Nucleon-Nucleon Interactions at Energies Greater than 10^{12} ev

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A primary jet of type $2+16p$ and a high-energy secondary jet of the same type as that of the primary, i.e., $2+16p$, were observed in nuclear emulsion. A very high energy electromagnetic cascade was also observed in the primary interaction. Both the primary and the secondary interactions were considered as nucleon-nucleon interaction for their analysis. The energy as calculated from the angular distribution of the shower particles of the primary and of the secondary events is 3.1×10^{12} and 2.5×10^{12} ev, respectively. The estimate of the energy of the primary event is also obtained from its total energy dissipated. The validity of the energies of the primary as well as of the secondary events as obtained from their angular distribution is discussed. The average transverse momentum of the shower particles is $\sim$0.30 Bev/c. The energy and the angles of shower particles of both these events are transferred to the c.m. system which shows sharp collimation of particles of high energy at small angles with the shower axis. The energy distribution in the c.m. system for both these events is peaked towards low values but shows a remarkably long tail at high energies extending up to 12 Bev. Analysis of both these events is consistent with the "two-fireball" model.

1. INTRODUCTION

DURING the past few years much interest has been shown in the study of high-energy ($\sim10^{12}$ ev) interactions. A number of laboratories are involved in this field all over the world. At these energies quite a few events had been reported. But in spite of a number of high-energy events recorded so far, only a very few events have been analyzed completely which can throw light on the details of the existing theories of high-energy nucleon-nucleon interactions.[1-6] The reason is that nuclear emulsions are generally used for the detection and for the analysis of the ultra-high energy nucleon-nucleon interactions. The most useful events for nucleon-nucleon interaction are those in which a nucleon collides with one single unbound nucleon of the emulsion. These types of events are found very rarely in nuclear emulsion, on account of its complex composition. Also for energy determination of the shower particles by means of scattering measurements, a very flat event is desired. The presence of secondary interactions produced by the shower particle helps a great deal in the analysis of the primary event. In order to observe all the secondary interactions along with the primary, one needs a very large size of emulsion stack. We shall report here two high-energy nuclear interactions in great detail which were observed in a medium size stack, as described in Sec. 2.

2. EXPERIMENTAL METHOD

A stack of 22 liters of Ilford G-5 emulsion, consisting of 200 pellicles, 60×30 cm, 600 μ thick, was exposed to cosmic radiation on a Skyhook balloon flight over Texas with the 60-cm side pointing in the vertical direction. The flight remained at an altitude of 116 000 ft for 13 hr. After development, each emulsion was cut into eight pieces of dimensions 15×15 cm. The stack was scanned for energetic electro-photon cascades which were traced backwards; all the other details are given in references 7 and 8.

We have observed in this stack an event[9] of type $2+16p$, as shown in Fig. 1. It is produced by a primary proton interacting with a single proton on the periphery of a heavy nucleus, as suggested by the even multiplicity of interaction. The incident proton of this event enters the stack with a zenith angle of 10° and a dip angle of about 3.5° with respect to the plane of the emulsion and travels about 13 cm in the emulsion before it makes an interaction with the emulsion nuclei.

By following the central core of the shower particles, which could be followed for more than 25 cm inside the stack within a cone of half-opening angle of 1×10^{-2} rad around the shower axis, an energetic secondary interaction of the same type as that of the primary, i.e., $2+16p$, shown in Fig. 2, was observed at a distance of 7.3 cm from the primary event. The distance between the shower axis of the primary and the secondary interaction is about 100 μ. It is assumed that the primary proton after making the first inelastic collision in the $2+16p$ event continues and makes a secondary collision once again of the same type, i.e., $2+16p$, as that of the primary event. In both the primary and the secondary event the inner and the outer cones have equal numbers of particles, i.e., 8 particles in each case. Apart from the inner tracks of the primary event, all the tracks in the outer cone were followed until they produced a secondary interaction or left the stack.

