Search for a Spin-1 Intermediate Meson in Neutral Pion Photoproduction

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If a spin-1 meson exists it might act as an intermediate particle in a meson current graph for neutral pion photoproduction. It is shown that the singularities from this graph are likely to be much closer to the physical region in the complex $x=\cos\theta$ plane than any other singularities with the possible exception of those coming from a pion-pion interaction. Thus an extrapolation can be carried out in x to determine the residue of the pole due to the spin-1 meson if it exists. This residue is calculated for vector (V) and pseudovector (PV) mesons and is found to have the same negative sign for both cases, unlike in the spin-0 case. Experimental data at 300, 450, 700, and 800 Mev are used for the extrapolation which is carried out as a function of intermediate meson mass. It is found that the results are consistent with zero residue for all energies with the possible exception of 450 Mev where there is a slight evidence for a positive residue. Thus present evidence favors the conclusion that there is no contribution to neutral photoproduction from a spin-1 meson. One can place an upper limit of about 0.01 for the V case and 0.05 for the PV case on the magnitude of $g^2(\mathfrak{M}/2m)^2$, where g is the coupling constant between the nucleon and the spin-1 meson, and the latter has a mass m and a magnetic moment $\mathfrak{M}$. A detailed analysis is also given of the error in the residue determined from a set of experimental data, as a function of the location and number of data points, the extrapolation distance, and the order of the extrapolating polynomial. It is concluded that the present error in the residue could be improved by a factor of 3 to 5 by an experiment measuring the differential cross section of neutral pion photoproduction at a photon energy of 800 Mev in the laboratory system at every 10° from 10° with an error of ± 0.2 μb/sr on the individual measurements.

I. INTRODUCTION

IN the past few years considerable effort has been directed to explore various aspects of the possible existence of a spin-1 boson. For a representative, although not necessarily complete, bibliography see references 1 through 26.[1–26] The paper adds to this list by exploring the effect of such a particle on neutral pion photoproduction. In particular, our aim will be to see if the existence of a spin-1 boson (henceforth called ω^0) as an intermediate particle can be detected in the differential cross section of neutral photopions.

The key to our discussion is the assertion that if the ω^0 exists, then also the Feynman diagram shown in Fig. 1 exists. The spin-1 property of the ω^0 figures heavily in the existence of such a graph, since only then can the neutral boson, through its magnetic moment (if any), interact with the electromagnetic field. The diagram in Fig. 1 resembles the conventional meson current graph of pion photoproduction (in which the intermediate particle is a pion) which, however, does not exist for the production of neutral pions. This is a very helpful fact, as it can be seen by investigating the location of singularities for the production amplitude at a fixed energy, as a function of the production angle. This has been discussed before[27] in connection with charged pion photoproduction. In our case the ordinary pole in the forward direction at β_π^{-1} (β_π being the pion velocity) is missing. The cut in the forward direction arises only if a pion-pion interaction exists, since it corresponds to a graph of the type shown in Fig. 2. Thus one has singularities in the forward direction at all only if a pion-pion interaction exists or, as we will now show, if the graph in Fig. 1 contributes.

Let us denote the four-momenta of the photon, pion, ω^0, incoming and outgoing protons by k, q, r, p, and p', respectively. We will use the metric $a\cdot b=a_0b_0-\mathbf{a}\cdot\mathbf{b}$, where a and b are four-vectors and $\mathbf{a}$ and $\mathbf{b}$ are three-vectors. Furthermore, μ denotes the pion mass and m the mass of the ω^0. Then the pole corresponding to Fig. 1 occurs at

$$r^2=(q-k)^2=0, \tag{1.1}$$

which in turn corresponds to

$$x\equiv\cos\theta=(2qk)^{-1}(2q_0k+m^2-\mu^2)=\beta_\pi^{-1}+(2qk)^{-1}(m^2-\mu^2), \tag{1.2}$$

or

$$\mathbf{q}\cdot\mathbf{k}=q_0k+\tfrac{1}{2}(m^2-\mu^2). \tag{1.3}$$

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[20] J. J. Sakurai, Phys. Rev. **119**, 1784 (1960).
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TABLE I. Position of singularities in $x=\cos\theta$ for neutral pion photoproduction.

Energy, Mev:		300	450	600	700	800
Conventional nucleon current pole:		-4.78	-3.29	-2.66	-2.41	-2.22
ω^0 pole, $m=$	$2\,\mu$	1.79	1.38	1.24	1.19	1.16
	$2.3\,\mu$	2.04	1.50	1.32	1.25	1.21
	$3\,\mu$	2.57	1.85	1.54	1.43	1.36
	$4\,\mu$	4.11	2.52	1.97	1.77	1.63

In the special case of $m=2\,\mu$ we get the same position as indicated for the forward branch point in Fig. 2 of reference 27.

Thus we have shown that any evidence of a singularity in the foreward direction would be of great interest since it would indicate the existence of an ω^0 particle or a pion-pion interaction. The latter would be represented by a cut, but if the multipion system has strong resonances, the cut would look very much like one or several poles.

Now, to make the situation even more favorable, we will show that not only do we have no conventional singularities in the forward direction, but even the conventional singularities in the backward direction are much more distant from the physical region than the ω^0 or pion-pion singularities. This statement of course depends on what mass we attribute to the ω^0 or what energy to the pion-pion resonance, but, in any case, for for $m\leq 3.5\,\mu$ the statement is true. Table I of the location of the singularities illustrates the point.

It follows from the above discussion that the situation is just about ideal for the application of the extrapolation procedure first suggested by Chew,[28] which has already been applied with fair success to various reactions.[27,29–34] Given the differential cross section as a function of production angle at a fixed energy, one can extrapolate to the conjectured pole. If the residue thus obtained is nonzero, evidence for the existence of an ω^0 or a pion-pion interaction has been found.

In order to carry out such a procedure, one has to calculate the residue one would expect if the ω^0 existed. This is done in Sec. II. Then one has to carry out the extrapolation on experimental data. This is done in Sec. III. It is also interesting to know how accurate must the experiments be to be able to extrapolate to a given point with a given error in the corresponding residue. This is discussed in Sec. IV.

Before we plunge into these details, it is worth mentioning three general points.

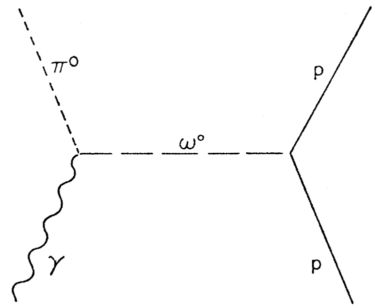

FIG. 1. Feynman diagram showing the contribution of a spin-1 particle to neutral pion photoproduction.

The first is related to the calculation of the residue. This residue can be obtained by performing the usual covariant lowest-order perturbation calculation, using the renormalized coupling constants. In the case of the spin-1 particle, however, the field theory is nonrenormalizable in the conventional sense. This is reflected by the fact that, as we will see in Sec. II, the residue as calculated by the procedure outlined above is large and increases with energy indefinitely. It is, however, finite at any fixed energy, which is the situation we are interested in. Its order of magnitude, however, at high energies can be much larger than that of the differential cross section itself. Since in dispersion theory one does not talk about renormalizability, one can say instead that the farther singularities in fact might partially cancel in order of magnitude the contribution of the nearby pole. This would occur mainly at high energies. The specific features of the pole contribution, however, presumably remain in evidence, and hence the extrapolation should point to the correct residue. In fact, the situation outlined above means that a negative result as to the existence of a residue could place quite stringent limits on the size of the coupling constants.

