

Temperature Dependence of the Surface Tension of He II

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It is shown that the temperature dependence of the surface tension of liquid helium II can be understood on the ideal gas model for the liquid also, provided we re-enumerate the eigenfunctions in a bounded continuum so as to take into account the surface term. The maximum deviation of our results from the experimental ones of Allen and Misener is about 7 percent, and from the theoretical calculations of Atkins only about 2 percent.

THE surface tension of liquid He II, starting from a value of $0.352 \text{ erg cm}^{-2}$ at 0°K , initially remains independent of temperature and falls off rapidly as the λ point is approached.¹ In order to explain this behavior Atkins² assumed that the effect arises from the thermal energy associated with the surface modes of vibration which he took to be similar to the surface tension waves. He calculated the deviation of the surface tension σ from its value at 0°K (σ_0) by using the relation³

$$\sigma - \sigma_0 = kT \sum_{\epsilon} \ln[1 - e^{-\epsilon/kT}]. \quad (1)$$

Here σ may be understood as the surface part of the Helmholtz free energy per unit area. Using the appropriate $\epsilon-p$ relation for the surface tension waves, he obtained

$$\sigma - \sigma_0 = -6.9 \times 10^{-3} T^{7/3} \text{ erg cm}^{-2}. \quad (2)$$

In this paper we wish to point out that the variation of the surface tension of liquid helium II with temperature can also be understood in a much simple manner, by using the ideal gas model for the liquid He II, provided we enumerate the eigenfunctions in a bounded continuum more exactly than is ordinarily done so as to take into account the surface term also.

The number of eigenfunctions in a bounded continuum of volume V and surface area A , with their momentum between p and $p+dp$, is⁴

$$a(p)dp = g \left[\frac{4\pi V}{h^3} p^2 dp + \frac{\pi A}{2h^2} p dp + \dots \right], \quad (3)$$

where g is the relevant weight factor and the eigenfunction ψ is assumed to have a stationary value at the boundary. Using Eq. (1), the surface part of Eq. (3), and the appropriate $\epsilon-p$ relation, we get for an ideal degenerate Bose gas

$$\sigma - \sigma_0 = -\pi m k^2 T^2 \zeta(2) / 2h^2 = -7.5 \times 10^{-3} T^2 \text{ erg cm}^{-2}. \quad (4)$$

In Fig. 1, we plot Eq. (4) with $\sigma_0 = 0.352 \text{ erg cm}^{-2}$. The experimental results of Allen and Misener and the theoretical curve from Atkins [Eq. (2)] are also given for comparison. We find that up to $T \approx 1.3^\circ\text{K}$ our results agree with Allen and Misener and also with Atkins. At higher temperatures the variation given by Eq. (4) is less rapid. However, the maximum deviation of our results (at the λ point) from Allen and Misener is about 7% and that from those of Atkins is only about 2%.

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⁴ D. S. Kothari and F. C. Auluck, *Nature* **159**, 204 (1947).

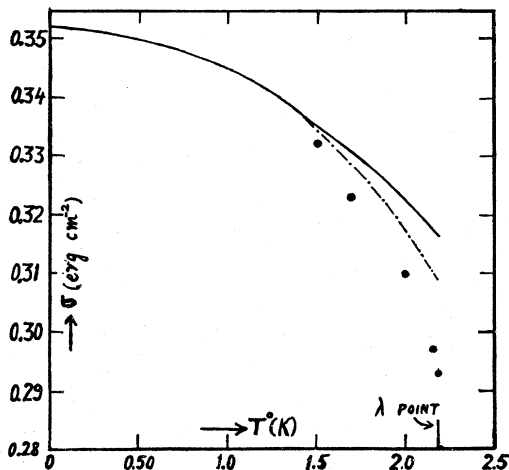


FIG. 1. Variation of the surface tension of liquid He II with temperature. Solid line, theoretical curve from present calculations; dot-dashed line, theoretical curve from Atkins²; ●, experimental points of Allen and Misener.¹

¹ J. F. Allen and A. D. Misener, *Proc. Cambridge Phil. Soc.* **34**, 299 (1938).

² K. R. Atkins, *Can. J. Phys.* **31**, 1165 (1953).

³ L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Pergamon Press, New York, 1958), p. 460.