Study of Zr^{92} Levels Excited in the (d,p) Reaction*

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(Received July 27, 1961; revised manuscript received October 23, 1961)

Seven energy levels in Zr^{92} were excited by the (d,p) reaction using 10.85-Mev deuterons. These levels were at excitation energies of 0, 0.94, 1.49, 1.86, 2.06, 2.91, and 3.28 Mev. The last two levels appear to be complex. Angular distributions were measured for all of these levels except the one at 1.86-Mev excitation. The distributions for the three lowest levels were d wave; the distributions for the three highest levels were s wave. Cross-section comparisons were made with earlier work on Zr^{91} levels. These comparisons indicated that the dominant configuration of the lowest levels is $(d_{5/2})^2$ and that the 2.91-Mev level is largely $(d_{5/2}s_{1/2})$. A calculation of the neutron-neutron interaction, using vacuum-like, singlet forces between the nucleons, gives fair agreement with the experimental results.

THE element Zr^{90} has 40 protons and a closed shell of 50 neutrons. Energy levels of the 51st neutron are excited by the $Zr^{90}(d,p)Zr^{91}$ reaction. A study of this reaction is reported in an earlier paper[1] and gives the positions and single particle assignments of the neutron energy levels in Zr^{91}. A natural continuation of this work is the study of the $Zr^{91}(d,p)Zr^{92}$ reaction presented in this paper. This study is combined with experimental measurements from the earlier work on the Zr^{91} energy levels to give spin assignments and possible configurations for Zr^{92} two-neutron states excited by the (d,p) reaction. A shell model calculation of the two-neutron states is also presented and is compared with the experimental results.

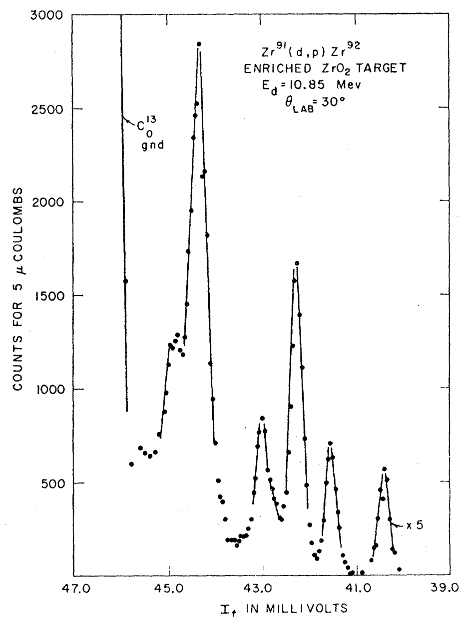

FIG. 1. A proton momentum spectrum for the ZrO_2 target. Angular distributions were measured for all of the prominent proton groups. The abscissa is inversely proportional to the proton momentum and the spectrum covers proton energies between 13 and 18 Mev.

I. EXPERIMENTAL PROCEDURES

ZrO_2, enriched to 86.9% in Zr^{91}, was obtained from the Oak Ridge National Laboratory. The oxide powder was suspended in ethylene dichloride. Formvar and Aquadag were added, the former to serve as a binder and the latter to strengthen the target under beam bombardment. This suspension was dried on a glass plate and then peeled. The total thicknesses of the targets were about 2 mg/cm^2.

These targets were of limited usefulness. They contained large amounts of carbon and oxygen; these contaminants obscured parts of the proton momentum spectrum. The targets also had a general background that made measurements on weak states difficult. Efforts to prepare metallic targets from small amounts of ZrO_2 were unsuccessful.

Useful comparisons of the $Zr^{91}(d,p)Zr^{92}$ data with the $Zr^{90}(d,p)Zr^{91}$ data of reference 1 involve relative cross sections. The Zr^{91} enriched targets and the Zr^{90} enriched targets (of reference 1) were normalized by comparison with a natural zirconium target. The errors on relative cross sections between the targets are $\pm 15\%$. Errors on the absolute cross sections are $\pm 40\%$.

