Elastic Scattering in the N-N, C-N, and C-C Systems*

A. M. Smith† and F. E. Steigert
Sloane Physics Laboratory, Yale University, New Haven, Connecticut
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Differential cross sections have been measured for the elastic scattering of N^{14} from C^{12}, N^{14} from N^{14}, and C^{12} from C^{12} to a relative precision of about 5%, and an absolute accuracy of 15%. Incident energies of nominally 10 Mev/amu were employed, and an angular range was covered from approximately 4° to 24° in the center-of-mass system. The use of solid-state detectors rendered energy resolution of the order of 1.5%. Theoretical cross sections calculated from both the simple-diffraction model and the sharp-cutoff model were fitted to the experimental data, giving interaction radii for the three reactions of 6.44, 6.68, and 6.23 fermi, respectively.

INTRODUCTION

OVER the past several years an increasing number of elastic scattering reactions have been studied using as projectiles alpha particles and heavier ions. Hopefully, the accumulation of these data when analyzed in terms of the several recipes now extant[1–3] will give information on the nature of the nuclear surface. Accordingly, Yavin and Farwell[4] have employed both sharp cut off and diffraction analysis to interpret their data on the scattering of alpha particles from heavier nuclei. Similarly, the nitrogen scattering results of the Oak Ridge group[5] have been analyzed in terms of both the sharp cutoff model and the more complete optical model.[6] Since these initial attempts several groups have reported results using heavy ions at higher energies.[7–10] While these latter experiments afford the advantage of observing the scattering at energies equal to and above the Coulomb barrier, by the same token they present the difficulty of discriminating against inelastic processes. This paper reports the results of the scattering of N^{14} from both N^{14} and C^{12}, and C^{12} from C^{12} in this same energy region, but with the added feature of utilizing the higher resolving power of silicon junction detectors to reduce the non-elastic background. The parameterization of the experimental data has been performed so as to best reproduce the locations of the observed extrema.

EXPERIMENTAL APPARATUS

Figure 1 shows the essential features of the experimental apparatus which is described in detail elsewhere.[9] A few remarks will explain the main features. A magnetically analyzed beam of heavy ions enters the beam collimating system through a 4.8-mg/cm² nickel foil, and is scattered at the target center. The detector and the detector collimating system are contained in a stainless steel tube which can be rotated from $-20°$ to $+80°$ (laboratory system) about the scattering volume. Grazing angles expected on the basis of the sharp cutoff model are in the neighborhood of 4° 30′ in the center of mass system. Since work was to be done in this angular region it was deemed impractical to design a system in which a detector and a Faraday cup could be used simultaneously. A beam monitor, consequently, is used to obtain relative cross sections. It consists of a shielded CsI crystal mounted to a photomultiplier, the pulses from which are subsequently amplified and fed into an integral discriminator and scaler. The monitor could be calibrated by simply swinging the vacated detector tube to the $\theta=0$ position, fitting a Faraday cup on the end of the tube, and then establishing the ratio of monitor counts to the charge accumulated in the Faraday cup. Absolute cross sections were determined in this manner.

The only essential difference in the apparatus from that described in reference 9 is found in the detector. In this experiment a silicon junction detector is used. It was biased sufficiently positive that the sensitive region extended beyond the maximum residual range of the

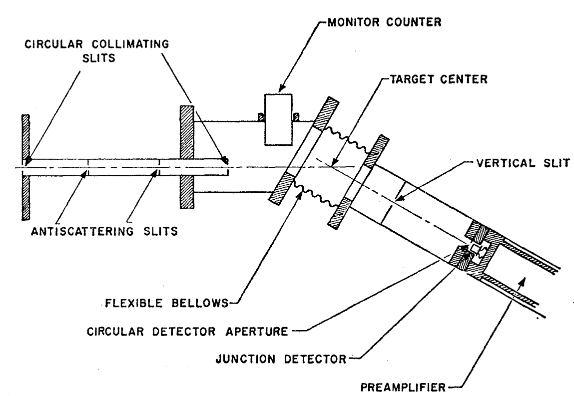

FIG. 1. A schematic view of the experimental apparatus in the plane of scattering. This differs from that shown in reference 9 only in the detector assembly.

