01	0.48 ± 0.11
0.571	0.12 ± 0.02	0.50 ± 0.12
0.581	0.05 ± 0.02	0.19 ± 0.10
0.600	-0.06 ± 0.02	-0.25 ± 0.11
0.625	-0.04 ± 0.02	-0.15 ± 0.09
0.650	-0.01 ± 0.02	-0.06 ± 0.08
0.700	-0.04 ± 0.02	-0.15 ± 0.07

^a Measurement actually made at $\theta_{c.m.} = 127^\circ$.

have been shown to be consistent with those of others.¹⁻³ Tables II through VI list for each of the five scattering angles, the energies, the corrected experimental products $P_1(51^\circ)P_2(\theta)$, and the resulting $P_2(\theta)$, each of the latter two with its respective error. The points in Fig. 1 show $P_2(\theta)$ as a function of θ (c.m. system) for various laboratory energies. In Fig. 2, $P_2(\theta)$ is plotted (as the points) as a function of the energy in the laboratory system for each of the angles (in the c.m. system) used in the experiment. The errors shown are based on counting statistics only. At energies of 0.40, 0.45, and 0.50 Mev the neutron energy spread was ≈ 40 kev, while for all other energies it varied from 8 to 15 kev. At 0.500 and 0.625 Mev the background counting rates at the largest angle were about 60% of the gross counts. This leads to large uncertainties in these cases. (For all other energies and angles the background counting rates were ≈ 10 – 30% of the gross counts.)

TABLE VI. Polarization results for silicon at a scattering angle of $\theta_{c.m.} = 151^\circ$.

E_{lab} (Mev)	$P_1(51^\circ)P_2(151^\circ)$	$P_2(151^\circ)$
0.400	-0.05 ± 0.03^a	-0.12 ± 0.08^a
0.450	-0.03 ± 0.04^a	-0.10 ± 0.16^a
0.500	0.09 ± 0.04^a	0.42 ± 0.20^a
0.525	0.01 ± 0.02	0.06 ± 0.07
0.530	-0.01 ± 0.02	-0.06 ± 0.07
0.534	-0.02 ± 0.01	-0.07 ± 0.06
0.538	0.01 ± 0.01	0.03 ± 0.06
0.543	0.02 ± 0.01	0.10 ± 0.06
0.549	0.03 ± 0.01	0.12 ± 0.06
0.560	0.04 ± 0.01	0.17 ± 0.06
0.565	0.05 ± 0.01	0.22 ± 0.06
0.571	0.04 ± 0.01	0.16 ± 0.06
0.581	0.03 ± 0.02	0.10 ± 0.10
0.600	-0.03 ± 0.03	-0.10 ± 0.11
0.625	-0.09 ± 0.02	-0.35 ± 0.12
0.650	0.00 ± 0.03	0.00 ± 0.10
0.700	0.00 ± 0.02	0.02 ± 0.08

^a Measurement actually made at $\theta_{c.m.} = 160^\circ$.

