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Decay of Sb^{115}†

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Sb^{115} (32 min) decays by electron capture and positron emission to Sn^{115}, the positron branch accounting for $(12\pm3)\%$ of the disintegrations. Of the total disintegrations 94% lead directly to a previously known 499-kev level in Sn^{115}. The remaining 6% of the disintegrations are to an excited state in Sn^{115} and are followed by two gamma rays in cascade whose energies are 980 kev and 1240 kev, respectively. The crossover gamma ray of 2220 kev has also been seen, with an intensity equivalent to 1% of the disintegrations. The cascade and crossover gamma rays lead to the 499-kev level and coincidences have been observed between these gamma rays. An upper limit of 0.4% was obtained for a beta branch from In^{115m} to the 499-kev level in Sn^{115}. These levels in Sn^{115} seem to be too high in energy to correspond to the predicted low-lying levels in Sn^{115} of Kisslinger and Sorensen.

INTRODUCTION

THE interest in the tin isotopes stems in part from the fact that the protons are in a major closed shell and calculations of excited levels are simplified by virtue of this fact.[1] Kisslinger and Sorensen,[1] for example, have made calculations for single closed shell nuclei considering the effects of a pairing correlation, and they have predicted in detail the low-lying excited levels of the tin isotopes. Sn^{115} in particular is of additional interest from the point of view of the systematics of isomerism in the odd tin isotopes. The odd isotopes of tin[2] from Sn^{117} to Sn^{125} exhibit isomerism and this has been attributed to the fact that in these isotopes two low-lying adjacent levels have spins and parities of $11/2-$ and $3/2+$. In Sn^{113} an isomer has recently been reported,[3] but the ground and first excited levels have been assigned the spins and parities $1/2+$ and $7/2+$, respectively. No isomer has as yet been reported in Sn^{115}. The question thus arises whether an isomer exists in Sn^{115} similar in kind to those of the heavier odd-mass isotopes of tin or whether the level order has been changed by the appearance of new low-lying levels ($7/2+$ for example) between the $11/2-$ and $3/2+$ levels which prevent effectively the existence of isomers.

Although no one has reported the existence of an isomer of Sn^{115}, some information does exist on its low-lying levels through the decay of neighboring elements. Chikhladze *et al.*,[4] reported a 30-min activity in Sb_{11_9} which decays to Sn^{115} through the emission of positrons and through orbital electron capture. The positrons have an end-point energy of 1.51 ± 0.02 Mev, and a $\log ft$ value of 4.25. The positron is followed by a gamma ray of 499 ± 2 kev which has a K conversion coefficient α_K equal to 0.00625 and a K to L ratio of about 6. The theoretical values for α_K for a gamma ray of this energy in tin are 0.0065 and 0.0060 for $M1$ and $E2$ transitions, respectively. Markowitz and Wilson[5] reported 1.55 ±0.08 Mev for the end-point energy of the positron.

Recently, Beard and Kelly[6] measured the end-point energy of the beta spectrum emitted in the decay of the ground state of In^{115} (6×10^{14} yr) and obtained a value of 625 kev. In^{115m} (4.5 hr) which is 335 kev above the ground state[2] is reported to emit a beta particle with an end-point energy of 830 kev. This leaves a discrepancy of the order of 100 kev which Beard and Kelly point out could be due either to large errors in the beta end-point measurements or the indication of a level $\sim$100 kev above the ground state of Sn^{115}.

EXPERIMENTAL WORK

Sb^{115} was produced in the Brookhaven cyclotron by irradiating enriched[7] In^{113} (65.4%) with 40-Mev α particles through a 15-mil thick aluminum foil which was chosen to emphasize the $(\alpha,2n)$ reaction over other competing reactions. Due to the presence of In^{115}, the irradiated sample contained, besides the Sb^{115} (32 min) activity, other known activities which were present as background in the sources. Figure 1 shows a singles spectrum of the gamma rays soon after irradiation. The radiations were detected with a 3-in.$\times$2-in. NaI(Tl) crystal and the counts were recorded with a 256-channel pulse-height analyzer. The spectrum was repeated as a function of time and the decay of each peak was plotted (Fig. 2). The peaks at 500, 980, 1240, and 2220 kev all exhibit a component with a 32-min half-life and are attributed to the decay of Sb^{115} (32 min). The low-energy peaks and the long-lived components of the abovementioned peaks may all be assigned to one of the following activities: Sb^{114}, Sb^{116}, Sb^{117}, and Sb^{118}.