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† I.B.M. Fellow 1960–1961.
1 J. S. Blair, Phys. Rev. **95**, 1218 (1954).
2 G. P. Placzek and H. A. Bethe, Phys. Rev. **57**, 1075 (1940).
3 Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
4 A. I. Yavin and G. W. Farwell, Nuclear Phys. **12**, 1 (1959).
5 H. L. Reynolds and A. Zucker, Phys. Rev. **102**, 1378 (1956).
6 C. E. Porter, Phys. Rev. **112**, 1722 (1958).
7 E. Goldberg and H. L. Reynolds, Phys. Rev. **112**, 1981 (1958).
8 J. A. McIntyre, S. D. Baker, and T. L. Watts, Phys. Rev. **116**, 1212 (1959).
9 E. Newman, P. G. Roll, and F. E. Steigert, Phys. Rev. **122**, 1842 (1961).
10 P. G. Roll, E. Newman, and F. E. Steigert, Nuclear Phys. **25**, (1961).

elastically scattered particles. A typical resolution of about 1.5% was obtained. The output pulses were amplified, shaped, and subsequently analyzed by an R.I.D.L. 400 channel analyzer.

Two cases of geometry were used. One incorporated a set of beam collimating apertures and detector aperture of 0.032 in., affording an angular resolution (full width at half-maximum) of about 18′. The other utilized beam collimating apertures of 0.100 in. in radius, and a detector aperture of 0.050 in. in radius. In this case the angular resolution was about 40′. In both cases the vertical slit in the detector collimating system was 0.064 in. wide.

Standard commercial grade nitrogen and methane were used for targets. The gases were flowed through the apparatus at a rate of approximately 2 cubic centimeters per minute, and maintained at a pressure of 50 mm Hg in the cases of N-N and N-C, and 100 mm Hg in the case of the C-C reaction.

To determine the absolute zero of the scattering angle, data points were taken for both negative and positive angles. It was found that the average path followed by the beam of ions through the collimating systems varied during the course of time. To correct for this, calibration runs were made at appropriate intervals at a fixed angle. The detector count to monitor count ratios thus obtained were corrected for target density variation, and compared to an approximate relative cross-section curve. From this comparison a correction curve was made giving the variation in scattering angle vs time, and used to correct the data. This procedure reduced the uncertainty in angle to about $\pm 5'$ in the case of large-angle geometry, and $\pm 3'$ in the case of small-angle geometry.

After appropriate dead-time corrections were applied, the detector yield to monitor yield ratio was converted to a relative cross section by using the measured target density and a geometrical factor (G) calculated by the formula of Critchfield and Dodder.[11] The second- and third-order corrections of this formula were usually 2 to 3%, and not more than 4%.

The energy of the incident ions at the center of the scattering volume and in the Faraday cup was de-

TABLE I. Estimated experimental errors.

Source	Small-angle geometry	Large-angle geometry
Apparatus	2.5%	2.0%
G factor	0.5%	1.5%
Counting statistics	2.0%	2–4%
Target density	1.5%	1.5%
Angle	$\pm 3'$	$\pm 5'$
Energy	2.0%	2.0%
Relative error	5%	6%
Absolute error	15%	15%

[11] C. L. Critchfield and P. C. Dodder, Phys. Rev. **75**, 419 (1949).