Q-value measurements were similar to those described in previous work.[1-3] The ground-state Q value was measured with the natural Zr target, using (d,p) peaks from oxygen and carbon contaminants to determine the beam energy. Relative Q values were measured with the ZrO_2 target. These measurements give the difference in the Q values of two proton groups leading to adjacent states in Zr^{92}. This means that a shift in the measured excitation energy of one of the Zr^{92} states will shift the energy of all the higher excited states by the same amount. Errors on the Q values are ± 30 kev.

The experimental arrangements have been discussed

* This work was supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.
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[1] R. L. Preston, H. J. Martin, Jr., and M. B. Sampson, Phys. Rev. **121**, 1741 (1961).
[2] M. T. McEllistrem, H. J. Martin, D. W. Miller, and M. B. Sampson, Phys. Rev. **111**, 1636 (1958).
[3] H. J. Martin, M. B. Sampson, and D. W. Miller, Phys. Rev. **121**, 877 (1961): G. B. Holm, J. R. Burwell, and D. W. Miller, *ibid.* **118**, 1247 (1960).

extensively in earlier papers.[2,4] The deuteron bombarding energy was 10.85 Mev. A typical proton momentum spectrum is shown in Fig. 1.

II. EXPERIMENTAL RESULTS AND DISCUSSION

A. Energy Levels

The work of reference 1 gave the states of the 51st neutron in Zr^{91}. The pertinent results of this work are shown in Figs. 2 and 3(a). Figure 2 shows two of the $Zr^{90}(d,p)Zr^{91}$ angular distributions. These are the distributions for excitation of the ground state and the first-excited state of Zr^{91}. They have been fitted with Butler curves and are representative of d-wave neutron capture and s-wave neutron capture, respectively.

The neutron states in Zr^{91} are shown in Fig. 3(a). Assignments for these states were made on the basis of angular distributions and relative cross sections.

If the interaction between the two extra neutrons in Zr^{92} is ignored, then, as a first approximation, the energy states of each neutron are the Zr^{91} states and all of the states of a particular two-neutron configuration have the same energy. These states are the Zr^{92} "zero order" levels shown in Fig. 3(b). The next higher approximation considers the interaction between the two neutrons, removing the degeneracy in the states of a particular configuration and allowing configuration mixing. The effect of the neutron-neutron interaction is shown, qualitatively, in Fig. 3(b), and will be discussed in more detail in the next section of this paper.

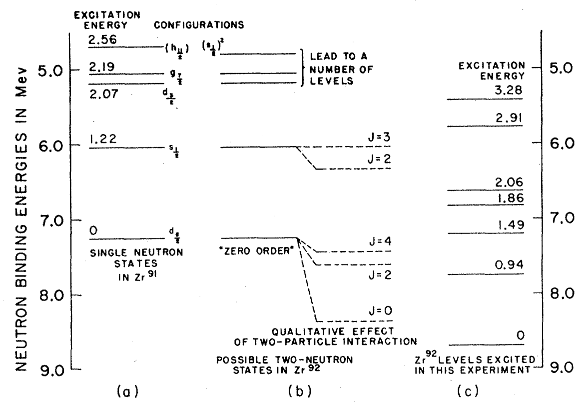

FIG. 3. Part (a) shows the single particle neutron states in Zr^{91}. The assignments are based on the work in reference 1. Part (b) shows the lowest possible two-neutron states in Zr^{92}. Each state consists of a $d_{5/2}$ neutron and a neutron of the same configuration as the adjacent Zr^{91} state. The qualitative effect of the neutron-neutron configuration is shown. A more quantitative calculation is discussed in the text. Part (c) shows the energy levels of Zr^{92} that are studied in the present work.