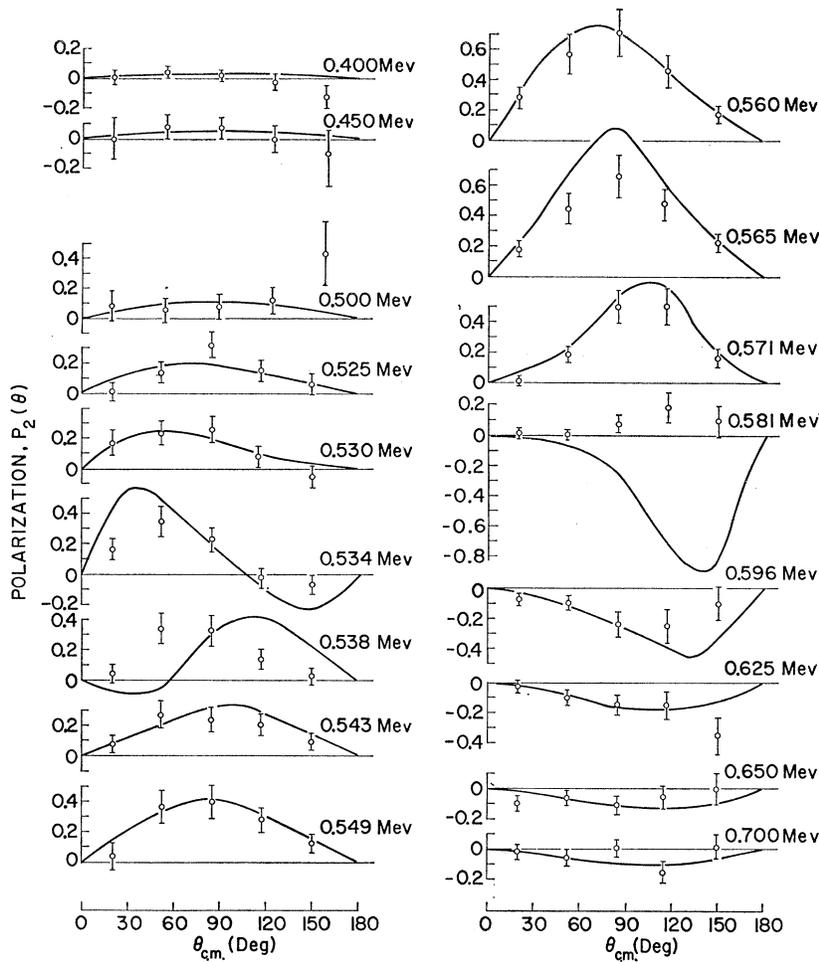


FIG. 1. Polarization $P_2(\theta)$ of neutrons scattered from silicon. Energies are in the laboratory system. The points are obtained from the experiment on natural silicon and the solid curve is computed from the phase shifts of the R -function analysis of resonances in $\text{Si}^{28}+n$.

At any given angle θ , the sum of the properly normalized scattered intensities from the magnet-off and magnet-on runs is proportional to the unpolarized differential cross section at θ . It was possible therefore to obtain unpolarized differential cross sections from the present work. (The effects of nonuniformity in the response of the 5 detectors and the energy dependence of their efficiencies were considered and concluded to be negligible.) Comparison of these results with previous measurements⁷ in the energy range from 0.48 to 0.76 Mev showed good agreement except in the region of the neutron resonances at 0.536 Mev and 0.571 Mev where the measurements with better resolution (≈ 6 -kev energy spread) of reference 7 should be superior.

The present results for the unpolarized differential cross section in the off-resonance regions were combined with the results from reference 7 for neutron energies from 0.48 to 0.76 Mev, and with earlier measurements of both total¹³ and differential cross section¹⁴ from 0.01

to 0.48 Mev. The measured unpolarized differential cross sections were then analyzed in terms of Legendre polynomials by use of the expansion

$$\sigma(\theta, E) = \sum_{L=0}^4 B_L(E) P_L(\cos\theta), \quad (2)$$

where the $B_L(E)$ are the coefficients of the Legendre polynomials in the c.m. system and θ is the angle in the c.m. system. The variation of $B_L(E)$ as a function of laboratory energy is shown by the points in Figs. 3 and 4. It has been shown⁷ that terms in $L \geq 5$ are negligible in this energy region.

The R -matrix formalism of Lane and Thomas⁶ was utilized to calculate the phase shifts to be expected on the basis of the assumed level parameters for states in Si^{29} that correspond to observed resonances in the scattering cross section of Si^{28} . In this case of neutron scattering from a zero-spin nucleus (Si^{28}), the R matrix reduces to an R function for a single channel. It was assumed that the R function for a given spin and parity could be represented by a single level and a

¹³ R. E. Fields and M. Walt, Phys. Rev. **83**, 479 (1951).

¹⁴ R. O. Lane (previously unpublished results obtained at an earlier time with unpolarized neutrons).