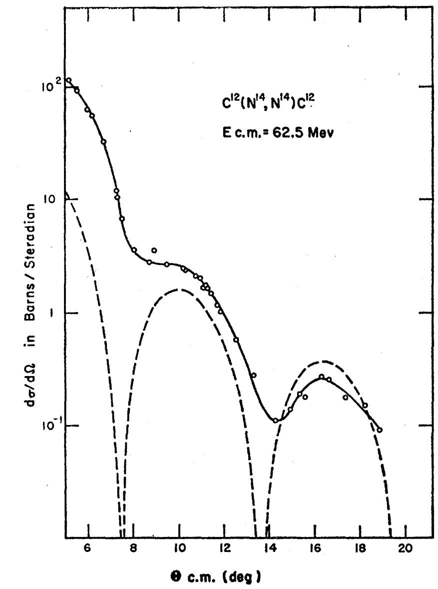

FIG. 2. The differential elastic scattering cross section of N^{14} from C^{12}. The dashed curve is the theoretical prediction by the diffraction model for a value of $r_0 = 1.37$ fermi.

termined by range energy curves of Roll and Steigert,[12] and Martin.[13] To obtain absolute cross sections the ratio of monitor yield to charge collected in the Faraday cup was determined. Then without changing the monitor discriminator level several data points were taken using the detector by the usual procedure. Combining these measurements, an absolute cross section for these

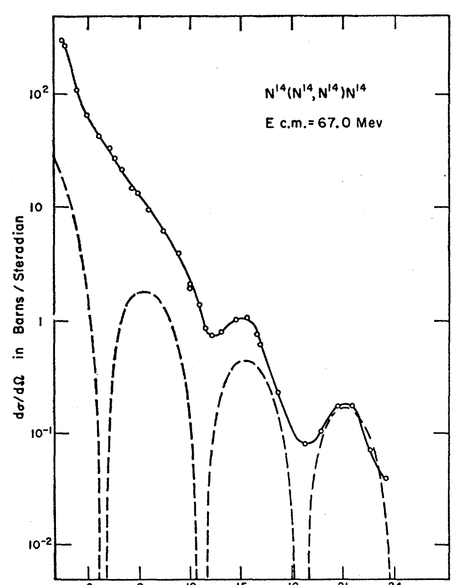

FIG. 3. The differential elastic scattering cross section of N^{14} from N^{14}. The dashed curve is the theoretical prediction by the diffraction model for a value of $r_0 = 1.39$ fermi.

[12] P. G. Roll and F. E. Steigert, Nuclear Phys. **16**, 534 (1960).
[13] F. W. Martin (private communication).

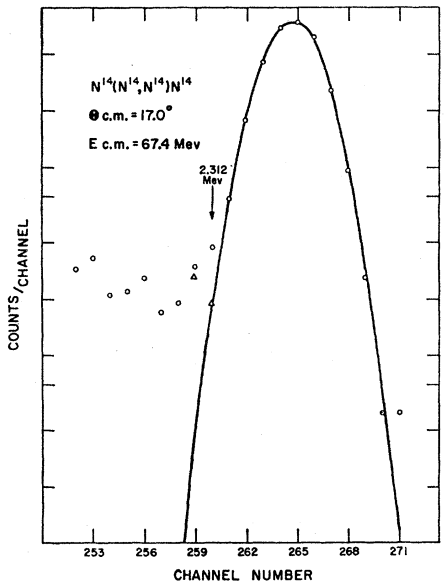

FIG. 4. A portion of the detector energy spectrum for the N^{14}-N^{14} reaction. The peak, due to elastic scattering, is shown with a resolution sufficient to eliminate the first excited state of N^{14} indicated by the arrow. The triangles represent the hypothetical value of the background count in their respective channels.

data points was thus established to which the relative cross sections were then normalized.

Table I gives the estimated errors. The error in the G factor is due to uncertainty in the condition that the entrance aperture foil produces an isotropic source of rays over its entire surface—a requisite of the G-factor calculation. Counting statistics for the target angles at which data were collected in the N-N, and C-C runs were necessarily increased from running time considerations. It was not felt that counting losses due to variation in beam intensity could be accurately estimated and since they are known to be small they were omitted.[9] Corrections for slit edge scattering and multiple scattering have also been omitted since their estimate is again small.[9]

For relative cross sections therefore the total error was estimated to be 5% in the case of small geometry and 6% in the case of large geometry. The absolute cross section is believed to be accurate to 15%.