The second remark also pertains to the calculation of the residue. The present paper calculates only the effect on an ω^0 meson. If an ω^0 does not exist but some kind of a pion-pion interaction causes a cut in the forward direction, the effect on the differential cross section is more complicated. Even if a resonance is assumed, a model is needed to make specific predictions. A calculation of this kind for the energy dependence of

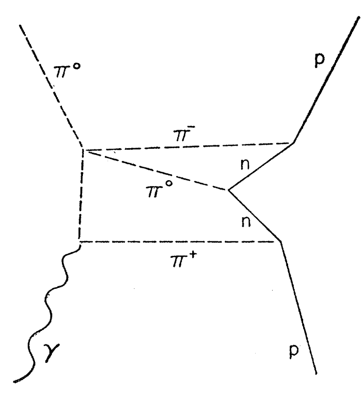

FIG. 2. Feynman diagram showing the contribution of a pion-pion interaction to neutral pion photoproduction.

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[33] N. S. Amaglobeli, B. M. Golovin, Yu. M. Kazarinov, S. V. Medvedev, and N. M. Polev, J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 660 (1960) [translation: Soviet Phys.-JETP **11**, 474 (1960)].
[34] R. Larsen, University of California Radiation Laboratory Report UCRL-9292, 1960 (unpublished).

the 90° differential cross section has been made recently.[35] It to some extent complements the present paper.

Finally, one should make a remark on the extrapolation procedure itself. It can lead to interesting and meaningful results, but only if careful statistical criteria are used in the analysis. An example of how sloppy statistics can lead to absurd results has been given by Feldman and Fulton.[36] It is also true, however, that experiments contain, beside random errors, some systematic errors which of course are not accounted for by statistics. Such statistical errors can invalidate all conclusions drawn from experiments. Whether the danger is larger for the extrapolation procedure than for any other way of extracting information from experimental data is not clear. In any case, one should try to eliminate such systematic errors by carrying out the extrapolation at various energies and for various sets of data, thus randomizing the nonrandom errors. This is at least attempted in Sec. III.

II. CALCULATION OF THE RESIDUE

We will now proceed to calculate the residue from the graph in Fig. 1, assuming both a vector and a pseudovector ω^0. Part of our notation has been explained in Sec. I. We will denote by $\boldsymbol{\varepsilon}$ and $\boldsymbol{\eta}$ the polarization vectors of the photon and ω^0, respectively; M will be the proton mass. We also have

$$a\cdot b=a_\mu b_\nu g_{\mu\nu}, \tag{2.1}$$

where $g_{00}=1$, $g_{ii}=-1$ ($i=1,2,3$), and $g_{\mu\nu}=0$ ($\mu\neq\nu$). We use $\gamma_0^2=\beta^2=1$, $\gamma_i^2=-1$ ($i=1,2,3$), and $\gamma_5^2=(\gamma_0\gamma_1\gamma_2\gamma_3)^2=-1$. We will denote by ψ, ϕ, and χ the wave functions of the nucleon, pion, and ω^0, respectively. The nucleon spinor will be called u. Finally g is the ω^0-nucleon coupling constant and $\mathfrak{M}$ the anomalous magnetic moment of the ω^0.

Now let us choose the basic interactions. For the ω^0-nucleon vertex the simplest couplings are

$$\begin{aligned}&g\bar{\psi}\gamma_\mu\psi\chi_\nu g_{\mu\nu} && \text{if } \omega^0 \text{ is a vector particle} && (V),\\ &g\bar{\psi}\gamma_\mu\gamma_5\psi\chi_\nu g_{\mu\nu} && \text{if } \omega^0 \text{ is a pseudovector particle} && (PV).\end{aligned} \tag{2.2}$$

In the vertex containing the photon, pion, and ω^0, the interaction must contain $F_{\mu\nu}$ and not A_μ, since we are talking about a magnetic moment interaction. Thus the simplest couplings are

$$\begin{aligned}&e(\mathfrak{M}/2m)\phi F_{\mu\nu}\epsilon_{\sigma\upsilon\tau\omega}G_{\lambda\rho}g_{\mu\sigma}g_{\nu\upsilon}g_{\lambda\tau}g_{\rho\omega}, && (V)\\ &e(\mathfrak{M}/2m)\phi F_{\mu\nu}G_{\lambda\rho}g_{\mu\lambda}g_{\nu\rho}, && (PV)\end{aligned} \tag{2.3}$$

where $\epsilon_{\mu\nu\lambda\rho}$ is the Levi-Civita tensor, totally antisymmetric such that $\epsilon_{\mu\nu\lambda\rho}=0$ if any of the two indices are equal, $\epsilon_{\mu\nu\lambda\rho}=-(-1)^P$ if all indices are different and P is the parity of the permutation $\mu\nu\lambda\rho$, and where

$$\begin{aligned}F_{\mu\nu}&=\partial_\mu A_\nu-\partial_\nu A_\mu=i(k_\mu\epsilon_\nu-k_\nu\epsilon_\mu),\\ G_{\mu\nu}&=\partial_\mu\chi_\nu-\partial_\nu\chi_\mu=i(r_\mu\eta_\nu-r_\nu\eta_\mu),\end{aligned} \tag{2.4}$$

with

$$\partial_\mu\equiv\partial/\partial x_\mu.$$

For the photon-ω^0-pion vertex one might also think, e.g., in the PV case, of the interaction

$$e(\mathfrak{M}/2m)(\partial_\mu\phi)\chi_\nu F_{\rho\sigma}g_{\mu\rho}g_{\nu\sigma}, \tag{2.5}$$

which is just as simple as the one we chose. One can show, however, that the two couplings are in fact identical except for sign. We have

$$\begin{aligned}\partial_\mu(F_{\rho\nu}\phi\chi_\sigma g_{\mu\rho}g_{\nu\sigma})=0=&(\partial_\mu F_{\rho\nu}g_{\mu\rho})\phi\chi_\sigma g_{\nu\sigma}\\ &+F_{\rho\nu}g_{\rho\mu}(\partial_\mu\phi)\chi_\sigma g_{\nu\sigma}+F_{\rho\nu}\phi g_{\rho\nu}(\partial_\mu\chi_\sigma g_{\nu\sigma}),\end{aligned} \tag{2.6}$$

and $\partial_\mu(g_{\mu\rho}F_{\rho\nu})=0$, which establishes the identity.

