Induced Pseudoscalar Interaction and the Low-Energy Anomalies in Beta Spectra*

J. M. PEARSON†
Université de Montréal, Montréal, Canada
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An unsuccessful attempt is made to interpret in terms of the induced pseudoscalar interaction the $(1+a/W)$ anomaly factors which have been found by Langer and co-workers in the shapes of all the allowed Gamow-Teller beta spectra that they have measured. Tadic's recent observation that the Hamiltonian of the induced pseudoscalar interaction in nuclear beta decay is radically different from that of the more familiar "basic" pseudoscalar interaction is taken into account. The shape factors for the transitions in question are derived and found to display an anomaly of the required form. In the case of pure Gamow-Teller transitions arbitrarily large distortions are possible, provided that an appropriate destructive interference takes place between the axial vector and induced pseudoscalar terms. However, for a mixed Fermi-Gamow-Teller transition there is a definite upper limit to the amount of distortion which the induced pseudoscalar interaction can give rise to. This limit is greatly exceeded by the Zr^{89} spectrum and hence the anomaly must have some other origin. It is furthermore concluded that until the reason for these anomalies is understood it would be unsafe to draw any conclusion concerning the strength of the induced pseudoscalar interaction on the basis of low-energy spectrum shapes.

I. INTRODUCTION

STUDIES of allowed nuclear beta decays have shown that of the five possible forms for the beta decay interaction Hamiltonian, linear in the field operators, the vector and axial vector are certainly present, while the scalar and tensor are hardly present at all.[1] On the other hand very little is known about the existence of a pseudoscalar interaction in nuclear beta decay.

In the case of bare fermions, the phenomenon of muon beta decay[2] shows that the vector and axial vector coupling constants, g_V and g_A respectively, have equal magnitude, while the branching ratio of the $\pi \rightarrow e+\nu$ decay mode to the $\pi \rightarrow \mu+\nu$ mode shows that the pseudoscalar interaction is effectively absent.[3] All this is in accordance with the theory of Feynman and Gell-Mann.[4] However, for nuclear beta decay and muon capture the pion dressing of the fermions has to be taken into account and Goldberger and Treiman[5] have shown that this actually leads to the generation of a further weak interaction term, the so-called induced pseudoscalar interaction, of which the coupling constant, b_P, is a function of $(p_\lambda-n_\lambda)^2$, where $(p_\lambda-n_\lambda)$ is the nucleon four-momentum transfer in the transition. These workers were able to show that for muon capture $b_P\approx 8g_A$, whence for a beta decay in which the same momentum transfer takes place the result $b_P\approx 8g_A m_e/m_\mu \approx 8g_A/207$ follows quite readily in their theory. Extensive investigations have been made of muon capture[6] but without any definite conclusion being reached concerning the induced pseudoscalar interaction.

However, the four-momentum transfer never can be the same in beta decay and in muon capture; in fact for the latter it is space-like, since $(p_\lambda-n_\lambda)^2\approx m_\mu^2$, while for beta decay it is always time-like, it being simple to show that

$$(p_\lambda-n_\lambda)^2=-[1+2q(W-p\cos\theta)]. \qquad (1)$$

Here p is the electron momentum; $W=(p^2+1)^{\frac{1}{2}}$ is the electron energy, q is the energy of the emitted neutrino, and θ is the angle between the directions of emission of the electron and the neutrino. Actually, in the theory of Goldberger and Treiman b_P is only a slowly varying function of $(p_\lambda-n_\lambda)^2$ so that radical departures from a value of 8/207 would not be anticipated. However, the point of view which we adopt in this paper is to regard b_P as an unknown parameter to be determined phenomenologically, if possible. Throughout this work we make the assumption that b_P/g_A is constant over a particular spectrum, whence it will be seen from Eq. (1) that this effective value of b_P/g_A can really be expected to be the same only for decays having the same energy and angular distribution of leptons. However, the only alternative, to write b_P as an unknown function of q, W and θ and then to attempt to determine the functional form phenomenologically, would introduce impossible complications.