RESULTS

Figures 3–5 show the absolute cross sections which are characterized by the usual oscillations. From the simple diffraction model the following expression can be derived,[2] and used to extract interaction radii.

$$d\sigma/d\Omega = K^2R^4\cos^2(\Theta/2) \times[J_1(2KR\sin(\Theta/2))/2KR\sin(\Theta/2)]^2.$$

The derivation assumes a model in which the nucleus acts as a totally absorbing disk whose radius is defined by the intersection of the nuclear sphere with the focal axes of classical hyperbolic orbits. Here K is the momentum of the incident wave, Θ the center-of-mass scattering angle, and R the interaction radius. The cross section of identical particles is of course

$$d\sigma/d\Omega = |f(\Theta)+f(\pi-\Theta)|^2.$$

The theoretical curves predicted by these formulas are fitted to the data by a proper choice of KR. From this value R is calculated, and presented in Table II. The total error in R was estimated to be $\pm 2.6\%$.

The cross section derived from the sharp-cutoff model can be written in the usual notation[14] for nonidentical particles as

$$\frac{d\sigma}{d\Omega} = \left|\frac{ZZ'e^2}{2Mv^2}\csc^2(\Theta/2)\exp[-2i\eta\ln\sin^2(\Theta/2)] - \frac{i}{K}\pi^{\frac{1}{2}}\sum_{l=0}^{l=l'}(2l+1)^{\frac{1}{2}}e^{2i(\sigma_l-\sigma_0)}Y_{l,0}(\Theta)\right|^2.$$

For the case of identical particles, the cross sections must be evaluated using both $f(\Theta)$ and $f(\pi-\Theta)$, and the necessary spin factors. An IBM 650 computer was used to compute the ratios of the sharp cut off cross section to the Coulomb cross section both for the nonidentical particle case of N-C, and the identical particle cases of C-C and N-N. Similar ratios of experimental data to the Coulomb cross section were calculated, and are plotted along with the best-fit computed curves in Fig. 6. From this model the radii are determined using the equation

$$E_{\mathrm{c.m.}} = ZZ'e^2/R + \hbar^2 l'(l'+1)/2MR^2,$$

where M is the reduced mass and l' the sharp-cutoff

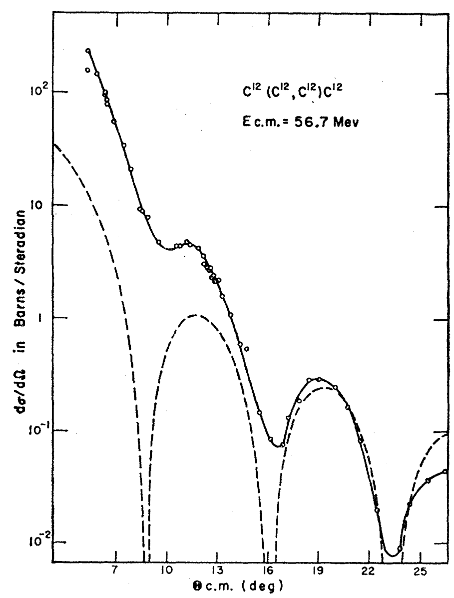

FIG. 5. The differential elastic scattering cross section of C^{12} from C^{12}. The dashed curve is the theoretical prediction by the diffraction model for a value of $r_0 = 1.36$ fermi.

[14] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. 8.

value of l. The optimum values of l' are given in Table II.

The case of N-C can be compared to the C-N reaction performed recently by Newman.[9] The reported cross section, and his quoted radii of 6.45 fermi from the diffraction model and 6.99 fermi from the sharp-cutoff model are in good agreement with the values recorded here. The differences between the two sets of data would appear to be accounted for by the method used here in correcting for the beam drift. Zucker[15] has recently analyzed the elastic scattering of N^{14} from C^{12} at 27.3 Mev in terms of the optical model giving an $r_0=1.275$ fermi. Blair has pointed out that the optical model radius can be expected to be less than the sharp-cutoff radius, as it is in this case, by an amount d, the surface "thickness" parameter; and that this is due to the attractive tail of the nuclear potential.