Now we are in the position of writing down the matrix element for the process in Fig. 1, using lowest-order perturbation theory and the rules as given, for instance, by Schweber[37]

$$\begin{aligned}T=&\frac{i\hbar c}{(2\pi)^4}\frac{1}{r^2-m^2}\frac{(\hbar c)^{\frac12}}{(2\pi)^{\frac32}}\frac{1}{(2k)^{\frac12}}\frac{(\hbar c)^{\frac12}}{(2\pi)^{\frac32}}\frac{1}{(2q_0)^{\frac12}}\frac{1}{(2\pi)^{\frac32}}\\ &\times\left(\frac{M}{p_0}\right)^{\frac12}\frac{1}{(2\pi)^{\frac32}}\left(\frac{M}{p_0'}\right)^{\frac12}\\ &\times\left(\frac{-i}{\hbar c}\right)^2(2\pi)^4(2\pi)^4 e\frac{\mathfrak{M}}{2m}\sum_{\text{int}}gI\bar{u}\Gamma u\\ =&-\frac{i}{(2\pi)^2}\frac{M}{2(kq_0p_0p_0')^{\frac12}}\frac{1}{(r^2-m^2)}e\frac{\mathfrak{M}}{2m}\sum_{\text{int}}gI\bar{u}\Gamma u,\end{aligned} \tag{2.7}$$

where

$$\left.\begin{aligned}I=&-(k_\mu\epsilon_\nu-k_\nu\epsilon_\mu)g_{\mu\tau}g_{\nu\upsilon}\epsilon_{\sigma\upsilon\tau\omega}\\ &\qquad\times g_{\lambda\tau}g_{\rho\omega}(r_\lambda\eta_\rho-r_\rho\eta_\lambda),\\ \Gamma=&\gamma_\beta\eta_\delta g_{\beta\delta},\end{aligned}\right\}\quad(V) \tag{2.8}$$

and

$$\left.\begin{aligned}I&=-(k_\mu\epsilon_\nu-k_\nu\epsilon_\mu)g_{\mu\rho}g_{\nu\sigma}(r_\rho\eta_\sigma-r_\sigma\eta_\rho),\\ \Gamma&=\gamma_\beta\gamma_5\eta_\delta g_{\beta\delta}.\end{aligned}\right\}\quad(PV) \tag{2.9}$$

The symbol $\sum_{\text{int}}$ refers to summing over the possible polarization directions of the intermediate meson. This sum can be carried out easily if we remind ourselves of a few properties of vector particles. We have in a covariant normalization

$$g_{\mu\nu}\eta_\mu\eta_\nu=-1. \tag{2.10}$$

[35] B. De Tollis and A. Verganelakis, Phys. Rev. Letters **6**, 371 (1961).

[36] G. Feldman and T. Fulton, Phys. Rev. Letters **3**, 64 (1959). The four pieces of data they used in their example are never enough for a statistical analysis.

[37] S. S. Schweber, H. A. Bethe, and F. De Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. I, pp. 242, 247, 248.

Furthermore, we have

$$\partial_\mu g_{\mu\nu}\chi_\nu = 0 = ir_\mu g_{\mu\nu}\eta_\nu, \tag{2.11}$$

and hence we can write for each of the three polarization vectors

$$g_{\mu\nu}r_\mu\eta_\nu{}^{(s)}=0, \quad (s=1,2,3). \tag{2.12}$$

In fact, if we make $s=3$ the longitudinal vector, we have

$$\eta_\mu{}^{(s)}=(0,\eta_i{}^{(s)}), \quad (s=1,2), \quad \sum_i \eta_i{}^2=1,$$

and

$$\eta_\mu{}^{(3)}=\left(\frac{r}{m},\frac{r_0}{m}\frac{\mathbf{r}}{r}\right). \tag{2.13}$$

To derive this last result we used Eq. (2.12). Then the following sum can be evaluated[23,38]

$$\sum_{(s)=1}^{3}\eta_\mu{}^{(s)}\eta_\nu{}^{(s)}=\frac{r_\mu r_\nu}{m^2}-g_{\mu\nu}.$$

With the help of these relations, we can sum over the intermediate spin polarization, obtaining after some algebra

$$I\Gamma=4\{(\mathbf{k}\times\boldsymbol{\varepsilon}\cdot\mathbf{q})\gamma_0-(\mathbf{k}\times\boldsymbol{\varepsilon}\cdot\boldsymbol{\gamma})q_0 -(\boldsymbol{\varepsilon}\times\mathbf{q}\cdot\boldsymbol{\gamma})k\}, \quad (V) \tag{2.14}$$

$$I\Gamma=-2\gamma_5\{\mathbf{k}\cdot\mathbf{q}\boldsymbol{\gamma}\cdot\boldsymbol{\varepsilon}-\boldsymbol{\varepsilon}\cdot\mathbf{q}\mathbf{k}\cdot\boldsymbol{\gamma}\}. \quad (PV)$$

Since the residue involves $|T|^2$, we now have to calculate it. We have

$$|T|^2=\frac{1}{(2\pi)^4}\frac{M^2}{4kq_0p_0p_0'}\frac{1}{(\mathbf{r}^2-m^2)^2}e^2\left(\frac{\mathfrak{M}}{2m}\right)^2 g^2 \times\tfrac{1}{2}\,\mathrm{Tr}\left\{I\Gamma\frac{(\mathbf{p}\cdot\boldsymbol{\gamma}+M)}{2M}I\Gamma\frac{(\mathbf{p}'\cdot\boldsymbol{\gamma}+M)}{2M}\right\}. \tag{2.15}$$

The trace can be calculated by standard methods. It gives

$$\tfrac{1}{2}\,\mathrm{Tr}\left\{I\Gamma\frac{\mathbf{p}\cdot\boldsymbol{\gamma}+M}{2M}I\Gamma\frac{\mathbf{p}'\cdot\boldsymbol{\gamma}+M}{2M}\right\}$$

$$=\frac{8}{M^2}\{2(p_0+k)^2(\mathbf{k}\times\boldsymbol{\varepsilon}\cdot\mathbf{q})^2-(k\cdot q-\tfrac{1}{2}\mu^2) \times[-q_0{}^2k^2+2q_0k\mathbf{k}\cdot\mathbf{q}-k^2q^2 +k^2(\mathbf{q}\cdot\boldsymbol{\varepsilon})^2+(\mathbf{k}\times\boldsymbol{\varepsilon}\cdot\mathbf{q})^2]\} \quad (V)$$

$$=\frac{2}{M^2}\{2k^2(p_0+k)^2(\mathbf{q}\cdot\boldsymbol{\varepsilon})^2 +(kq_0-\mathbf{q}\cdot\mathbf{k}-\tfrac{1}{2}\mu^2+2M^2)(kq_0-\mathbf{q}\cdot\mathbf{k})^2\}. \quad (PV) \tag{2.16}$$

The residue Q_0 in the $x=\cos\theta$ plane can be calculated from this by the relationship

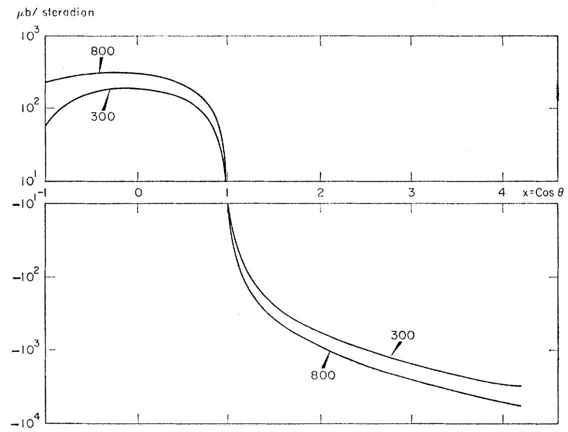

FIG. 3. $Q(x)$ vs x at 300- and 800-Mev incident photon energies in the laboratory system, for vector intermediate particles, using $g^2(\mathfrak{M}/2m)^2=1$. Note the peculiar scale on the ordinate.

$$Q_0=(2\pi)^2qq_0(1+q_0/q_0')^{-1}(1+k/p_0)^{-1} \times\left(1+\frac{m^2-\mu^2}{2q_0k}-\beta_\pi\cos\theta\right)^2|T|^2\Big|_{\cos\theta=\beta_\pi{}^{-1}+(2qk)^{-1}(m^2-\mu^2)}. \tag{2.17}$$