Figure 3(c) shows the levels excited in the present work. An adjustment has been made in these measurements to bring them into agreement with a study at Los Alamos of the Y^{92} beta decay.[5] A level at 1.49-Mev excitation energy was seen in the beta decay and is apparently the same level measured at 1.54 Mev in this (d,p) work. The relative Q-value of this level was adjusted to give agreement with the beta decay work,

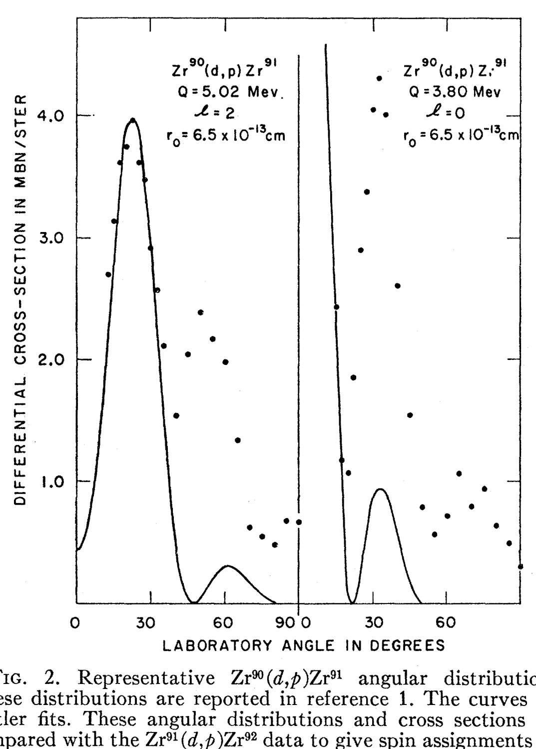

FIG. 2. Representative $Zr^{90}(d,p)Zr^{91}$ angular distributions. These distributions are reported in reference 1. The curves are Butler fits. These angular distributions and cross sections are compared with the $Zr^{91}(d,p)Zr^{92}$ data to give spin assignments for the Zr^{92} states.

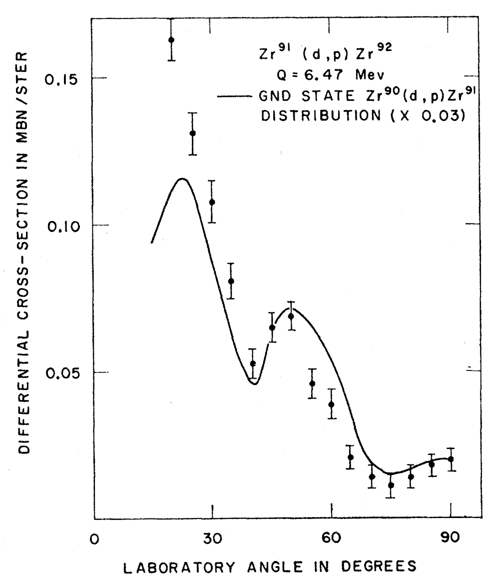

FIG. 4. The angular distribution of the proton group leading to the ground state of Zr^{92}. The solid curve is an experimental d-wave angular distribution.

[4] V. K. Rasmussen, D. W. Miller, and M. B. Sampson, Phys. Rev. **100**, 181 (1955); J. R. Rees and M. B. Sampson, *ibid.* **108**, 1289 (1957).

[5] M. E. Bunker (private communication).

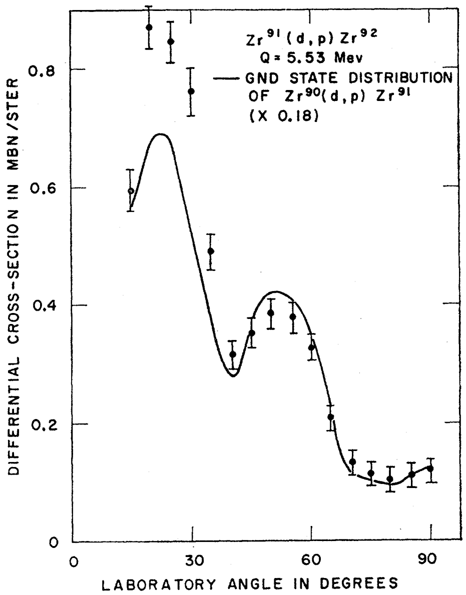

FIG. 5. The angular distribution of the proton group leading to the first excited state of Zr^{92}. The solid curve is a d-wave angular distribution.