In the case of the N^{14}-N^{14} system a comparison can again be made to work of Reynolds and Zucker.[5] For a laboratory energy of 21.7 Mev an interaction radius deduced from the sharp-cutoff model of $2(4.0\pm0.2)$ fermi is quoted. One expects a smaller sharp cut off radius at higher energies, however, because for this model the interaction distance R is roughly inversely proportional to the system energy. A typical N-N spectrum is shown in Fig. 8 of reference 5. As can be seen, the 2.312 Mev, $T=1$ level in nitrogen, can just be resolved, and no observable contribution from this level is found. An upper limit to scattering from this level can be assigned as being approximately 1.8% of the elastic scattering cross section.

The fit of the sharp-cutoff calculations to the experimental data for the case of C^{12}-C^{12} is perhaps the worst of the three reactions studied. The strong rise of the ratio $\sigma/\sigma_{\mathrm{Coul}}$ above unity at small angles is particularly puzzling. If experimental in origin, this could arise from one of three sources. An error in the reaction energy and hence the computed Mott cross section, or an erroneous normalization with the Faraday would both shift the entire experimental curve vertically. Repeated checks would seem to limit such a vertical scaling to a maximum of about 14% (or a root mean square of 11%). An

TABLE II. Extracted interaction radii.[a]

Reaction	Radius by diffraction model	Radius by sharp-cutoff model	l'
N-C	6.44±0.17 1.37±0.04	7.00 1.49	28
N-N	6.68±0.15 1.39±0.04	7.20 1.49	31
C-C	6.23±0.16 1.36±0.04	6.59 1.44	24

[a] All radii are in fermi. The upper number of each group is R the interaction radius, the lower number is $r_0=R/(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$.

[15] A. Zucker, and M. L. Halbert, *Proceedings of the Second Conference on Reactions Between Complex Nuclei, Gatlinburg, Tennessee, 1960* (John Wiley & Sons, Inc., 1960), pp. 144 ff.

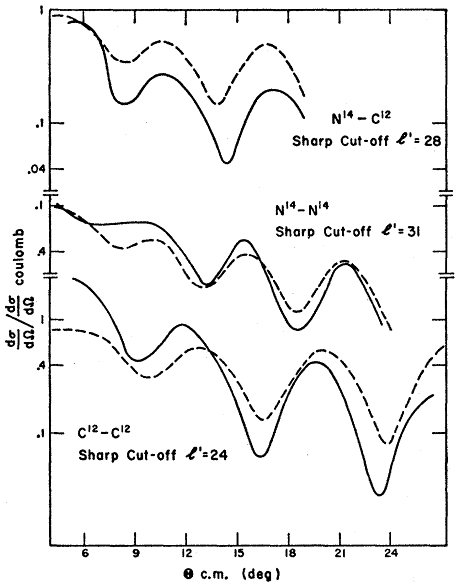

FIG. 6. The ratio of the experimental elastic scattering cross section to the Coulomb scattering cross section for all three reactions studied. The dashed curves represent the same ratio calculated from the sharp-cutoff model for the l' values given in Table II.

underestimate of the effects of multiple scattering would on the other hand result in an anomalous increase at the forward angles. A series of runs using a constant beam energy and fixed scattering angle but with a target density varying from 5 mm to 100 mm of mercury gas pressure showed no variation in yield to within the net 3% counting statistics. If multiple scattering were indeed the source of the discrepancy, this variation of a factor of 20 in gas pressure would be expected to produce a noticeable monotonic increase in the yield rather than the random variation obtained. Additional data will be taken to further investigate this apparent anomaly in magnitude.

The value of the interaction radius reported here is considerably smaller than that found by Bromley[16] using energies on the order of 10 Mev per ion. He explains the difference in terms of a deformation in the colliding nuclei to form a resonance structure similar to that of a metastable molecule. At energies used in this experiment, apparently no such effect occurs.

ACKNOWLEDGMENTS

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[16] D. A. Bromley, J. A. Kuehner, and E. Almquist, reference 15, pp. 151 ff.