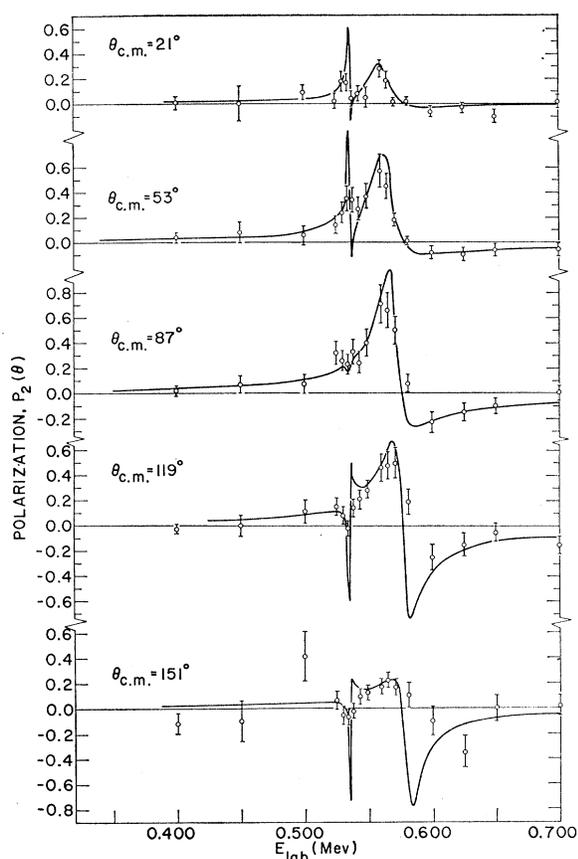


FIG. 2. Polarization $P_2(\theta)$ of neutrons scattered from silicon. Points are obtained from the experiment on natural silicon, and the solid curve is computed from the phase shifts of the R -function analysis of resonances in $\text{Si}^{28}+n$.

constant term, i.e.,

$$R_{lJ}(E) = \frac{\gamma_{lJ}^2}{E_{lJ} - E} + R_{lJ}^0, \quad (3)$$

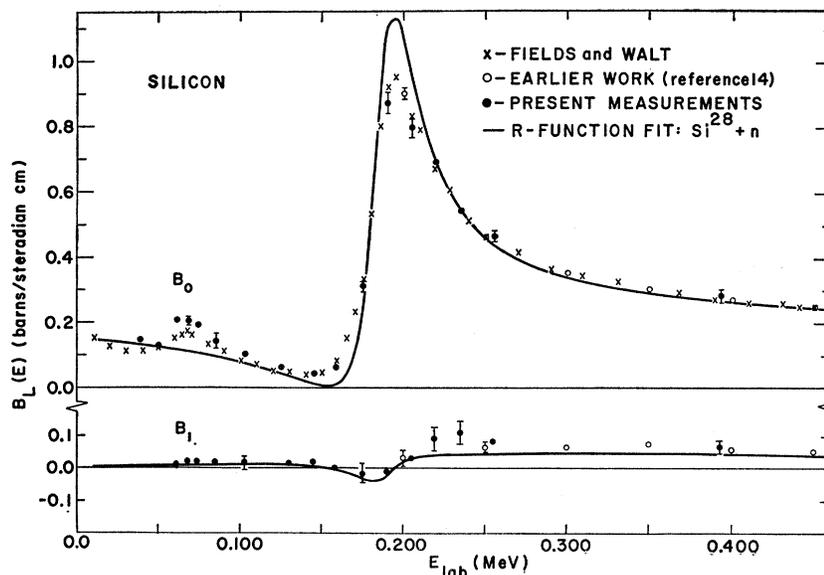
where γ_{lJ}^2 and E_{lJ} are the reduced width and characteristic energy, respectively, of the level. The scattering phase shifts are expressed as

$$\delta_{lJ}(E) = \tan^{-1} \left[\frac{P_l(\rho)R_{lJ}(E)}{1 - S_l(\rho)R_{lJ}(E)} \right] - \phi_l(\rho), \quad (4)$$

where $P_l(\rho)$ is the penetration factor, $S_l(\rho)$ the shift factor, $\phi_l(\rho)$ the hard-sphere phase shifts, and $\rho = kA$, k being the wave number and A the interaction radius. The coefficients $B_L(E)$, based on assumed level parameters, are then calculated from the phase shifts [Eq. (4)] according to the theory of Blatt and Biedenharn.¹⁵ A program to calculate the phase shifts and coefficients $B_L(E)$ was written for the IBM-704 computer. The solid lines on Figs. 3 and 4 show the best fit to the experimental $B_L(E)$ that was obtained after a number of trials.