The Hamiltonian $H_P{}^0$ of the "basic" pseudoscalar interaction, introduced *a priori* as one of the five possible types of weak coupling acting between bare fermions, is, in the usual notation

$$H_P{}^0=g_P(\psi_f{}^\dagger\beta\gamma_5\psi_i)(\psi_e{}^\dagger\beta\gamma_5\psi_\nu). \qquad (2)$$

Here, we have written g_P in place of b_P to indicate the different origin of this coupling form; ψ_i and ψ_f are respectively the initial and final nuclear states, while ψ_e and ψ_ν are the electron and neutrino states. Until recently, it had been assumed that the Hamiltonian

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[1] See E. J. Konopinski, Ann. Rev. Nuclear Sci. 9, 99 (1959).
[2] R. J. Plano, Phys. Rev. 119, 1400 (1960), and references cited therein.
[3] See J. J. Sakurai, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1960).
[4] R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).
[5] M. L. Goldberger and S. B. Treiman, Phys. Rev. 111, 354 (1958).
[6] G. Flamand and K. W. Ford, Phys. Rev. 116, 1591 (1959), and references cited therein.

of the induced pseudoscalar interaction, H_P^{Ind}, had the same form. Then the effects of the induced pseudoscalar interaction arising in all nuclear beta decays would be identical to those which would have arisen from a basic "a priori" pseudoscalar interaction, had not the existence of the latter been disproven, as already mentioned.

Now, Tadic[7] has shown that this assumption is not correct, but that rather a supplementary term, dependent upon the Coulomb field of the nucleus, must be included. In place of (2), the relevant Hamiltonian becomes[8]

$$H_P^{\text{Ind}}=b_P(\psi_f\beta\gamma_5\psi_i) \times\{(\psi_e^\dagger\beta\gamma_5\psi_\nu)-V_c(\psi_e^\dagger\gamma_5\psi_\nu)\}. \quad (3)$$

Here V_c is the Coulomb energy of the decay electron or positron and we shall take for it an average value determined at the nuclear radius, R, i.e., $V_c=\mp\alpha Z/R$, respectively. For muon capture the first term, which just has the form of H_P^0, has to be weighted by the muon mass, and it is therefore unlikely that Tadic's considerations would have an appreciable effect on any of the calculations that have been made. On the other hand, the extra term will actually be dominant in the case of nuclear beta decay, and thus the various spectral analyses which have hitherto been undertaken in the (inconclusive) search for a pseudoscalar interaction must now be regarded as irrelevant, since they can refer only to the nonexistent "basic" pseudoscalar interaction.

In the conventional terminology the lowest degree of forbiddenness in which a pseudoscalar interaction can contribute is the first, this being in the first forbidden $\Delta I=0$ (yes) transitions. The pseudoscalar contribution to allowed transitions is said to be second forbidden, except for the pure Fermi transitions, where there can be no pseudoscalar contribution at all. Thus it is frequently said that the former transitions offer the best chance of revealing any pseudoscalar interaction that may exist and for this reason the search has been confined almost entirely to such transitions, especially the $0^-\rightarrow 0^+$ decay of Pr^{144}. The most recent and elaborate analysis of this decay in terms of a "basic" pseudoscalar interaction is that of Bhalla and Rose,[9] while Tadic[7] has made a similar analysis in terms of an induced pseudoscalar interaction. This latter work shows $b_P=0$ to be consistent with experiment, but some ambiguity is introduced by the appearance of a nuclear matrix element ratio whose value is rendered uncertain by its dependence on nuclear radial wave functions.