Thus

$$Q_0=(2\pi)^{-2}e^2g^2(\mathfrak{M}/2m)^2q(4k^3q_0{}^2)^{-1}(p_0+k)^{-2} \times[\text{rhs of Eq. (2.16)}]|_{\cos\theta=\beta_\pi{}^{-1}+(2qk)^{-1}(m^2-\mu^2)}. \tag{2.18}$$

This would be the residue for the photoproduction with polarized photons. Present-day experiments suitable for extrapolation, however, utilize unpolarized gammas. Thus we average over $\boldsymbol{\varepsilon}$ and obtain

$$Q_0=(2\pi)^{-2}e^2g^2(\mathfrak{M}/2m)^2q(4k^3q_0{}^2)(p_0+k)^{-2} \times C\{[q^2k^2-(q_0k+\tfrac{1}{2}(m^2-\mu^2))^2](p_0+k)^2 +[-\tfrac{1}{2}(m^2-\mu^2)+D][\tfrac{1}{2}(m^2-\mu^2)]^2\}, \tag{2.19}$$

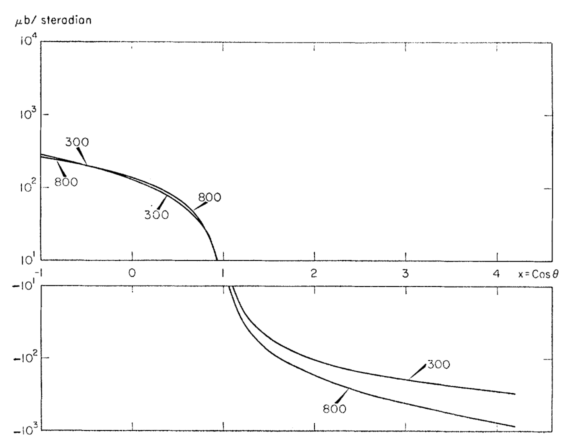

FIG. 4. $Q(x)$ vs x at 300- and 800-Mev incident photon energies in the laboratory system, for pseudovector intermediate particles, using $g^2(\mathfrak{M}/2m)^2=1$. Note the peculiar scale on the ordinate.

[38] R. P. Feynman, Phys. Rev. **76**, 769 (1949), Sec. 10 and footnote 27.

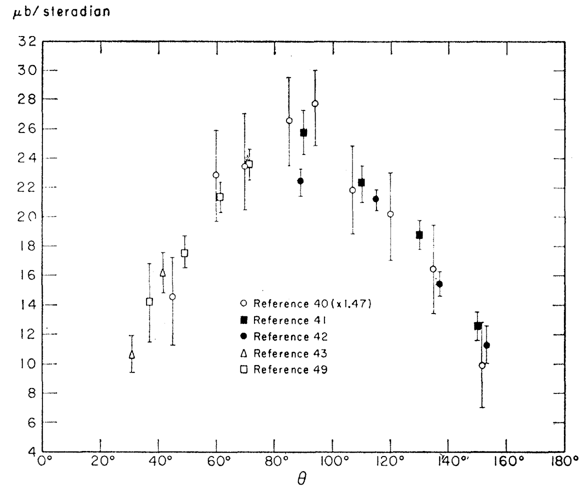

FIG. 5. Experimental data at 300-Mev incident photon energy in the laboratory system, as used in the present analysis.

where

$$C=8,\quad D=-\mu^2/2, \qquad (V)$$
$$C=2,\quad D=-\mu^2/2+2M^2. \qquad (PV) \tag{2.20}$$

This is our final result.

Numerical results are given in Figs. 3 and 4. The quantity plotted is

$$Q(x)=\left(1+\frac{m^2-\mu^2}{2q_0k}-\beta_\pi x\right)^2(2\pi)^2 \times qq_0(1+q_0/p_0')(1+k/p_0)|T|^2, \tag{2.21}$$

which is independent of m. Thus the value of the residue for a given m and at a given photon energy is given by the value of $Q(x)$ at that photon energy and that value of x which corresponds to the location of the pole for the value of m in question. The function $Q(x)$ is given

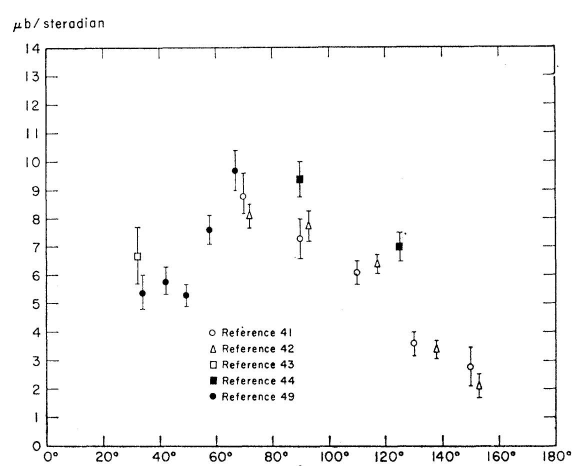

FIG. 6. Experimental data at 450-Mev incident photon energy in the laboratory system, as used in the present analysis.

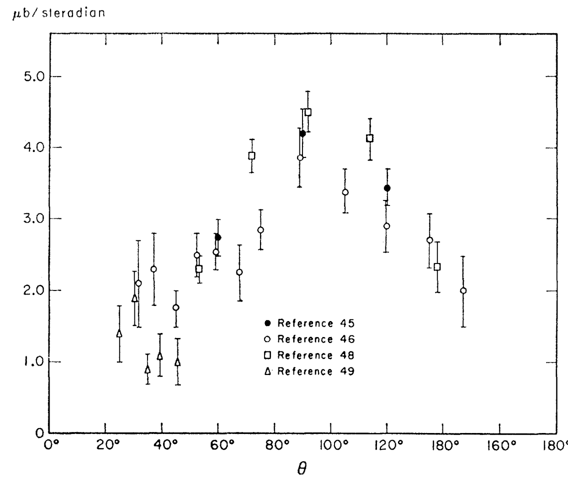

FIG. 7. Experimental data at 700-Mev incident photon energy in the laboratory system, as used in the present analysis.

for V and PV, at photon energies of 300 and 800 Mev in in the laboratory system.

One of the qualitative features of the results is that the residue is always negative for both V and PV. This is unlike the spin-0 case where the sign of the residue can be used to tell the parity of the intermediate particle. It is clear from the proof,[39] however, that such a property is somewhat fortuitous and cannot be expected to hold for arbitrary spin. In particular, in our case a term arises which is the same for V and PV and which, at least for the actual pion-nucleon mass ratio, is always the dominant one. The other term changes sign according to whether the intermediate particle has even or odd parity, much the same way as in the spin-0 case.

We see that the angular dependence of $Q(x)$ is peaked around 90° for the V case, and around 180° for the PV case. In differential cross section this corresponds to a strong peak for the V case somewhere in the middle of

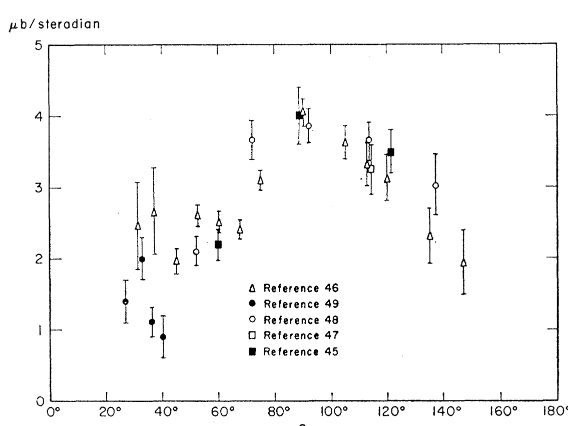

FIG. 8. Experimental data at 800-Mev incident photon energy in the laboratory system, as used in the present analysis.