moving all of the higher levels down 50 kev. The adjusted positions of these levels are shown in Fig. 3(c) and are in good agreement with the beta decay work.[5] The proton groups to levels at excitation energies of 2.91 Mev and 3.28 Mev are both broader than the experimental resolution and presumably feed several levels at the two excitation energies.

b. Angular Distributions

Angular distributions for proton groups exciting the three lowest Zr^{92} levels are shown in Figs. 4–6. The distributions for the 0.94-Mev level and the 1.49-Mev level (Figs. 5 and 6) can be fitted with Butler curves for d-wave neutron capture. The ground-state distribution (Fig. 4) is similar to the other two distributions except at the most forward angles and also appears to be d wave. All three distributions are compared with the $Zr^{90}(d,p)Zr^{91}$ ground-state distribution.

Figures 7–9 show the angular distributions for proton groups exciting Zr^{92} levels at excitation energies of 2.06-Mev, 2.91-Mev, and 3.28-Mev. All of these distributions can be fitted with s-wave Butler curves and they are all compared with the $Zr^{91}(d,p)Zr^{92}$ first excited state distribution.

MacFarlane and French have published a detailed review of the use of stripping reactions in nuclear spectroscopy.[6] They feel that informative and useful results can be obtained by analyzing the angular distributions in terms of the original Butler theory. This cross section can be written[7]

$$\frac{d\sigma}{d\omega}=\frac{2J+1}{2J_0+1}f_B(l_n,Q,\theta)\Theta^2,$$

where J_0 is the spin of the target nucleus, J is the spin of the final nucleus, $f_B(l_n,Q,\theta)$ is the Butler cross section divided by the reduced width, and Θ^2 is the reduced width for the reaction. The reduced width can then be written as the product of two factors[6,7]

$$\Theta^2=\mathcal{S}\Theta_0^2,$$

where the spectroscopic factor $\mathcal{S}$ depends only on the nuclear wave functions and the single particle reduced width Θ_0^2 is regarded as an empirical parameter.

The Θ_0^2 parameters for this experiment are given by the earlier $Zr^{90}(d,p)Zr^{91}$ work. A comparison of the cross sections of that work with the cross sections measured in this work allows extraction of experimental values of $(2J+1)\mathcal{S}$. The Butler theory is used to estimate the Q-value dependence of the reaction cross section. There is evidence in heavier nuclei, where deviations from the Butler theory are more pronounced, supporting this type of estimate.[8]

Maximum values of the spectroscopic factor $\mathcal{S}$ depend upon whether or not the transferred nucleon is or is not equivalent to some of the target nucleons. In particular, $\mathcal{S}\leq n$ where n is the number of nucleons in the final nucleus that are equivalent to the transferred nucleon.[6]

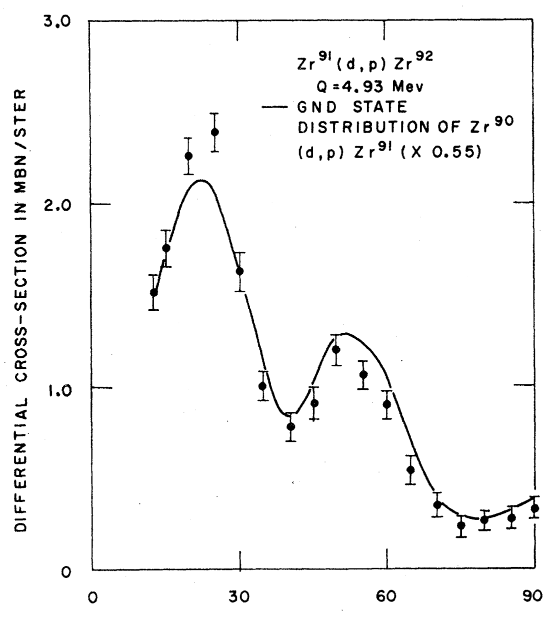

FIG. 6. The angular distribution of the proton group leading to the 1.49-Mev state of Zr^{92}.