The levels included in the calculations correspond to three neutron resonances in $(\text{Si}^{28}+n)$: the $S_{1/2}$ level at 0.195 Mev, the $D_{5/2}$ level at 0.536 Mev, and a $P_{3/2}$ state at 0.571 Mev. Fields and Walt¹³ previously determined that the resonance at 0.195 Mev has the character of an $S_{1/2}$ state, but could only place a lower limit of $J > \frac{1}{2}$ for the resonance at 0.571 Mev, and did not notice the resonance at 0.536 Mev because of the large experimental energy spread in the neutron beam. (Actually, the state did cause one high point in their data.)

FIG. 3. Legendre polynomial coefficients of Eq. (2) for scattering from Si. Experimental points are for natural silicon. The solid curve is the fit to the data when the level parameters given in Table I were used in the R -function calculation for resonances in $\text{Si}^{28}+n$. These calculations did not take into account any averaging over energy spreads. The $B_0(E)$ from the present polarization measurements were normalized to those of the earlier work (references 13 and 14). $B_L(E) = 0$ for $L \geq 2$ at these energies.



¹⁵ J. M. Blatt and L. C. Biedenharn, *Revs. Modern Phys.* **24**, 258 (1952).

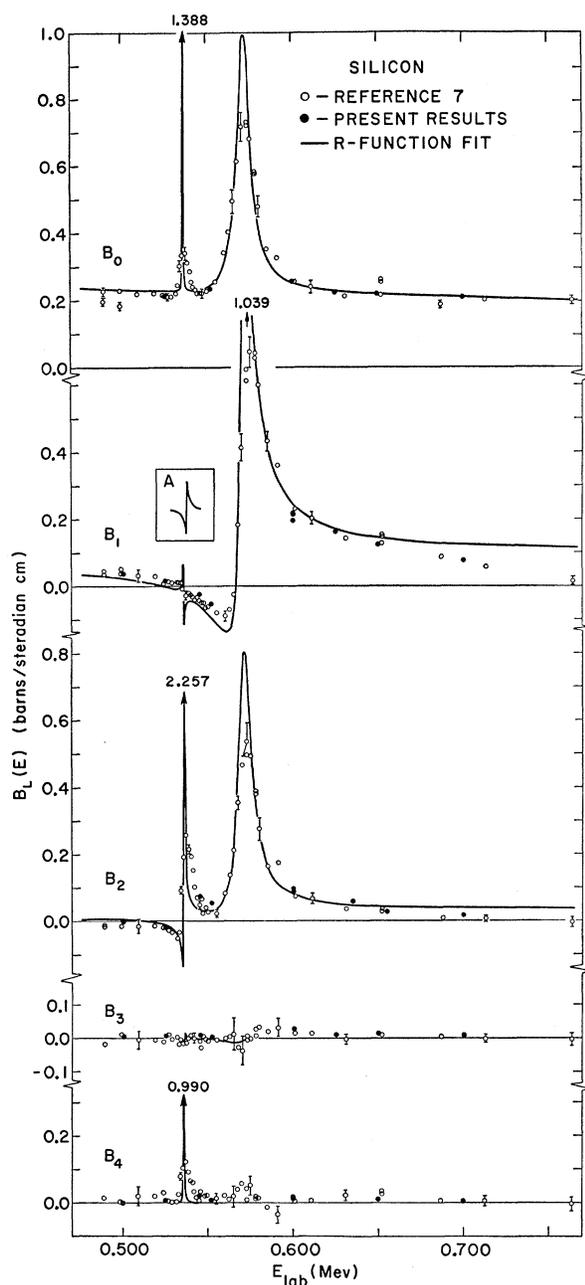


FIG. 4. Legendre polynomial coefficients of Eq. (2) for scattering from Si. Experimental points are for natural silicon. The solid curve is the fit to the data when the level parameters given in Table I were used in the R -function calculation for resonances in $\text{Si}^{28}+n$. These calculations did not take into account any averaging over energy spreads. Inset A shows the change in the variation of the interference term $B_1(E)$ with energy near the $D_{3/2}$ resonance when the $P_{3/2}$ resonance is omitted from the R -function calculations.