The 500-kev peak is a mixture of the 499-kev gamma

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[1] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 9 (1960): R. A. Sorenson, Nuclear Phys. **25**, 674 (1961).

[2] Nuclear Data Sheets, prepared by the Nuclear Data Group, National Academy of Sciences.

[3] M. Schmorak, G. T. Emery, and G. S. Goldhaber, Phys. Rev. **124**, 1186 (1961).

[4] V. L. Chikhladze, D. E. Khulelidze, and I. P. Selinov, Zhur. Eksptl. i Teoret. Fiz. **38**, 1353 (1960) [translation; Soviet Phys.—JETP **11**, 974 (1960)].

[5] S. S. Markowitz and F. Wilson, University of California Radiation Laboratory Report UCRL-9093, 1960 (unpublished), p. 53.

[6] G. B. Beard and W. H. Kelly, Phys. Rev. **122**, 1576 (1961).

[7] Obtained from Oak Ridge National Laboratory, Oak Ridge, Tennessee.

ray and annihilation radiation resulting from the positron. In order to determine the relative proportions of the two components in this peak, gamma-gamma coincidences were studied with a fast-slow coincidence circuit with a resolving time of 0.25 μsec. Coincidences were counted with the detectors in two geometries: first with 180° between the detectors, then 90°. Standard sources were used to calibrate the efficiencies of the crystals and the geometries. The percentage of positron emission relative to the 499-kev gamma ray was determined to be $12\pm3\%$.

Coincidences were studied also between various combinations of the 500-kev peak and the three other peaks which have a 32-min component (980, 1240, and 2220 kev). Since these gamma-ray peaks contain measurably large amounts of long-lived gamma rays of about the same energy as that of the Sn^{115} gamma rays, coincidences were repeated as a function of time and the coincidence efficiencies (number of coincidences per singles counts) were recorded as a function of time.

Coincidences between 499-kev gamma rays and each of the three higher energy gamma rays were found to exist. The 980-kev gamma ray, in addition, coincides with the 1240-kev gamma ray but not the 2220-kev gamma ray. Coupled with the facts that the 2220-kev gamma ray coincides with the 499-kev gamma ray and that its energy is equal to the sum of the 980-kev and 1240-kev gamma rays, the data on the 2220-kev gamma ray make it highly probable that this is the crossover transition from the third excited state to the first excited state. The coincidence efficiency for the 980 kev-1240 kev pair of gamma rays did not change as a function of time even though measurable amounts of long-lived impurities were present in both gamma-ray peaks. The main impurities in these peaks are probably due to Sb^{118} (5 hr) which has among its radiations two gamma rays (1030 and 1220 kev) which are not resolved from the Sn^{115} gamma-ray peaks and which coincide with one another. Therefore as the 32-min Sn^{115} activity decays and the sample becomes predominantly 5-hr Sb^{118}, the coincidence efficiency of the 980 kev–1240 kev pair remains unchanged. On the other hand, the efficiency for 499 kev–980 kev coincidences and 499 kev–1240 kev coincidences goes to zero in time because the Sb^{118} (5 hr) background does not have a 500-kev gamma ray coinciding with the 1030-kev and 1220-kev gamma rays. Thus, the data on the coincidences as a function of time are satisfactorily in agreement with the assumption that the 5-hr background in the sample is Sb^{118}. The relative positions of the 980-kev and 1240-kev gamma rays is unknown. The level at 2.72 Mev agrees in energy with the level at 2.68 Mev in Sn^{115} reported by Willard *et al.*[8] from the study of the (p,n) reaction on In^{115}.