Now the conventional degree of forbiddenness terminology overlooks the peculiarities[10] arising from the $\beta\gamma_5$ *nuclear* operator, which occurs in both forms of pseudoscalar interaction. For this reason we are able to argue (in the next section) that the degree of forbiddenness nomenclature should be modified for a contribution from a pseudoscalar interaction of either kind and that the allowed (except pure Fermi) transitions would be the more fruitful source of evidence for any pseudoscalar interaction that may exist. Not only that, but for these transitions the single matrix element ratio that is involved is independent of radial functions and can therefore be calculated fairly reliably in the region of pure *j-j* coupling. There will thus be a minimum of ambiguity in the interpretation of these transitions.

We now take notice of the fact that the Langer group has observed anomalous spectrum shapes for precisely these spectra, the distortion being a low energy one of the form $(1+a/W)$.[11] These distortions are remarkable in that a has the same sign for both electron and positron emitters, thereby eliminating an explanation in terms of a Fierz interference between the scalar and vector or tensor and axial vector interactions. For the same reason electron screening effects cannot be held responsible. That these anomalies have their origin in an induced pseudoscalar interaction is made even more plausible by the fact that the only undistorted allowed spectrum which this group has observed is a pure Fermi one.[12] This paper, then, is concerned with an attempt at such an interpretation of the anomalies.[13]

2. NONRELATIVISTIC LIMIT OF THE PSEUDOSCALAR INTERACTION

Since H_P^0 and H_P^{Ind} differ only in their lepton covariants, it is possible to write both (2) and (3) in the nuclear matrix element form

$$H_P=\langle\psi_f|L\beta\gamma_5|\psi_i\rangle, \quad (4)$$

where L is the appropriate lepton covariant and the coupling constants have been omitted. The difficulties which arise from this essentially relativistic matrix element having to be evaluated in terms of nonrelativistic nuclear wave functions are well-known. The problem is one of reducing the odd nuclear operator, $\beta\gamma_5$, to even form; clearly, the same difficulties will be equally present for either kind of pseudoscalar interaction and in the following remarks we shall not have to distinguish between the two forms. There is no unique way of proceeding to the nonrelativistic limit

[7] D. Tadic, Nuclear Phys. **26**, 338 (1961).

[8] Since we are concerned only with spectrum shapes we do not have to consider parity nonconservation explicitly.

[9] C. P. Bhalla and M. E. Rose, Phys. Rev. **120**, 1415 (1960).

[10] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1315 (1954).

[11] O. E. Johnson, R. Johnson, and L. M. Langer, Phys. Rev. **112**, 2004 (1958); J. H. Hamilton, L. M. Langer, and W. G. Smith, Phys. Rev. **112**, 2010 (1958) and Phys. Rev. **119**, 772 (1960).

[12] L. M. Langer, D. C. Camp, and D. R. Smith, Bull. Am. Phys. Soc. **6**, 334 (1961).

[13] In a previous work by the author [J. M. Pearson, Bull. Am. Phys. Soc. **6**, 34 (1961)], it was claimed that these anomalies could be explained in terms of what was effectively the "basic" pseudoscalar interaction. However, this conclusion is vitiated by a serious error in the calculation.

of (4) and so different authors have proposed different approximations with the result that there are no clear predictions as to what the observed effects of a given pseudoscalar interaction would be. To analyze the observed spectra in terms of these different possibilities would probably be inconclusive. Rather, we make the following crude examination of the different nonrelativistic limits of (4) and try to establish the domain of validity of each.

In the absence of any knowledge of relativistic nuclear wave functions, the simplest procedure is to regard the nucleons as Dirac particles moving independently in a shell model potential $V(r)$ and express the small components v in terms of the large components u. We write the Dirac equation for the nucleon as

$$[-\boldsymbol{\alpha}\cdot\mathbf{p}-\beta M+V(r)]\psi=W\psi,$$

where we have used the usual notation with

$$\beta=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

and

$$\psi=\begin{pmatrix}v\\ u\end{pmatrix}.$$

Then

$$v=-(\varphi/2M)\boldsymbol{\sigma}\cdot\mathbf{p}u \tag{5}$$

where

$$\varphi=\left(1+\frac{W-M-V}{2M}\right)^{-1}\approx 1+\frac{V-W+M}{2M}. \tag{6}$$