[6] M. H. MacFarlane and J. B. French, Revs. Modern Phys. 32, 567 (1960).

[7] J. B. French and B. J. Raz, Phys. Rev. **104**, 1411 (1956).

[8] B. L. Cohen, R. E. Price, and S. Mayo, Nuclear Phys. **20**, 370 (1960).

For this experiment, $\mathcal{S}\leq 2$ if a $d_{5/2}$ neutron is transferred, while $\mathcal{S}\leq 1$ for all other neutron captures. These limits indicate the spin assignments for the Zr^{92} levels. These assignments are shown in Table I, along with a summary of the other data. The spins are in agreement with spin assignments from the study of the Y^{92} beta decay.

Table I also shows the experimental values of $\mathcal{S}$ for the different states. All of the states are strongly excited except for the 1.86-Mev level. The angular distribution for this level was not measured because of target difficulties. A partial distribution showed a broad forward peak of about 0.4 mb/sr, falling off to 0.2 mb/sr near 45°. The values of $\mathcal{S}$ for the other states support a two-neutron description of the low-lying Zr^{92} states, indicating a $(d_{5/2})^2$ configuration for the first three levels and a $(d_{5/2}s_{1/2})$ configuration for the levels at 2.91-Mev excitation.

III. ENERGY-LEVEL CALCULATION

There are a number of different approaches to the problem of considering low-lying nuclear states within the framework of the shell model. Talmi and Unna[9] assume that the potential energy of the nucleus is effectively due to two-body interactions between nucleons. They avoid assuming a specific interaction or specific wave functions and, instead, use experimental data to evaluate matrix elements. The strength of the calculation lies in the assumption that the matrix elements are the same in neighboring nuclei so that the number of parameters is much less than the amount of possible experimental data.

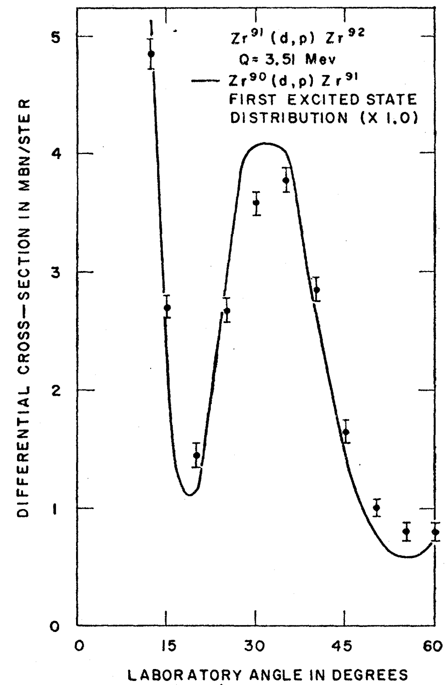

FIG. 8. The angular distribution of the proton group leading to the 2.91-Mev state of Zr^{92}.

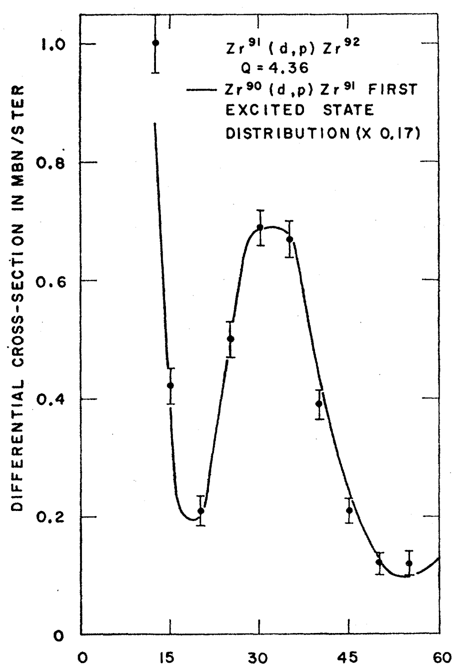

FIG. 7. The angular distribution of the proton group leading to the 2.06-Mev state of Zr^{92}. The solid curve is an experimental s-wave angular distribution.