[39] M. J. Moravcsik, in *Dispersion Relations* (Oliver and Boyd, Ltd., Edinburgh, 1961).

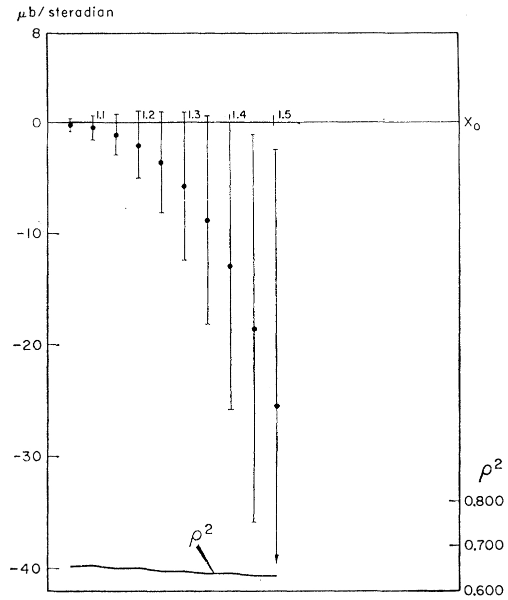

FIG. 9. Residue at 300-Mev as determined from a fourth-order extrapolation, as a function of the position x_0 of the pole in the x plane. The values of ρ^2 are also shown.

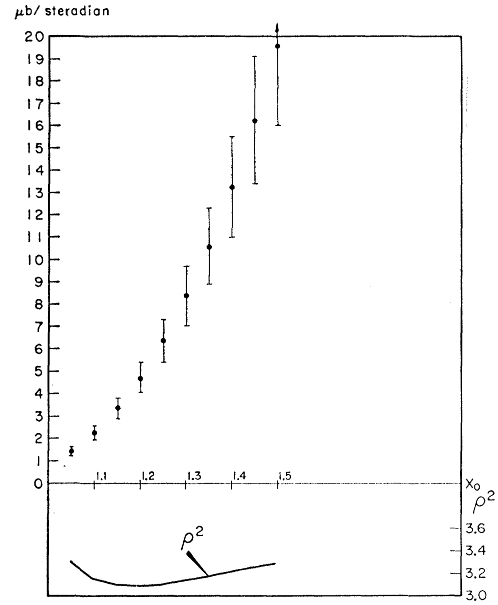

FIG. 11. Residue at 450 Mev as determined from a third-order extrapolation, as a function of the position x_0 of the pole in the x plane. The values of ρ^2 are also given.

the angular region and a relatively flat angular distribution for the PV case. This qualitative feature resembles the spin-0 case. Since, however, a large term of the type under consideration would also have large cross terms, the above features would not necessarily be patently present in the actual experimental differential cross section even if the intermediate spin-1 meson contributed significantly. In fact, this is the reason why it is not sufficient to glance at the qualitative features of angular distributions but rather one has to resort to extrapolation procedures to detect the presence of the spin-1 meson current term.

It is to be noted that the residue increases as the distance of the pole increases, and in fact the magnitude of the residue is very roughly (within a factor of 3 or so) proportional to its distance from the edge of the physical

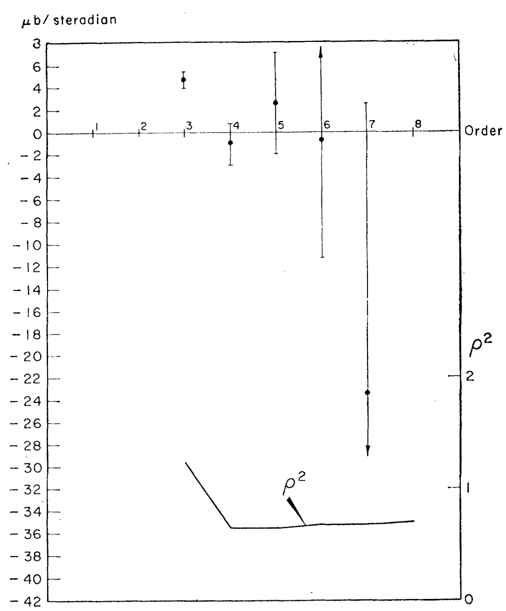

FIG. 10. Residue at 300-Mev at a pole position of $x_0=1.15$ as a function of the order of the extrapolating polynomial. The values of ρ^2 are also given.

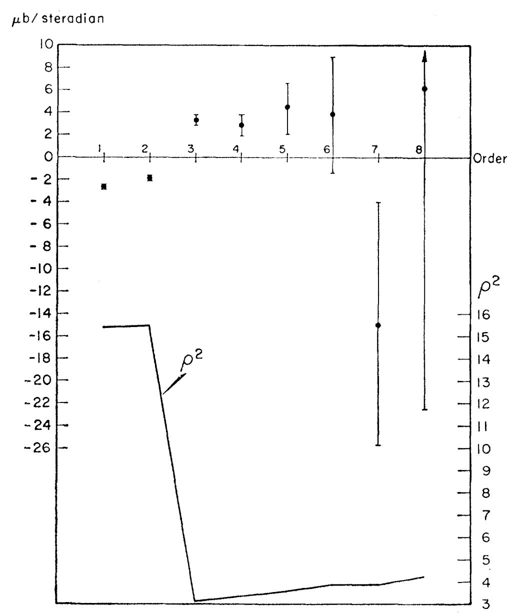

FIG. 12. Residue at 450 Mev at a pole position of $x_0=1.15$, as a function of the order of the extrapolating polynomial. The values of ρ^2 are also given.

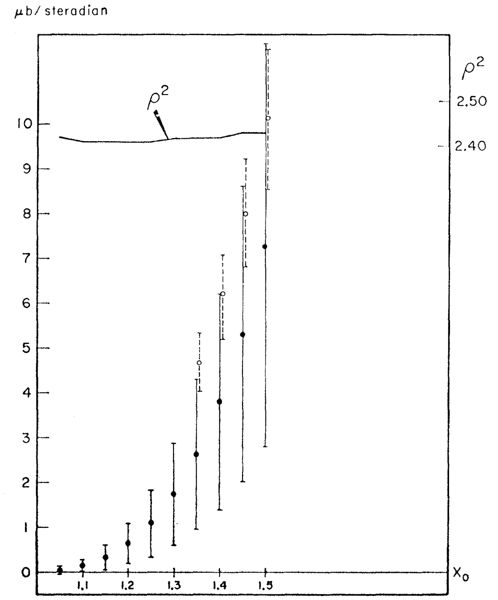

FIG. 13. Residue at 700 Mev as determined from the 4th-(solid line) and 3rd-(broken line) order polynomials, as a function of the position x_0 of the pole in the x plane. The values of ρ^2 are also given.

region. Using the electrostatic analogy for dispersion relations one would expect, therefore, that the effect of a pole, if far enough from the physical region, would be about the same for any combination of strength and distance which leaves the ratio of these two quantities constant. Thus we can conclude that if we could detect an effect in the differential cross section the product $g^2(\mathfrak{M}/2m)^2$ deduced from the effect would be insensitive

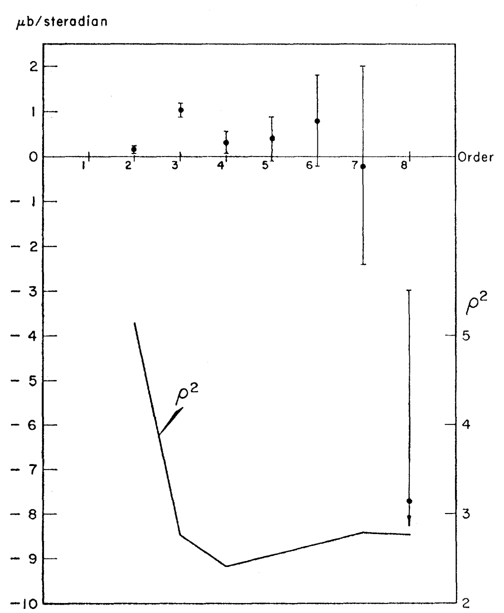

FIG. 14. Residue at 700 Mev at a pole position $x_0=1.15$, as a function of the order of the extrapolating polynomial. The values of ρ^2 are also given.

to uncertainties in our knowledge of the mass of the intermediate particle.