Rose and Osborn[10] made the approximation $\varphi=1$; whereupon they found that H_P vanished if the traditional procedure of regarding the lepton covariant as constant over the nucleus was followed. Hence, the lepton covariant must be kept inside the matrix element of (4), the nonrelativistic limit of which we write as $\langle u_f|\mathcal{H}_P|u_i\rangle$. In the approximation of these authors, then, we have

$$\mathcal{H}_P=\{\boldsymbol{\sigma}\cdot\mathbf{p}L\}/2M\equiv\mathcal{H}_P{}^L, \tag{7}$$

for $\gamma_5=i\alpha_1\alpha_2\alpha_3$. The important feature of (7) is that $\boldsymbol{\sigma}\cdot\mathbf{p}$ is essentially a gradient operator acting on the lepton covariant alone.

In contrast to this, Konopinski[14] regarded the lepton covariant as a constant, to be evaluated at some mean effective nuclear radius $r=R$, but did not put $\varphi=1$. Neglecting small terms, we are left with a potential gradient term

$$\mathcal{H}_P=L(\boldsymbol{\sigma}\cdot\mathbf{p}V)/4M^2\equiv\mathcal{H}_P{}^V. \tag{8}$$

Let us now take for V the harmonic oscillator potential

$$V=-V_0+\tfrac{1}{2}M\omega^2r^2, \tag{9}$$

[14] E. J. Konopinski, Phys. Rev. **94**, 492 (1954).

where $\omega\approx 75/A^{\frac{1}{3}}$.[15] Then Konopinski's treatment gives

$$\mathcal{H}_P{}^V=-(i/4M)\omega^2L\boldsymbol{\sigma}\cdot\mathbf{r}. \tag{10}$$

If the variations of both L and φ are considered, then, after neglecting further small terms, we have simply

$$\mathcal{H}_P=\mathcal{H}_P{}^L+\mathcal{H}_P{}^V. \tag{11}$$

We first compare the role of these two terms in different transitions; essentially, we shall make a degree of forbiddenness analysis. This involves in the usual way, a multipole expansion of the lepton covariant, the various terms of which have the familiar form $F_\lambda(r)Y_\lambda{}^\mu(\theta,\varphi)$, where F_λ varies as $r^\lambda\{1+O(r^2)\}$ for small r. The only important term will be the one having the lowest possible value of λ consistent with a nonvanishing matrix element. This will be given by $\lambda=\Delta I$ or $\Delta I+1$, where $\Delta I\equiv|I_i-I_f|$ is the nuclear spin change in the transition. The actual choice of λ will be determined by the nuclear parity change according to $\pi_i\pi_f=(-)^{\lambda+1}$. In the case of either I_i or I_f being zero λ can take only the value ΔI whence there can be no pseudoscalar contribution at all if $\pi_i\pi_f=(-)^{\Delta I}$. Otherwise the pseudoscalar contribution is conventionally designated as having degree of forbiddenness $n=\lambda+1$, while for the same transition the axial vector (and possibly the vector) interaction will give an $(n-2)$th forbidden correction. The only exception is the $\Delta I=0$ (yes) transitions, for which both the axial vector and pseudoscalar interactions give first forbidden contributions.

For a given λ the term $\mathcal{H}_P{}^V$ varies as $r^{\lambda+1}$, because of the $\boldsymbol{\sigma}\cdot\mathbf{r}$ factor, while because of the gradient operation $\mathcal{H}_P{}^L$ varies as $r^{\lambda-1}$. The only exception to this latter statement is the case of $\lambda=0$ for which $\mathcal{H}_P{}^L$ must vary as r, the first term in the derivative of F_0 vanishing. However, the $\lambda=0$ term can only give a nonvanishing contribution in the $\Delta I=0$ (yes) decays; these form a special case to be discussed later. In general, we have the result that for a pseudoscalar contribution that is conventionally designated as nth forbidden $\mathcal{H}_P{}^V$ varies as r^n while $\mathcal{H}_P{}^L$ varies as r^{n-2}. Although $\mathcal{H}_P{}^V$ is weighted by a factor of $\omega^2/2$, this is not sufficient to prevent it being dominated by the lepton gradient term, so that $\mathcal{H}_P$ varies essentially as r^{n-2}.