Since the experiment shows that $B_4(E)$ is resonant and that $B_L(E)=0$ for $L \geq 5$ for the 0.536 Mev resonance, the selection rules on the angular-momentum coupling coefficients set the lower limits of $J \geq \frac{5}{2}$, $l \geq 2$ for this resonance. If this resonance were $F_{7/2}$ then both $B_3(E)$ and $B_6(E)$ should be strongly resonant, the

TABLE VII. Level parameters of Si^{29} . These were determined for a radius of 4.8 fermis by making an R -function fit to differential-scattering data for $(\text{Si}^{28}+n)$.

E_{res} (Mev, lab)	Char- acter of reso- nance	$\Gamma_{lJ}(E_{\text{res}})^a$ (kev, lab)	γ_{lJ}^2 (Mev, c.m.)	E_{lJ} (Mev, c.m.)	R_{lJ}^0	$\frac{\gamma_{lJ}^2}{(\frac{3}{2}\hbar^2/MA^2)}$
0.195	$S_{1/2}$	32.5	0.0350	0.181	0	0.0125
...	$P_{1/2}$	0.2	...
0.571	$P_{3/2}$	11.8	0.020	0.540	0.2	0.0072
0.536	$D_{5/2}$	0.32	0.0075	0.504	0	0.0027

^a Determined from the expression $\Gamma_{lJ}(E) = 2P_l(E)\gamma_{lJ}^2$.

first from the interference term $(\delta_{0\ 1/2} - \delta_{3\ 7/2})$ and the second from the squared term in $\delta_{3\ 7/2}$. Neither of these is observed. The assignment $F_{5/2}$ would predict $B_6(E)=0$, as is observed, but still a large $B_3(E)$ from the interference term $(\delta_{0\ 1/2} - \delta_{3\ 5/2})$. This latter again is not observed. On the other hand, the assignment of $D_{5/2}$ does agree very well with the available data for this resonance.

Arguments similar to those above can be given for the resonance at 0.571 Mev. In this case $B_L(E)=0$ for $L \geq 3$ and $B_2(E)$ is resonant, so that $J \geq \frac{3}{2}$ and $l \geq 1$ for this resonance. A D -wave assignment would not give the large value of $B_1(E)$ observed. Rather, this is produced by the interference term $(\delta_{0\ 1/2} - \delta_{1\ 3/2})$ which is predicted by the $P_{3/2}$ assignment. This assignment agrees quite well with all $B_L(E)$ near this resonance. Thus the results being reported here, together with those of reference 7, establish the assignments as those given in Table VII. The effect of higher energy P -wave resonances required the inclusion of a positive term $R_{1\ 1/2}^0 = R_{1\ 3/2}^0 = 0.2$ in the P -wave R functions. A radius of 4.8 fermis gave the best fit to S - and P -wave backgrounds, and was employed throughout the calculations.

From the curves of Figs. 3 and 4 it is seen that the calculations for Si^{28} fit the data for natural silicon reasonably well except at energies below 0.200 Mev and above 0.660 Mev. Below 0.180 Mev the effects of the minor isotopes Si^{29} and Si^{30} become evident. They can be seen as a residual contribution to $B_0(E)$ where the nearly complete interference in the dip practically eliminates all contribution from Si^{28} . The small peak near 0.06 Mev has been assigned to Si^{28} by Newson *et al.*,¹⁶ the state having an $S_{1/2}$ character. The width was given as 1.5 kev and hence was neglected in these calculations since it has little effect at energies at which the polarization is of interest. It should be mentioned that the fitting was done primarily with emphasis on the region from 0.500 Mev to 0.650 Mev and that this lower energy region is included in the fitting only to ensure that the effects of levels in this region are taken into account to a good approximation.

¹⁶ H. W. Newson, R. C. Block, P. F. Nichols, A. Taylor, A. K. Furr, and E. Mezbacher, *Ann. Phys.* **8**, 211 (1959).

The fitting at these lower energies therefore was not treated as critically as in the region above.

Above 0.65 Mev the effects of levels (probably P states) at higher energy are too energy dependent to be taken into account by a constant in the R function. Because of the lack of information to date concerning such resonances, the fitting is only meaningful up to ≈ 0.650 Mev.