3. ENERGY DETERMINATION

One of the most serious problems in the study of high-energy nuclear interactions is that of a reliable

[1] E. Fermi, Progr. Theoret. Phys. **5**, 570 (1950), Phys. Rev. **81**, 683 (1951).
[2] W. Heisenberg, Z. Physik **126**, 569 (1949); **133**, 65 (1952).
[3] S. Belenki and L. Landau, Suppl. Nuovo cimento **3**, 15 (1956).
[4] P. Ciok, T. Coghen, J. Gierula, R. Holynski, A. Jurak, M. Miesowicz, T. Saniewska, O. Stanisz, and J. Pernegr, Nuovo cimento **8**, 166 (1958).
[5] G. Cocconi, Phys. Rev. **111**, 1699 (1958).
[6] K. Niu, Nuovo cimento **10**, 994 (1958).
[7] A. Barkow, B. Chamany, D. M. Haskin, P. L. Jain, E. Lohrmann, M. Teucher, and M. Schein, Phys. Rev. **122**, 617 (1961).
[8] P. L. Jain, Phys. Rev. **122**, 1890 (1961).
[9] P. L. Jain, Bull. Am. Phys. Soc. **5**, 24 (1960).

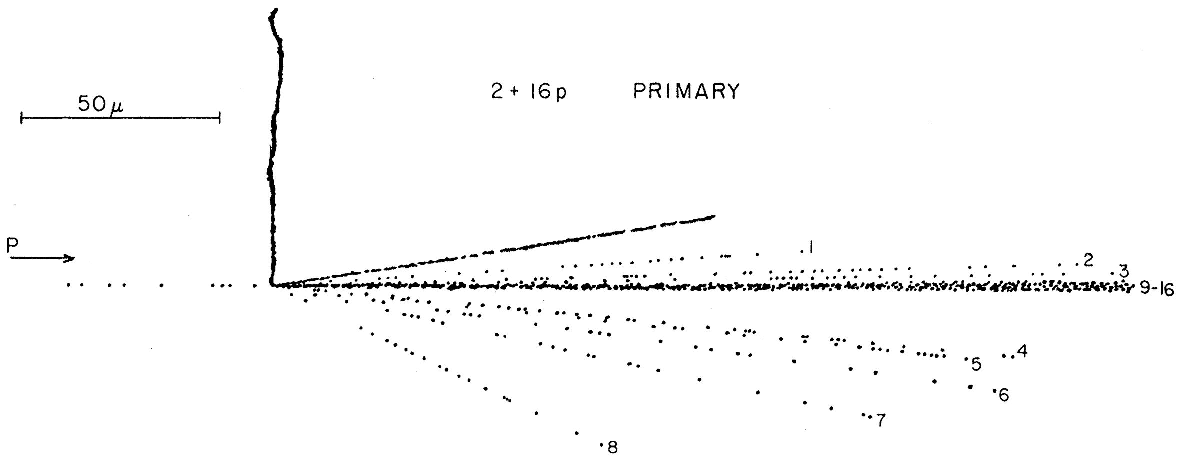


FIG. 1. Projection drawing of the primary event $2+16p$.

determination of the energy of the primary particle. The most common practice for the determination of the primary energy is to make use of the angular distribution of the shower particles which are produced in nucleon-nucleon or in nucleon-nucleus interactions. As yet there exists no procedure by which it could be ascertained without any doubt whether a particular collision is a nucleon-nucleon interaction or not. We shall follow here the generally accepted practice for selecting nucleon-nucleon jets, i.e., to chose only those events which have not more than three or four heavy prongs in the jet. For the energy determination of the primary event from the angular distribution of shower particles we shall use Castagnoli's[10] method

$$\log(C\gamma_c) = -\langle \log \tan\theta_i \rangle, \tag{1}$$

where C is roughly energy independent and is equal[11] to 1.4, γ_c is connected with the primary energy, E_p, per nucleon in the laboratory system by

$$E_p = (2\gamma_c^2 - 1)Mc^2, \tag{2}$$

θ_i is the angle which the ith shower particle makes with the direction of the primary particle in the laboratory system, and γ_c is the energy of the primary particle in the center-of-mass system in units of its rest energy, Mc^2 the rest energy of a nucleon. The angular distribution of shower particles of the primary and of the secondary event in the laboratory system is shown in Fig. 3 in terms of log $\tan\theta_L$ and will be discussed in Sec. 7. The energy of the primary and of the secondary events, as calculated from the angular distribution of shower particles from Eqs. (1) and (2), are 3.1×10^{12} and 2.5×10^{12} ev, respectively. We may mention here that the energy values obtained for individual cases from the above equations give a rough estimate of the true energies.