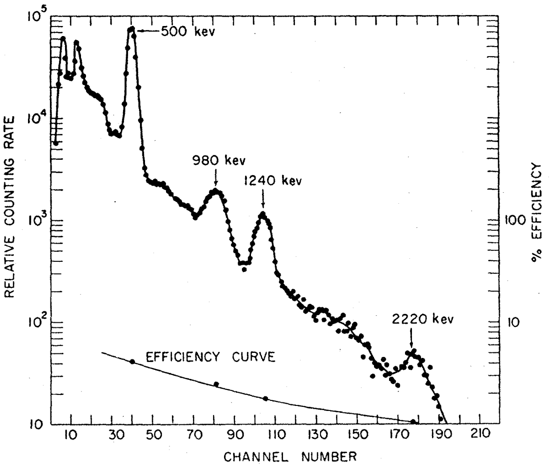

FIG. 1. Gamma-ray spectrum of Sb^{115} (32 min) taken with a 3-in.×2-in. NaI(Tl) crystal. The spectrum was taken at three different geometries to detect possible addition peaks. The peaks at 500, 980, 1240, and 2220 kev decayed with 32-min half-life. Several low-energy peaks are seen, all of which are attributable to known Sb activities. The efficiency (solid angle×the intrinsic photoelectric efficiency) as a function of energy is plotted below the spectrum.

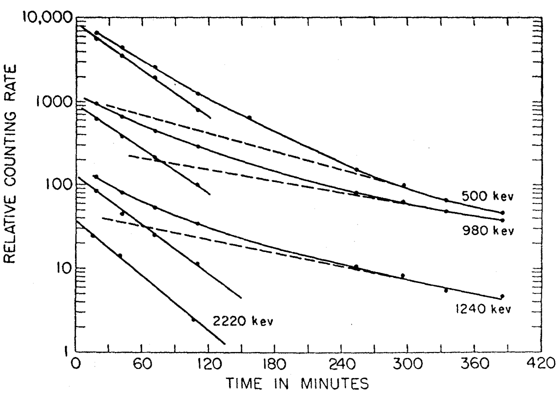

FIG. 2. Lifetime curves for the various peaks in the gamma-ray spectrum following the decay of Sb^{115}.

The relative intensities of the various γ rays were determined from the singles spectrum shown in Fig. 1. The efficiency of the 3-in.×2-in. NaI(Tl) crystal used in this determination was calibrated as a function of energy with various standard sources. Both the 980-kev and 1240-kev gamma rays appear to be about 5% as intense as the 499-kev ground-state transitions, while the 2220-kev gamma ray is only 1% as intense as the 499-kev transition.

The end-point energy of the positron spectrum was measured with a scintillation spectrometer using an anthracene crystal detector, and the value given by Chikhladze *et al.*[4] (1.51 Mev) was confirmed. An upper limit of 1% was placed on a positron branch to the ground state. Since all disintegrations of Sb^{115} lead eventually to the 499-kev level, the intensity of the 499-kev gamma ray along with the intensities of the K x ray and positron branches allows one to calculate the ratio of $\beta+$ emission to electron capture and the ratio

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[8] H. B. Willard, J. K. Bair, and J. D. Kington, Oak Ridge National Laboratory Report ORNL-1278, 1951 (unpublished).

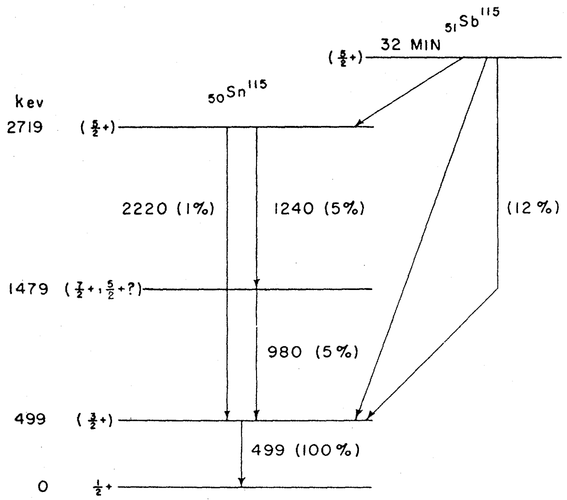

FIG. 3. Decay scheme of Sb^{115} (32 min). Note that the relative order of the 980-kev and 1240-kev gamma rays is unknown, so that the second excited state could be either at 1479 kev or at 1739 kev.

of $L+M$ capture to K capture. Both these experimentally determined ratios were in reasonable agreement with the assumption that the transition from the Sb^{115} ground state to the 499-kev level in Sn^{115} is allowed.