It is also of interest that the magnitude of the residue, while increasing indefinitely with energy, is not a strong function of energy, particularly not for the V case.

III. ANALYSIS OF EXPERIMENTS

Experimental information on neutral pion photoproduction is relatively scanty compared to charged photopions. Nevertheless, a considerable body of information has been building up on the differential cross section anywhere between 180 and 985 Mev.

For reasons outlined in Sec. I, the extrapolation has been carried out at four different energies. These were selected so that many data points should be available and also so that high energies should be well represented. The reason for the latter is that the higher the energy, the closer the pole is to the physical region for a given ω^0 mass. One might think that this will be counterbalanced by the increased number of powers of x needed to describe the angular variation. This, however, need not be necessarily so. In the region between 500 and 800 Mev we know that the main contribution beside S and P waves is the second resonance state which is in a $J=\frac{3}{2}$ state and hence does not increase the power of cosine beyond that needed for S and P waves Thus one might hope that the extrapolation at 800 Mev, *ceteris paribus*, would be actually easier than, say, at 400 Mev. This assumption was to some extent verified by the results. For a discussion of this question, see also Sec. IV.

The four energies chosen were 300, 450, 700, and 800 Mev. The data were taken from references 40–49. They are shown in Figs. 5–8. The extrapolation was carried out at each energy to $x=1.05, 1.10, \cdots, 1.50$. It was found that generally the error in the extrapolation of present-day data got to be too large if the procedure was carried beyond $x=1.50$. Of course, a given value of x corresponds to different values of the ω^0 mass at different energies. For each position of the presumed pole the extrapolation was carried out using polynomials of the 1st, 2nd, $\cdots$, 10th order and the standard statistical methods[50] were used to select the "right" order. It turned out to be in all cases the 3rd, 4th, or 5th order, in general agreement with expectations based on

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[41] R. L. Walker, D. C. Oakley, and A. V. Tollestrup, Phys. Rev. **97**, 1279 (1955).
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[43] W. S. MacDonald, V. Z. Peterson, and D. R. Corson, Phys. Rev. **107**, 577 (1957).
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[45] P. C. Stein and K. C. Rogers, Phys. Rev. **110**, 1209 (1958).
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[48] R. M. Worlock, Phys. Rev. **117**, 537 (1960).
[49] K. Berkelman and J. A. Waggoner, Phys. Rev. **117**, 1364 (1960).
[50] See, e.g., P. Cziffra and M. J. Moravcsik, University of California Radiation Laboratory Report UCRL-8523 (Rev.), 1959 (unpublished).

S, P, $D_{\frac{3}{2}}$ states being dominant at the energies under investigation. The residues as functions of the location of the pole are shown in Figs. 9, 11, 13, and 15, while the residues as functions of the order of the polynomial are shown in Figs. 10, 12, 14, and 16. In all these figures the corresponding ρ^2 (which is χ^2 divided by the number of degrees of freedom) is also given.

The selection of the proper order of the polynomial could be carried out with fair confidence. At 300 Mev it is the 4th and 5th order, and the prediction of the two orders agree quite well. At 450 Mev the 3rd order is the best, but the predictions of the 4th and 5th orders also give the same result within errors. At 700 Mev the 3rd and 4th orders give the best fits, in agreement with each other (see Fig. 14). At 800 Mev the 4th order is singled out, although there the plateau in ρ^2 as a function of the order is a bit bumpy (see Fig. 16). In general, therefore, there are no serious ambiguities in selecting the proper order.

One might remark in this connection that if the differential cross section itself demands a polynomial of order n, one cannot necessarily conclude that the order of polynomial giving the best fit to $Q(x)$ will be $n+2$. Firstly, the fitting of $Q(x)$ by a polynomial corresponds to a fitting of the differential cross section itself by a nonpolynomial, so that strictly speaking no direct comparison can be made. Secondly, the pole term might account for some of the higher powers of x required in the simple polynomial fit of the cross section. Thus, the physical arguments concerning the expected order of the best fitting polynomial are only approximate and have to be supplemented by statistical criteria.

Once the problem of selecting the proper order is taken care of, one would like to (a) investigate if the residues are significantly different from zero, and (b) if so, select the right location of the pole and thereby determine the mass of the ω^0.

At 300 Mev one cannot say that the residue is significantly nonzero. The slight tendency toward a negative residue is not convincing. At 450 Mev, on the other hand, a fairly definite positive residue is evident. The same is true to a lesser degree at 700 Mev. At 800 Mev the residue is zero within error. One would conclude therefore that the residue is zero or perhaps positive, but that no very definite conclusions can be drawn.

The indecisiveness of the conclusion from present data is underlined if one looks at the ρ^2 as a function of location. Firstly, the absolute values of ρ^2, with one exception, is significantly larger than unity, probably indicating that the data leave something to be desired. Furthermore, one would hope that if a pole really existed, one would reach a minimum in ρ^2 at its location. Such minimum, however, is either absent (300 and 800 Mev) or is very shallow (450 and 700 Mev) and occurs at distances corresponding to $m=1.5\,\mu$ and $m=2.3\,\mu$, respectively. The former is too low compared with expectations, while the latter is the right order of magnitude. The sign of the residue, however, is opposite

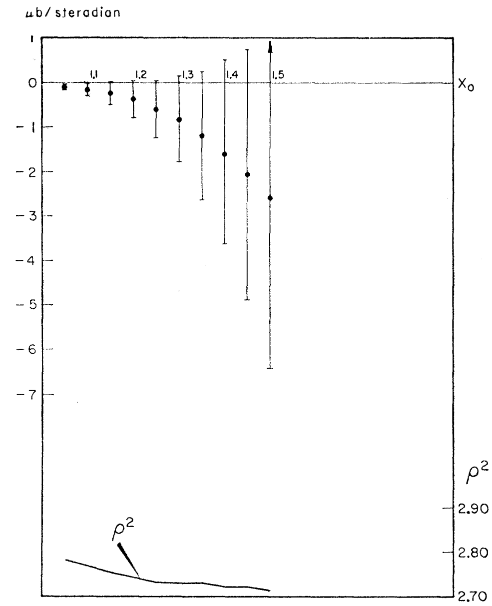

FIG. 15. Residue at 800 Mev, as determined from a 4th-order extrapolation, as a function of the position x_0 of the pole in the x plane. The values of ρ^2 are also shown.