For the $\Delta I=0$ (yes) transitions both $\mathcal{H}_P{}^L$ and $\mathcal{H}_P{}^V$ vary as r, so that the radial behavior is appropriate to the conventional statement that $\mathcal{H}_P$ gives a first forbidden contribution. But then by the same token all the other so-called nth forbidden pseudoscalar contributions to $(n-2)$th forbidden transitions should themselves be regarded as $(n-2)$th forbidden. Of course, a considerable retardation is introduced by the nuclear mass factor $1/M$ but this occurs in the $\Delta I=0$ (yes) case also, so if the pseudoscalar contribution there is called first forbidden it would be misleading to

[15] See S. A. Moszkowski, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39.

call the pseudoscalar contribution (if it exists at all) to any other nth forbidden transition as anything but nth forbidden itself. Thus, it appears that the relative role of the pseudoscalar interaction will be the same in all transitions to which it can give any contribution at all.

However, the latter statement requires qualification because of the unique property of the $\Delta I=0$ (yes) transitions in having a significant, in fact dominant, contribution from $\mathcal{H}_P{}^V$. Since this term is essentially $\lambda=0$ it will tend to give rise to an allowed spectrum shape. Hence of all transitions these will be the *least* likely to show pseudoscalar distortions of the spectrum shape. Rather one should examine the allowed (other than pure Fermi) and the unique spectrum shapes. The nonunique nth forbidden spectra (other than those of $0\longleftarrow\longrightarrow n$ transitions) should also show the effects of a pseudoscalar interaction, but an unambiguous interpretation will be impossible because of the several nuclear matrix elements that will be involved. All these transitions may presumably be adequately treated by the methods of Rose and Osborn. Only in the special case of the $\Delta I=0$ (yes) transitions would their methods seem inappropriate.

The statement that the pseudoscalar interaction gives an nth forbidden contribution in transitions with $\Delta I=n$ (not $n\longleftarrow\longrightarrow 0$) or $n\pm 1$, and parity change $=(-)^n$ shows a strong similarity with Gamow-Teller interactions. The only difference is that for the latter it is only the $0\rightarrow 0$ transitions that are completely prohibited. It is interesting to make the comparison between the pseudoscalar and Gamow-Teller interactions in the familiar picture in which the leptons are considered nonrelativistically. Then the degree of forbiddenness becomes equal to the total orbital angular momentum l_L carried off by the leptons and the distinction between Fermi and Gamow-Teller interactions is that the former sends the leptons off in a singlet state, whereas the latter sends them off in a triplet state. For the Gamow-Teller interaction, then, the total lepton angular momentum J_L can have three values, l, $l\pm 1$, except of course for $l=0$, i.e., allowed transitions, where J_L must be 1. On the other hand, for the pseudoscalar interaction, J_L can only have the two values $l\pm 1$. It is as though the lepton *pair* sent off by a pseudoscalar interaction has only two degrees of orientation with respect to its orbital angular momentum, despite the intrinsic spin being 1.