The levels of Sn^{115} were also looked at from the decay of In^{115m} to Sn^{115}. Normal cadmium containing Cd^{114} (28.8%) was irradiated with slow neutrons to give Cd^{115m} (44 days) and Cd^{115} (54 hr). Cd^{115} (54 hr) decays to In^{115m} (4.5 hr). About 24 hr after irradiation, indium activity was separated from the cadmium activity by the standard ion exchange method. The separated indium activity showed a lifetime of 4.5 hr. The end-point energy of the In^{115m} beta ray was checked and a value of (830±30 kev) in agreement with Langer *et al.*[9] was obtained. In order to determine whether any of the beta decay was to an excited state in Sn^{115}, beta-gamma coincidences were performed, with negative results. No evidence was found for levels in Sn^{115} below the 499-kev level and an upper limit of 0.4% of the total decay of In^{115m} was put on a β branch to the 499-kev level.

DISCUSSION

A decay scheme for Sb^{115} (32 min) is proposed in Fig. 3. The ground-state spin and parity of Sn^{115} have been measured[10] and are 1/2+. A spin and parity of 5/2+ have been assigned to Sb^{115} (32 min) on the basis of the systematics of nuclei, although the less likely assignment of 7/2+ cannot be entirely excluded. In the tin region (single closed shell) the various possible low-lying spins on the basis of the shell model are $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$. The K conversion coefficient of the 499-kev γ ray is 0.00625 as reported by Chikhladze *et al.*[4] The theoretical values for a K conversion coefficient for a gamma ray of this energy for the tin isotopes are 0.0065 and 0.0060 for $M1$ and $E2$ transitions, respectively. That is, the spin of the 499-kev level may be either 3/2+ or 5/2+. Since the first excited levels in both Sn^{117} and Sn^{119} have spin 3/2+ assigned to them, 3/2+ may be the more probable assignment in this case also. The spin assignment of 5/2+ to Sb^{115} (32 min) is in agreement with our observation that there is little or no $\beta+$ emission to the ground state of Sn^{115}, while 94% of the transitions from Sb^{115} go directly to the 499-kev level. The 2.72-Mev level is fed by electron capture from Sb^{115} 6% of the time. For this transition to compete with an allowed transition to the 499-kev level, it is almost mandatory that a spin of 5/2+ or 3/2+ be assigned to the 2.72-Mev level. If the spin were 3/2+, then one should normally expect a crossover transition from this level to the ground state, which is not observed. Although a spin of 7/2+ for this level cannot be completely ruled out, it seems a much less likely assignment than 5/2+, because the transition to the 2.72-Mev level would then be l-forbidden, for which the $\log ft \approx 6$. For $\log ft = 6$, the partial lifetime of the transition to the 2.72-Mev level would become about 20 days, which means that the transition to this level from Sb^{115} would become about 0.1% of the total disintegration; this is against the experimental observation of 6%. Therefore, the likely spin of the 2.72-Mev level is 5/2+. The $\log ft$ values for the transitions to the 2.72-Mev and 499-kev levels seem to be 4.1 and 4.5, respectively. The spin of the remaining level could in principle be one of the several available choices, i.e., 3/2+, 5/2+, 7/2+. Actually, an 11/2− level is possible in this region but it would not be fed from any of the observed levels in the Sb^{115} decay. Moreover, if the second level were 11/2−, one should have observed the lifetime corresponding to an $M4$ transition. Of the available spins 3/2+, 5/2+, and 7/2+, which are possible assignments to the second excited level, 7/2+ is to be preferred over 5/2+ since normally one might expect a direct transition to this level from Sb^{115} if the spin of this level were 5/2+, similar to the 2.72-Mev level. Since this transition does not take place one may suspect that this level has a character of $g_{7/2}$ which would again be l-forbidden. If the spin of this level were 3/2+, one might expect a crossover transition from this level to the 1/2+ ground state, which is not observed.

Kisslinger and Sorensen[1] have recently calculated the levels of the single closed shell nuclei including the effect of the pairing correlation. According to them the first 3/2+, 7/2+, and 5/2+ levels in Sn^{115} are at 140, 440 and 570 kev, respectively. The first excited level at 499 kev may be too high in energy for one to say with assurance that it corresponds to their predicted first

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[9] L. M. Langer, R. D. Moffat, and G. A. Graves, Phys. Rev. **86**, 632(A) (1952).