TABLE I. Angles, energies, and transverse momentum P_t of the secondaries of primary event $2+16p$. $\beta \equiv v/c$.

Track No.	θ	$P\beta$ (Bev/c)	P_t (Bev/c)	$P\beta$ determined from
1	6.3°	1.7 ±0.8	0.19	Scattering
2	3.5°	$8.5_{-3.1}^{+6.5}$	0.39	Scattering
3	1.5°	>5.1	>0.13	Scattering
4	6.4°	1.0 ±0.45	0.11	Scattering
5	8.0°	1.35±0.5	0.19	Scattering
6	10.0°	$1.3_{-0.6}^{+1.3}$	0.23	Scattering
7	12.7°	1.8 ±0.9	0.40	Scattering
8	34.5°	0.5 ±0.25	0.30	Scattering
9	1.6×10^{-3} rad	>45	>0.07	Scattering
10	1.6×10^{-3} rad	>50	>0.08	Scattering
11	1.0×10^{-3} rad	>70	>0.07	Scattering
12	2.6×10^{-3} rad	23 ± 9.5	0.06	Scattering
13	6.6×10^{-3} rad	27 ±10.7	0.18	Scattering
14	3.8×10^{-3} rad	160 ±65.0	0.61	$7+9p$ interaction
15	9.1×10^{-3} rad	15 ± 6.1	0.14	Scattering

The energies of the individual tracks in both these events were found in almost all cases by scattering measurements. Low-energy tracks in the outer cone were measured by the usual method of multiple scattering along the tracks. But for high-energy particles produced in the inner cone, the energy cannot be estimated by direct Coulomb scattering measurements. In those cases, however, where there are three or more shower particles very close to each other, relative scattering measurements over distances ~10 cm or more can be made which can give their energy values. In fact, relative scattering is the only method which can be used on these very fast tracks, since it eliminates the stage noise and the small distortions in the emulsions. In a few cases when the secondary track made an interaction with an emulsion nucleus and produced shower particles $n_s \geqslant 5$, the energy of this track was calculated from Eqs. (1) and (2). The experimental results for the energies and for the angles of all the tracks are shown in Tables I and II. In Table I, track number 16, which produces an energetic secondary event $2+16p$, is not included. In a few cases when the

[10] C. Castagnoli, G. Cortini, D. Moreno, C. Frazinetti, and A. Manfredini, Nuovo cimento 10, 1539 (1953).
[11] P. L. Jain, E. Lohrmann, and M. W. Teucher, Phys. Rev. 115, 643 (1959).

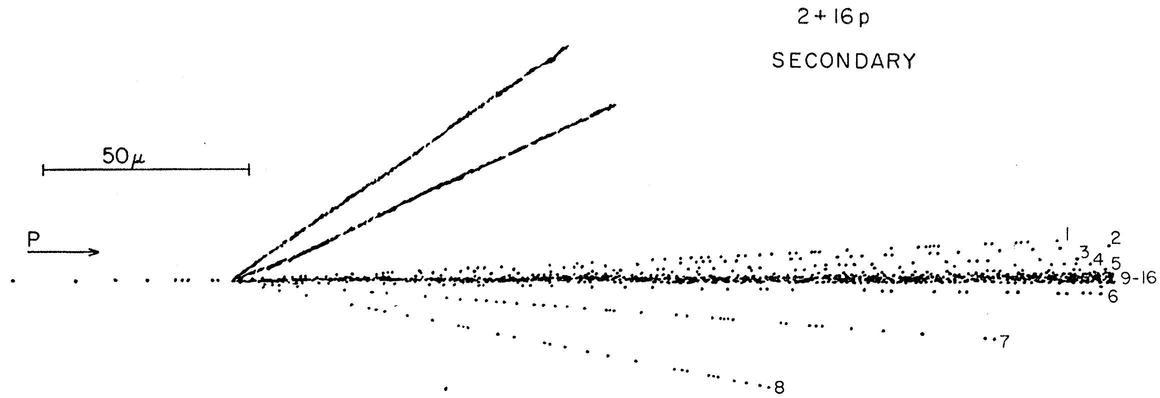


FIG. 2. Projection drawing of the secondary event $2+16p$.

track length available was not long enough or the energy was too high, or there was spurious scattering in the emulsion, only the lower limit of energy was given.