3. SHAPE FACTOR

Following the method of Rose and Osborn[10] it is a straightforward matter to derive the shape factor of allowed transitions in which there is a contribution from an induced pseudoscalar interaction. For a pure Gamow-Teller transition, i.e., $\Delta I=1$ (no), only the axial vector and pseudoscalar interactions can be effective, and the shape factor, denoted by $C_{AP}{}^{01}$, can be expressed in terms of the parameter $\Gamma=b_P/Mg_A$ and the nuclear matrix element ratio

$$\rho=\sqrt{2}\langle u_f|T_{12}{}^{-m}(\hat{r},\boldsymbol{\sigma})|u_i\rangle/\langle u_f|T_{10}{}^{-m}(\hat{r},\boldsymbol{\sigma})|u_i\rangle. \quad (12)$$

It is essential to note that the radial arguments appearing here in the spherical tensor operators, which are those of Rose and Osborn,[16] are unit ones. Thus, ρ depends only on the nuclear coupling scheme and in particular it is free from the ambiguities associated with nuclear radial wave functions.

For the transitions of interest we have

$$\xi\equiv\alpha Z/2R\gg W_0,$$

the energy end-point, in which approximation the shape factor can be written

$$C_{AP}{}^{01}=1-\frac{\Gamma}{3}\left[2(2+\rho)\xi^2\pm\rho\xi\frac{1}{W}\right]+\frac{\Gamma^2}{9}\left[(2+\rho)^2\xi^4\pm\rho(2+\rho)\xi^3\frac{1}{W}\right], \quad (13)$$

where in the choice of signs the plus sign refers to electron emission, and the minus sign to positron emission. In this approximation the AP cross term, i.e., the term linear in Γ, agrees with Eq. (A.50) of Tadic.[7] However, the term in Γ^2, which turns out to be of critical importance, is not considered by this author.

Our shape factor displays the required energy dependence, but it will be seen that in both the term in Γ and the one in Γ^2 there are constant terms whose order of magnitude is ξ times greater than that of the energy dependent terms. Thus the only way in which any appreciable spectral distortion could occur would be through an almost complete interference between the constant terms of the pseudoscalar contribution and that of the axial vector. This is certainly possible since Eq. (13) can be rewritten as

$$C_{AP}{}^{01}=\left\{1-\frac{\Gamma}{3}(2+\rho)\xi^2\right\}^2\mp\frac{\Gamma\xi\rho}{3}\left\{1-\frac{\Gamma}{3}(2+\rho)\xi^2\right\}\frac{1}{W}. \quad (14)$$

Thus the magnitude of the constant term has no lower limit and any observed $(1+a/W)$ distortion of a pure Gamow-Teller transition may be interpreted in terms of an induced pseudoscalar interaction with an appropriate value of Γ, the distortion parameter a being given by

$$a=\mp\frac{\Gamma\xi\rho}{3-\Gamma(2+\rho)\xi^2}. \quad (15)$$

Because of the dependence of a on both ρ and ξ we should expect very different distortions[17] in the different pure Gamow-Teller transitions, and in particular both signs of a should be possible.

[16] M. E. Rose and R. K. Osborn, Phys. Rev. **93**, 1326 (1954).
[17] Actually, for $\rho=0$, there will be no distortion at all.

In the case of the mixed Fermi-Gamow-Teller allowed transitions, i.e., $\Delta I=0$ (not $0\rightarrow 0$), there is a contribution from the vector interaction and the shape factor becomes

$$C_{VAP}^{00}=\{1+|P|^2C_{AP}^{01}\}/\{1+|P|^2\}, \qquad (16)$$

where $P=\langle u_f|\boldsymbol{\sigma}|u_i\rangle/\langle u_f|1|u_i\rangle$, the ratio of the Gamow-Teller to the Fermi matrix elements.[18] The distortion parameter is now

$$a=\mp\frac{\Gamma\xi\rho[1-\frac{1}{3}\Gamma(2+\rho)\xi^2]}{3\{|1/P|^2+[1-\frac{1}{3}\Gamma(2+\rho)\xi^2]^2\}}. \qquad (17)$$

Then for given values of P, ρ, and ξ there will be an upper limit on the magnitude of the distortion, since a complete interference among the constant terms is no longer possible. It is necessary to note that this conclusion depends upon the inclusion of the terms in Γ^2. Without these it is easy to see that with the appropriate value of Γ any value of a is possible. The importance of the terms in Γ^2 is due essentially to the fact that they acquire a significant magnitude when Γ is large enough to cause appreciable interference among the constant terms.