[10] M. Gurevitch, Phys. Rev. **75**, 767 (1949).

excited level at 140 kev. Their first excited level in Sn^{117} is also a little low in energy compared with the experimental value. The $7/2+$ and $5/2+$ observed states in Sn^{115} are quite high in energy in comparison with their lowest predicted $7/2+$, and $5/2+$ levels and may probably not be pure quasi-particle states. The second $7/2+$ excited level due to Kisslinger and Sorensen has a large phonon admixture and lies at 1.5 Mev above the $1/2+$ ground state. Our second excited level with a spin of $7/2+$ could be the second $7/2+$ excited level of their level structure of Sn^{115}.

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Use of Normal Coordinates in Nuclear Physics*

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It is shown that normal coordinates are not difficult to use in nuclear theory if it is assumed that the masses of the nucleons are equal. In the shell model with harmonic oscillator functions they have the advantage that they exclude the spurious states due to the motion of the center of mass.

The use of normal coordinates to describe a system of particles is usually restricted by cumbersome algebra. However, this algebra is much simplified in nuclear physics where the masses of the nucleons may often be taken as equal.

Consider a system of particles with equal masses moving in the field of a potential V. If $\mathbf{r}_i$ is the position vector of the ith particle relative to some fixed origin, the Schrödinger equation of the system is

$$[-\tfrac{1}{2}\textstyle\sum_i \nabla_i^2+V(\mathbf{r}_1,\mathbf{r}_2,\cdots)]\psi(\mathbf{r}_i)=E\psi(\mathbf{r}_i). \tag{1}$$

To separate the motion of the center of mass, a linear transformation is made to a set of normal coordinates, one of which is the position vector of the center of mass, $\mathbf{R}$. The matrix equation of the transformation can be written

$$\boldsymbol{\varrho}=B\mathbf{r}', \tag{2}$$

where $\boldsymbol{\varrho}$ represents the set of normal coordinates $\boldsymbol{\varrho}_j$, and $\mathbf{r}'$ the set of coordinates $\mathbf{r}_i$. The matrix, B, is nonsingular. If $\partial/\partial\boldsymbol{\varrho}$ and $\partial/\partial\mathbf{r}'$ represent the sets of gradients $\partial/\partial\boldsymbol{\varrho}_j$ and $\partial/\partial\mathbf{r}_i$, then

$$\frac{\partial}{\partial\boldsymbol{\varrho}_j}=\sum_i\left(\frac{\partial\mathbf{r}_i}{\partial\boldsymbol{\varrho}_j}\right)\frac{\partial}{\partial\mathbf{r}_i}.$$

So

$$\frac{\partial}{\partial\boldsymbol{\varrho}}=(B^{-1})^\dagger\frac{\partial}{\partial\mathbf{r}'},$$

and

$$\sum_i \nabla_i^2=\left(\frac{\partial}{\partial\mathbf{r}'}\right)^\dagger\left(\frac{\partial}{\partial\mathbf{r}'}\right)=\left(\frac{\partial}{\partial\boldsymbol{\varrho}}\right)^\dagger BB^\dagger\left(\frac{\partial}{\partial\boldsymbol{\varrho}}\right). \tag{3}$$

If there are to be no cross derivatives, $BB^\dagger$ must be diagonal. If $\boldsymbol{\varrho}_n$ is chosen as $\boldsymbol{\varrho}_n=n\mathbf{R}$, where n is the total number of particles in the system, then $B_{ni}=1$ for all i. It follows that

$$\textstyle\sum_i B_{ji}=0 \quad\text{if}\quad j\neq n,$$

and that

$$\textstyle\sum_i B_{ji}B_{ki}=0 \quad\text{if}\quad k\neq j.$$

These conditions may be satisfied by numerically simple matrices: for a single system of four particles the matrices take the form

$$B=\begin{bmatrix}1&-1&0&0\\1&1&-2&0\\1&1&1&-3\\1&1&1&1\end{bmatrix},$$

$$B^{-1}=\begin{bmatrix}1/2&1/2\times3&1/3\times4&1/4\\-1/2&1/2\times3&1/3\times4&1/4\\0&-1/3&1/3\times4&1/4\\0&0&-1/4&1/4\end{bmatrix}, \tag{4}$$

$$BB^\dagger=\begin{bmatrix}1\times2&0&0&0\\0&2\times3&0&0\\0&0&3\times4&0\\0&0&0&4\end{bmatrix}.$$

If the system consists of two (or more) groups, as in a collision problem, it is useful also to separate out the motion of one group relative to the other. This requires

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