More extreme calculations are quantitatively successful in explaining experimental data in the Pb region. These calculations assume specific nucleonic wave functions and specific two-body interactions. In particular, True and Ford[10] show that a large amount of data about Pb^{206} is explained by considering that the two neutron holes in the Pb^{208} core interact through forces with about the same strength and range as the forces between two free neutrons. Further work by Newby and Konopinski[11] indicates that the True-Ford approach is quite successful for interactions between like nucleons which interact largely through singlet central forces, e.g., the two protons outside of the Pb^{208} core in Po^{210}. Newby and Konopinski's extension of this work to the neutron-proton interaction is less successful, presumably because of the complexities of handling the triplet, exchange, and tensor effects.

The calculation presented here follows the True-Ford calculation. Energy levels in Zr^{91} give the interaction energies of the extra neutrons with the Zr^{90} core. Singlet forces are used for the neutron-neutron interaction and a Gaussian potential is assumed[10,11]

$$u(r)=-V_0\exp(-r^2/r_0^2).$$

[9] I. Talmi and I. Unna, Nuclear Phys. **19**, 225 (1960).

[10] W. True and K. Ford, Phys. Rev. **109**, 1675 (1958).

[11] N. Newby, Jr. and E. J. Konopinski, Phys. Rev. **115**, 434 (1959).

TABLE I. Summary of the data obtained from the $Zr^{91}(d,p)Zr^{92}$ reaction.

Level excitation energy (Mev)	l_n [b]	$(2J+1)\mathcal{S}$ [c]	J	$\mathcal{S}_{\text{expt}}$ [e]	$\mathcal{S}_{\text{max}}$	Dominant[g] neutron configuration
0	2	1.4	0[d]	1.4(1.7)	2	$(d_{5/2})^2$
0.94	2	7.2	2[d]	1.4(1.7)	2	$(d_{5/2})^2$
1.49	2	19.8	4[d]	2.2	2	$(d_{5/2})^2$
1.86	...	...	...	...	...	...
2.06	0	2.2	(2,3)	0.18[f]	1	
2.91[a]	0	11.6	(2,3)	0.97[f]	1	$(d_{5/2}s_{1/2})$
3.28[a]	0	3.7	(2,3)	0.31[f]	1	

[a] The experimental width of the proton groups indicate that a group of levels is excited instead of a single level.
[b] The angular momentum assignments are made by referring to earlier $Zr^{90}(d,p)Zr^{91}$ work. The Butler fits of this earlier work and the present work are with $r_0=6.5\times10^{-13}$ cm.
[c] These values are extracted by a comparison of the $Zr^{91}(d,p)Zr^{92}$ cross sections with the $Zr^{90}(d,p)Zr^{91}$ cross sections of reference 1. The Q-value dependence of the reaction cross section has been estimated by using the Butler theory. Cohen *et al.* (reference 8) use data from the $Pb^{207}(d,p)Pb^{208}$ reaction to obtain an experimental estimate of the Q-value dependence of the cross section. This estimate is within 10% of the value given by the Butler theory, although the angular distributions for the Pb case deviate strongly from Butler curves.
[d] These assignments are in agreement with the Y^{92} beta decay work of reference 5.
[e] The experimental spectroscopic factors $\mathcal{S}_{\text{expt}}$ are extracted by comparing cross sections over the complete angular distribution. A second value, enclosed in parentheses, is given when the comparison over the first peak of the angular distribution gives an appreciably different value of $\mathcal{S}_{\text{expt}}$.
[f] This is a minimum value of $\mathcal{S}_{\text{expt}}$. It assumes that the experiment excites a number of close-lying, unresolvable, $J=2$ and $J=3$ levels. The cross section for all of the levels with spin J is taken to be proportional to $(2J+1)(\Theta_1^2+\Theta_2^2+\cdots)=(2J+1)(\mathcal{S}_1+\mathcal{S}_2+\cdots)\Theta_0^2=(2J+1)\mathcal{S}\Theta_0^2$. $\mathcal{S}$ is assumed to be the same for the $J=2$ levels and for the $J=3$ levels. $\mathcal{S}_{\text{expt}}$ should be multiplied by 2.4 if the excited state or states are $J=2$ levels only. The multiplication factor is 1.7 for excitation of only $J=3$ states.
[g] This column lists the dominant neutron configurations of the Zr^{92} states as indicated by the (d,p) reaction data. Complete wave functions for the states require consideration of mixtures of other possible configurations when $\mathcal{S}_{\text{expt}}<\mathcal{S}_{\text{max}}$. Although some of the other possible configurations can be excited by the (d,p) reaction, the data in this paper do not cover an angular region that allows the detection of small admixtures. An extension of the data to smaller angles, for example, would allow an estimate of the amount of $(d_{5/2}s_{1/2})_{J=2}$ configuration in the wave function of the first excited state.