to what we obtained in Sec. II, no matter whether ω^0 is V and PV. It might be mentioned, of course, that if the singularity is due not to an ω^0 particle but to another model of pion-pion resonance, the sign of the residue need not necessarily be negative. One might also note that, as we argued before, if both the strength and the position of the pole is treated as unknown, one should expect to find a range of locations in which the strength

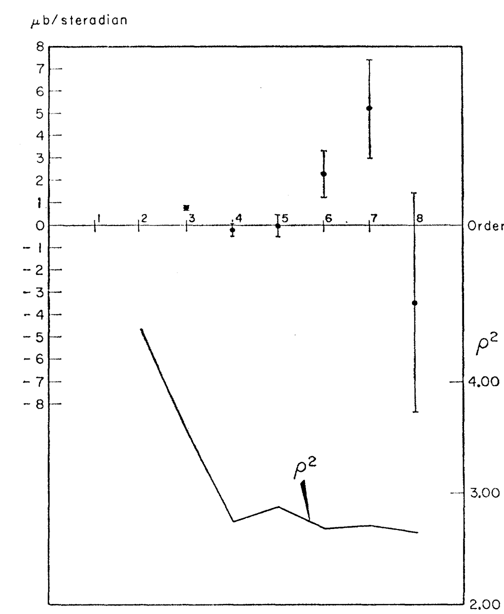

FIG. 16. Residue at 800 Mev at a pole position of $x_0=1.15$, as a function of the order of the extrapolating polynomial. The values of ρ^2 are also given.

is correlated with distance, and the couplets within this range should give approximately equally good fits. Hence, perhaps, the shallowness of the ρ^2 curves is no surprise. Finally it is also possible that, especially for the two lower energies, the location of the pole is beyond $x=1.50$. Nevertheless, the evidence presented here must be conservatively described as showing zero residue with a slight possibility of a positive residue.

Quantitatively we might summarize our results by saying that the 300-Mev data limit the order of magnitude of $g^2(\mathfrak{M}/2m)^2$ to be less than 0.03 to 0.1 for the vector case and 0.13 to 0.5 for the PV case, while the 700- and 800-Mev data impose the limits of 0.003 to 0.014 for the V case and 0.01 to 0.05 for the PV case. The ranges given here correspond to the various extrapolation distances giving slightly different upper limits.

Since the present experiments are indecisive, it is interesting to investigate the experimental requirements for a more accurate determination of the residue. This is done in the next section.

IV. SPECIFICATIONS OF EXPERIMENTAL REQUIREMENTS

In view of the tentative nature of the above results and in view of the increasing use of extrapolation techniques, it seemed advisable to carry out a detailed investigation of the relationship between the error in the residue, the number and distribution of experimental points, and the magnitude of the error in the extrapolated point. The magnitude of the measured differential cross sections themselves do not enter the problem.

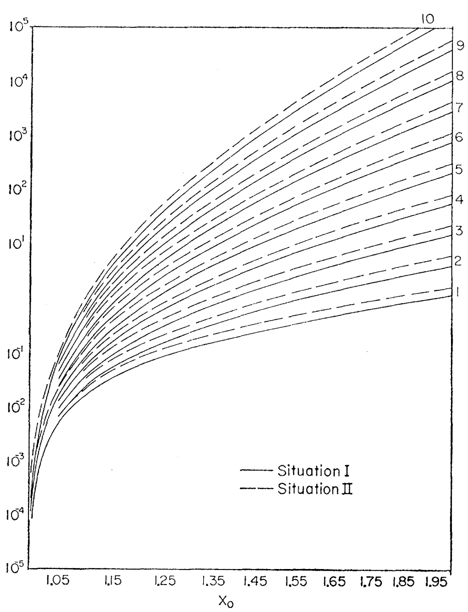

FIG. 17. Ratio of the error in the residue to the error in the experimental points, as a function of the position x_0 of the pole in the x plane for experimental situations I and II described in the text. The numbers 1, $\cdots$, 10 on the right refer to the order of the extrapolating polynomials.

Four hypothetical experimental situations have been considered. In situation I, experimental data are assumed to have been taken at every 5°, from 0° to 180°. In situations II and III the data are assumed to have been taken also in the whole angular interval, but every 10° and 15°, respectively. Finally, in situation IV it is assumed that we have data every 10° from 0° to 60° and every 15° thereafter up to 180°.

In all these cases absolute values of the errors in all the individual points were assumed to be equal. Since doubling the size of the errors on the individual points simply doubles the error in the residue, only the ratio of the error on the residue to the error on the individual points is of interest.

The above ratio is given for each of the four situations as a function of the location of the singularity. It might be mentioned in this connection that equal errors in the experimental data do not mean equal errors in $Q(x)$. The extrapolations have been performed by polynomials of 1st to 10th order. The results are shown in Figs. 17 to 19. They are summarized as follows.

The interpolation and extrapolation regions behave quite differently. In the figures only the boundary of the two regions is shown, but even this indicates that the error in the interpolation region is strongly dependent on the number of points, and that the variation with the order of the polynomial is much smaller for the many-point case than for the situations with fewer pieces of data.

The extrapolation region is quite different. It can be seen that after a short transition region (very small extrapolation distances) the pattern for all four situations investigated is the same except for a rather small over-all up-or-down displacement of the curves. This is an important property since it means that for any other combination of data points in the same angular range the error patterns in the extrapolation region can be established by computing only one point and normalizing the common set of curves against it.

We see that, *ceteris paribus*, the error depends very weakly on the number of data points. In particular, tripling the number of data decreases the error by less than a factor of two (compare situations I and III). We also see that the data points at the close end of the physical region are the most important. This is borne out by the errors in situations II and IV being almost completely identical.

On the other hand, the dependence on the distance of extrapolation is most critical, especially for small distances and high-order polynomials. Thus, e.g., for a 4th-order polynomial, the change of the position of the pole from 1.1 to 1.2 produces an increase in error by a factor of six.

The dependence on the order of the polynomial is uniform in the sense that at a given distance the error increases from order to order always by about the same factor. This factor increases as the distance increases.

The practical question often asked is whether it is

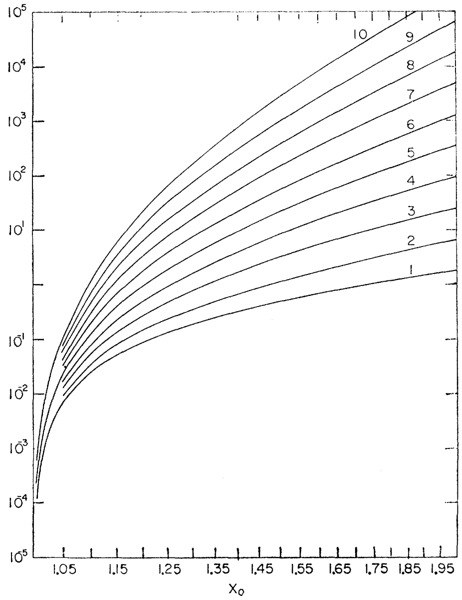

FIG. 18. Ratio of the error in the residue to the error in the experimental points, as a function of the position x_0 of the pole in the x plane for experimental situation III described in the text. The numbers 1, $\cdots$, 10 on the right refer to the order of the extrapolating polynomials.

more advantageous to do the experiment at high energies where the order might be large but the distance small, or at lower energies where the converse is true. In most particular situations one can make a decision now on the basis of the figures given here, but one can even state in general that it is very likely that the higher energy experiment will be preferable even at the cost of one or two additional parameters to be determined.