4. RESULTS AND CONCLUSION

The allowed pure Gamow-Teller spectra measured by the Langer group[11] are those of Na^{22}, P^{32}, and In^{114}, while a single mixed Fermi-Gamow-Teller spectrum, that of Zr^{89}, has been measured by this group. In all four cases $0.2<a<0.4$. However, because ρ can only be calculated reliably in the region of pure j-j coupling we have to confine our attention to the In^{114} and Zr^{89} decays.

Using the methods of Rose and Osborn,[16] we calculate ρ to have the value $-1/2$ and $8/11$, respectively, while for the latter case $|P|^2=11/(12\pi)$. We now find that the In^{114} spectrum shape can be fitted with $\Gamma\approx 0.02$, i.e., $b_P\approx 40g_A$. This interpretation of the observed value of a depends very much on a heavy cancellation taking place in the denominator of Eq. (15); otherwise the distortion will be very small. In fact in the limit of $\Gamma\rightarrow\infty$ we calculate $a=0.03$. On the other hand it is found that the maximum possible distortion for the Zr^{89} case corresponds to $a\approx 0.002$. Hence it is not at all possible to interpret this anomaly in terms of an induced pseudoscalar interaction. It is important to note, however, that if the terms in Γ^2 had not been included in the shape factor then a fit would have been possible with $\Gamma\approx 0.02$ again.

One must therefore look for an alternative origin of the Z^{89}, anomaly and it would be difficult to escape the conclusion that this, rather than an induced pseudoscalar interaction, was at least partially responsible for the In^{114} anomaly, and indeed for the other two, as well. Because one cannot make a reliable calculation of ρ for the Na^{22} and P^{32} cases one cannot rule out an explanation in terms of the induced pseudoscalar interaction, but it is a little surprising that these both have the same value of a as do the Zr^{89} and In^{114} spectra,[19] since this circumstance could only arise from a fortuitous coincidence in the values of ρ and ξ.

Although we have shown that the induced pseudoscalar interaction cannot be completely responsible for these anomalies there remains the problem of determining b_P, or at least setting an upper limit on it. But until such time as the alternative distorting mechanism responsible for at least the Zr^{89} anomaly is understood, it will be very difficult to learn anything about b_P from the allowed spectra. Furthermore, in the absence of knowledge to the contrary, one must assume that the $\Delta I=0$ (yes) transitions are likewise affected in an unknown way. Unfortunately, a recent investigation[20] of alternative explanations of the allowed anomalies was not successful. We therefore conclude that one cannot obtain any reliable upper limit on b_P from low-energy spectrum shapes.

One interesting conclusion emerges from the supposition that all allowed transitions other than pure Fermi ones are distorted. For, then, the standard f functions,[21] involving an integration over the allowed spectrum, will have to be modified. In particular, this will call for a re-evaluation of g_A, as determined from various allowed transitions, while g_V will be unaffected, since this is evaluated solely from pure Fermi transitions.[22] If for the neutron decay a were to be 0.4 then the value $|g_A/g_V|^2=1$ would lie within the limits of experimental error.

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[18] As a rough approximation we have assumed $|g_V|=|g_A|$.

[19] In connection with the constancy of the distortion parameter a, it should be recalled that $a=0$ has been obtained in the case of a pure Fermi transition (see reference 12). This should minimize any suspicion that the distortions are due to some instrumental error.

[20] B. Eman and D. Tadic, Glasnik mat.-fyz. i Astron., **16**, 89 (1961).

[21] S. A. Moszkowski and K. M. Jantzen, University of California at Los Angeles Technical Report UCAL-10-26-55 (unpublished).

[22] See O. C. Kistner and B. M. Rustad, Phys. Rev. **114**, 1329 (1959).