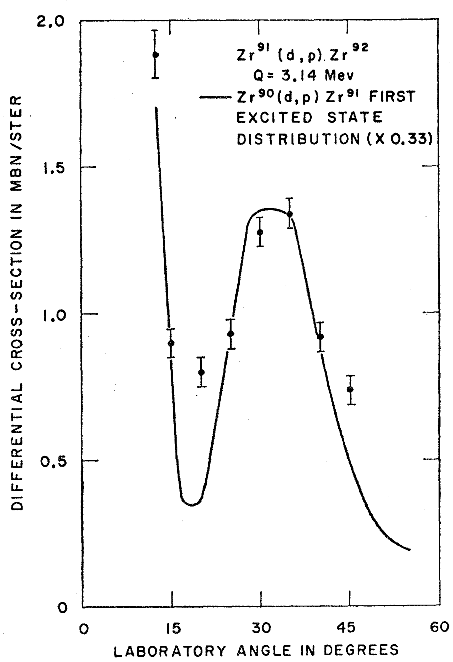

FIG. 9. The angular distribution of the proton group leading to the 3.28-Mev state of Zr^{92}.

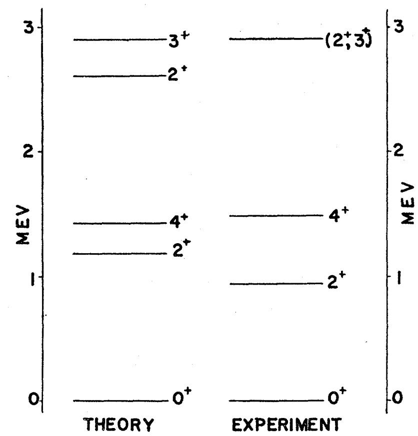

FIG. 10. A comparison of theory with experiment. The calculation of the neutron-neutron interaction used singlet central forces, a Gaussian potential, and harmonic oscillator wave functions. Configuration mixing was also included.

The parameters of this potential are then chosen to be consistent with nucleon-nucleon scattering, and with deuteron data, and are the parameters used by Newby and Konopinski[11]

$$V_0=46\text{ Mev},\quad r_0=1.6\text{ fermi}.$$

Harmonic-oscillator radial wave functions are used in the calculation. The parameter b, appearing in the wave functions $\psi\sim\exp(-r^2/2b^2)$, measures the spread of the radial distribution and can be related to the nuclear size. In particular, b is given by[10]

$$b^2=\langle r^2\rangle/2\langle(n+\tfrac{3}{2})\rangle,$$

where n is the radial quantum number and r is the nuclear radius. The best fit was found with $b=2.15$, corresponding to $r=1.27A^{\frac{1}{3}}$ fermis.

The matrix elements for the calculation are given by Newby and Konopinski.[11] Evaluation of the Slater integrals was carried out directly by using the formulas given by Ford and Konopinski.[12] The results of the calculation are shown in Fig. 10 and are given in Table

TABLE II. Calculated levels in Zr^{92}.