As seen in Table VII, the $D_{5/2}$ resonance has a natural width $\Gamma_{LJ}(E) \equiv 2P_1(E)\gamma_{LJ}^2 = 0.32$ kev if this energy-dependent term is evaluated at a neutron energy of 0.536 Mev. Hence the observed curve for $B_0(E) = \sigma(E)/4\pi$ in that region is a mirror image of the distribution of neutron energies in the beam. That this resonance is in Si^{28} is apparent from two facts: (1) even if they were completely resolved, the observed peaks in $B_0(E)$, $B_2(E)$, and $B_4(E)$ are much too high to be due to either of the minor isotopes, and (2) the $D_{5/2}$ resonance interferes coherently with the $P_{3/2}$ state in the term $B_1(E)$. This was demonstrated by repeating the R -function calculation with the $P_{3/2}$ state absent. The results of this are shown in inset A in Fig. 4, where the sign of the variation of $B_1(E)$ with energy is opposite to that when the $P_{3/2}$ state is present. Even though the energy spread in the neutron beam was large (relative to $\Gamma_{2\ 5/2}$), the sign of the variation of $B_1(E)$ was clearly evident from the data.

The reduced widths of the $D_{5/2}$ and $P_{3/2}$ states were adjusted so that the calculated values of the $B_L(E)$ at energies not far off resonance agreed with the experimental results where the energy spread of the neutron beam was not so critical. The $B_L(E)$ in these energy regions are still sensitive to interference effects and hence to changes in the γ_{LJ}^2 and E_{LJ} . Estimated averages of the calculated curve over the energy spread of the neutrons were also required to agree reasonably with the measured results. In the case of the $D_{5/2}$ resonance, the energy spread was so large relative to the width of the resonance that the value of $\Gamma_{2\ 5/2}$ is not well determined, but is probably not much over 0.5 kev. The energy spread also affects the $P_{3/2}$ resonance, but the effect is much less than on the $D_{5/2}$ resonance.

From the phase shifts obtained from the above analysis of the unpolarized differential cross sections, the polarization $P_2(\theta)$ for neutrons scattered from Si^{28} was then calculated according to the theory of Simon and Welton.¹⁷ These results are shown as the solid lines in Figs. 1 and 2. Near the resonances, the energy

spread in the neutron beam had considerable effect on the measured polarizations. In fact, we believe that this energy spread is sufficient to explain the discrepancies between the experimentally measured polarizations and the calculated ones, since the curves were plotted without averaging the calculated $P_2(\theta)$ over the energy spread. For example, near neutron energies of 0.536 Mev and 0.580 Mev the averaging effect was so large that it completely distorted the polarization curve from its true shape. However, in regions where the polarization varies more slowly with energy, the agreement between measured and calculated polarizations is rather good. The observed variation of $P_2(\theta)$ with θ near the $P_{3/2}$ resonance agrees quite well with the calculated polarization which varies approximately as $\sin\theta$, while the observed $P_2(\theta)$ near the $D_{5/2}$ resonance is in agreement with an approximate $\sin 2\theta$ dependence which is, to a large degree, expected. The polarization $P_2(\theta)$ was measured at energies between 0.200 and 0.400 Mev (not shown) and was found to be zero within the experimental error. This is also predicted by the R -function analysis of the scattering data. Above 0.650 Mev, the results of the analysis depart from the data and are no longer reliable in predicting polarization.

CONCLUSIONS

In regions of smoothly varying polarization, the predicted and measured values of $P_2(\theta)$ agree. This is further confirmation of the previous results for $P_1(51^\circ)$ for the $\text{Li}^7(p,n)\text{Be}^7$ reaction obtained by Striebel *et al.*¹ and by Elwyn and Lane⁴ in the energy range from 0.2 to 0.7 Mev. In silicon the only significant sources of polarization in this region are the resonances at neutron energies of 0.571 and 0.536 Mev in Si^{28} . These levels are assigned as $P_{3/2}$ and $D_{5/2}$, respectively. While the assumed level parameters can give a relatively good fit to the differential cross sections for the scattering of unpolarized neutrons and can predict polarizations reasonably well, the usefulness of silicon as an analyzer in this region is severely restricted by the necessity for very narrow energy spread in the neutron beam.

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¹⁷ A. Simon and T. A. Welton, Phys. Rev. **90**, 1036 (1953).