In any case, one of the important requirements is to take measurements at the close end of the physical region so as to decrease the extrapolation distance. For instance, if the pole is at 1.4, and a 4th-order extrapolation is needed, but the smallest angle point is at 25° instead of at 0°, the error is increased by a factor of two. Unfortunately, small angle differential cross section measurements often run into considerable experimental difficulties.

In view of these considerations as well as the results of the extrapolation from present data, it is suggested that an experiment be carried out around 800 Mev measuring the differential cross section of neutral pion photoproduction from 10° to 160° every 10°, with a relative error on the data points of $\pm 0.2\ \mu b$/sr. Such an experiment, if it confirms the choice of the 4th-order polynomial for the extrapolation, would reduce the error in the residue, compared to that given in this paper, by a factor of 3 to 5 depending on the precise location of the conjectured pole. It is needless to say that such a set of measurements would have other interesting uses also, since it would lie between the second and third-nucleon resonances.

If the above requirements turn out to be too stringent, the best compromise seems to be to thin out the data points around 90° to 120° (without, however, eliminating the endpoint at 160°) or measure them less accurately. If only a less ambitious experimental program is feasible, the 10° interval could be changed to 15° with a relatively small increase in the error on the residue.

It might be mentioned that in our case the high energy for the experiment is also suggested by the fact that the calculated residue itself, for given $g^2(\mathfrak{M}/2m)^2$, increases with energy and hence a negative result with a given error places stricter upper limits on $g^2(\mathfrak{M}/2m)^2$ at higher energies than it would at a lower.

It is perhaps needless to add that the results in Figs. 17 to 19 are in no way restricted to the particular reaction which is the topic of this paper but can be used in any extrapolation of angular distributions. In fact, the qualitative conclusions can also be used for the Chew-Low type[51] extrapolation which is also gaining in popularity.[52,53]

Note added in proof. Since this paper was submitted for publication, experimental evidence has accumulated for the existence of two vector mesons with masses around 5.3 and 5.6 pion masses. [For a recent summary of the status of vector mesons, see J. J. Sakurai, Phys. Rev. Letters **7**, 355 (1961).] At the lower energies considered in this paper, the pole corresponding to such a heavy intermediate meson would be so far from the physical region that its effect would likely be masked

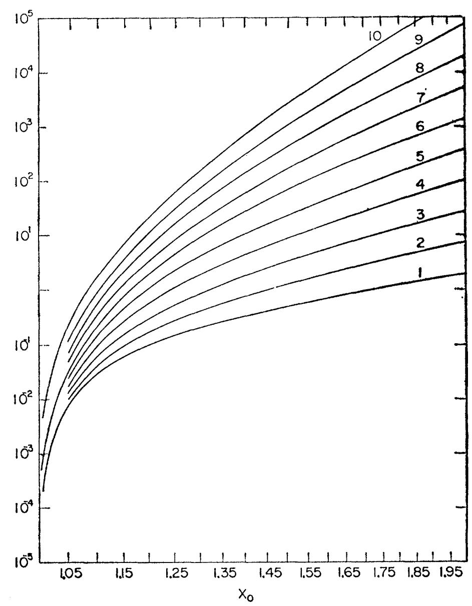

FIG. 19. Ratio of the error in the residue to the error in the experimental points, as a function of the position x_0 of the pole in the x plane for experimental situation IV described in the text. The numbers 1, $\cdots$, 10 on the right refer to the order of the extrapolating polynomials.

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[53] J. A. Anderson, Vo X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters **6**, 365 (1961).

by the nucleon pole (see Table I). At 800 Mev, however, this might not be the case, and the more precise experiment suggested in this paper, which would allow an extrapolation beyond 1.5, might show the effect.

ACKNOWLEDGMENTS

I am indebted to Dr. Sidney Bludman, Professor Geoffrey Chew, Dr. Bernard Lippmann, Mr. Stanley Schneider and Dr. James Young for various stimulating conversations. To the last of these I am grateful for letting me read his thesis prior to publication. I also want to thank Mr. Roy Clay and Miss Judy Ford for help in the numerical computations carried out on various electronic computers at the Lawrence Radiation Laboratory. The work was done under the auspices of the U. S. Atomic Energy Commission.

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Global Symmetry Resonance in Pion-Hyperon Scattering*

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Dispersion relations for fixed momentum transfer are applied to the problem of the Y_1^*, and it is shown that an s-wave resonance in π-Λ scattering does not appreciably influence the position of the p-wave π-Λ resonance predicted by global symmetry.

THE existence of the Y_1^* resonance now seems well established.[1] However, due to the effect of Bose statistics on the final two pions in the reaction

$$K^- + p \rightarrow Y_1^* + \pi \rightarrow \Lambda + \pi^+ + \pi^-,$$

the spin state of the Y_1^* is still an open question.[2] Theoretically, global symmetry[3] suggests that it should be $p_{\frac{3}{2}}$, while the calculation by Dalitz based of the s-wave $\bar{K}$-N scattering lengths shows that $s_{\frac{1}{2}}$ is an equally likely assignment.

The purpose of the present letter is to pursue a remark by Dalitz[4] that, if the resonance should turn out to be $s_{\frac{1}{2}}$, then perhaps the coupling of the $\bar{K}$-N system to the π-Y system might have shifted the resonance predicted by global symmetry to an extreme energy range, or that the coupling might destroy the resonance entirely. In this respect, two mechanisms for destroying global symmetry come to mind. First, there could exist a relatively strong nonglobally symmetric p-wave K interaction (e.g., due to a K meson-nucleon intermediate state); or second, recoil effects might couple a strong s-wave π-Λ interaction into the globally-symmetric p-wave equations. Concerning the first mechanism, there does not appear to be a very strong p-wave interaction in the reaction $\bar{K}+N \rightarrow \pi+\Lambda$.[5] However, since virtual effects may be expected to enter, this possibility is not really precluded. However, there does appear an *a priori* reason why the second mechanism might be significant. Chew *et al.*[6] have shown in π-N scattering that the resonating (3,3) p-wave produces a striking contribution to s-wave equations when recoil is taken into account. It is conceivable, therefore, that a corresponding effect might appear in the globally symmetric p-wave equations if the Dalitz s-wave resonance does in fact exist.

In this note we shall examine the latter possibility without investigating the effects of the first mechanism. A s-wave resonance is assumed for pion-hyperon scattering; it will be shown, however, that such a s-wave resonance has a negligible influence on the position of the $p_{\frac{3}{2}}$ resonance predicted by global symmetry.

As a first attempt, we use unsubtracted dispersion relations with a suitable cutoff. The technique used to couple the s-wave into the p-wave equations is that of Chew *et al.*,[6] and the notation used is identical with theirs. We write the basic dispersion relations for π-Λ scattering as

$$\mathrm{Re}A(\nu_L,\kappa^2) = \frac{P}{\pi}\int_1^\infty d\nu_L' \,\mathrm{Im}A(\nu_L',\kappa^2) \times\left(\frac{1}{\nu_L'-\nu_L}+\frac{1}{\nu_L'+\nu_L-2\kappa^2/M}\right), \quad (1)$$

and

$$\mathrm{Re}B(\nu_L,\kappa^2) = \mathrm{Poles} + \frac{P}{\pi}\int_1^\infty d\nu_L' \,\mathrm{Im}B(\nu_L',\kappa^2) \times\left(\frac{1}{\nu_L'-\nu_L}-\frac{1}{\nu_L'+\nu_L-2\kappa^2/M}\right), \quad (2)$$

* Supported in part by the National Science Foundation.
† National Science Foundation Predoctoral Cooperative Fellow.

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