Configuration[a]	J	Energy (kev)
$(d_{5/2})^2$	0	0
	2	1180
	4	1420
$(d_{5/2}s_{1/2})$	2	2610
	3	2900
$(d_{5/2}d_{3/2})$	1	3730
	2	3680
	3	3730
	4	3370
$(s_{1/2})^2$	0	5200

[a] Effects due to configuration mixing are included.

[12] K. W. Ford and E. J. Konopinski, Nuclear Phys. 9, 218 (1958/59).

II. Table II shows all of the energy levels involved in the calculation, while Fig. 10 shows only those levels that are pertinent to the experiment.

Fairly large deviations between the experimental and theoretical levels are shown in Fig. 10. The calculated effects of configuration mixing are not large enough to move the $J=2$ levels apart as much as is indicated by the experiment. This discrepancy between the experiment and the calculation also appears in a comparison of values of $\mathcal{S}$ for the first-excited state. The experimental value of $\mathcal{S}$ is 1.4, while the calculated value is 1.9. The results for the ground state are similar, but the errors on the experimental value of $\mathcal{S}$ are large for this case.

PHYSICAL REVIEW VOLUME 125, NUMBER 3 FEBRUARY 1, 1962

Low-Resolution Survey of (d,α) Reactions in Heavy Nuclei*

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(Received August 2, 1961)

A low-resolution survey of (d,α) reactions was made for about 28 heavy nuclei between $Z=28$ and $Z=83$ with an incident deuteron energy of 15 Mev. Energy distributions and cross sections were obtained at scattering angles of 30°, 60°, 90°, and 120°. The observed energy spectra are characterized by two strong peaks which vary regularly in energy and cross section with atomic weight. Energy distributions and cross sections of the low-energy peak are in accord with statistical theory. The principal features of the high-energy peak can be explained with the assumption of a process which involves the pickup of a proton and neutron from separate single-particle states.

INTRODUCTION

NUCLEAR reaction experiments are usually performed with two aims in mind: to study the reaction mechanism, and to gain insight into the problem of nuclear structure. Of particular interest are the energy distributions of the reaction products emitted at various scattering angles. From these data it is often possible to determine whether the reaction mechanism is direct in nature or whether it proceeds through the intermediate stage of a compound nucleus. It is also often possible to distinguish between various types of direct interactions and to determine parameters related to the reaction mechanism.

Previous experiments involving (d,α) reactions are notably sparse, except for some detailed work on light nuclei.[1–4] To date, the heavy-element region $(A>60)$ has been almost completely neglected. In view of this fact, it was decided to make a low-resolution survey of energy distributions and cross sections at several scattering angles for about 28 nuclei between mass numbers 59 and 209. Preliminary results for this experiment have appeared in a previous note.[5]

EXPERIMENTAL TECHNIQUES

Fifteen-Mev deuterons that initiated reactions studied in this experiment were obtained from the University of Pittsburgh cyclotron. The incident beam was momentum analyzed and contained a resultant energy spread of about 80 kev. The associated scattering system has been discussed in detail by previous workers[6–8] and will not be treated here. Beam current was collected by a Faraday cup; the accumulated charge was measured by a precision current integrator with an error less than 1%.

The detectors consisted of a CsI crystal scintillator and proportional counter used in coincidence as a $(dE/dX)-E$ particle separation system.[9] Pulses from each of the detectors were separately analyzed by single-channel pulse-height selectors (PHS) used as integral discriminators. The PHS outputs were presented to a coincidence circuit which subsequently gated a 256-channel analyzer (refer to Fig. 1). By proper setting of the PHS discrimination level associated with the (dE/dX) (proportional) counter, less ionizing reaction products could be rejected in favor of the remaining alphas.

As a result of the high Q values inherent in (d,α) reactions and the large Coulomb barriers for alphas, the spectra of outgoing particles were limited to an energy region where energy loss does not change rapidly with energy. In almost all cases the entire alpha energy spectrum could be obtained with a single PHS discriminator setting, thus obviating the need for a pulse

* This work was done at Sarah Mellon Scaife Radiation Laboratory and supported by the National Science Foundation.

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