While scanning along the forward cone of the primary event high-energy electromagnetic cascades were also observed. Within a distance of about 2 cm from the origin of the primary jet, four high-energy electron pairs were found, which were probably due to decay of π^0 mesons into two γ's, each of which is further materialized into an electron-positron pair. Two of these electron pairs originate rather very high energetic cascades in the stack. No high-energy cascade was observed in the secondary event.

One can determine the energies of pairs and of the parent π^0 meson by several methods. The energy of the π^0 meson can be determined by estimating the energies of pairs of electrons arising from γ rays ($\pi^0 \rightarrow \gamma+\gamma \rightarrow 2e^{\pm}$), by observing scattering measurements on individual tracks of each electron. On the average an electron of an associated pair will have only about one-quarter of the energy of the parent neutral pion. For high-energy pairs in the core, one can make relative scattering measurements, just as for charged shower particles in a jet. In general, the positron and electron have very unequal energies and their relative scattering is much larger than a pair of shower particles in the same region and can thus often be measured. The relative scattering measurements give only the mean energy of a large group of electron pairs. For photons of a given energy, the relative scattering of the pairs into which they convert will fluctuate widely, depending on the disparity in energy between the electron and positron in each case, and thus cannot be used to determine the energies of individual photons without the known disparity distribution function. Moreover, if the photon energy is high (>100 Bev), the analysis becomes complicated by the fact that a

TABLE II. Angles, energies, and transverse momentum P_t of the secondary event $2+16p$. $\beta\equiv v/c$.

Track No.	θ	$P\beta$ (Bev/c)	P_t (Bev/c)	$P\beta$ determined from
1	4.7°	>2.8	>0.23	Scattering
2	5.1°	2.0 ± 0.7	0.18	Scattering
3	3.5°	>4.0	>0.24	Scattering
4	4.8°	2.2 ± 0.8	0.18	Scattering
5	1.5°	$3.4_{-1.6}^{+3.2}$	0.08	Scattering
6	2.2°	$2.8_{-1.9}^{+2.8}$	0.09	Scattering
7	9.2°	>1.0	>0.15	Scattering
8	11.5°	1.2 ± 0.36	0.19	Scattering
9	6.2×10^{-3} rad	$20\ \pm9.0$	0.14	Scattering
10	3.8×10^{-3} rad	>50	>0.19	Scattering
11	3.5×10^{-3} rad	125_{-60}^{+120}	0.44	Interaction $4+10p$
12	5.2×10^{-3} rad	>32	>0.16	Scattering
13	8.0×10^{-3} rad	$10\ \pm4.7$	0.08	Scattering
14	3.0×10^{-3} rad	110_{-60}^{+100}	0.33	Interaction $7+7p$
15	3.9×10^{-3} rad	65_{-30}^{+60}	0.27	Scattering
16	5.0×10^{-3} rad	$35\ \pm15$	0.18	Scattering

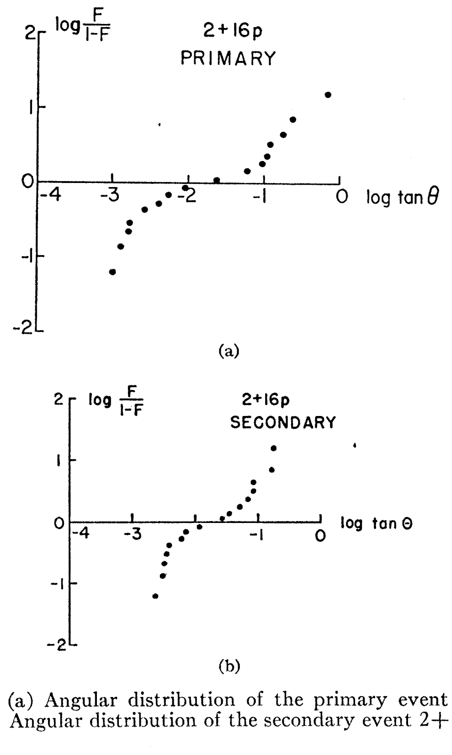


FIG. 3. (a) Angular distribution of the primary event $2+16p$. (b) Angular distribution of the secondary event $2+16p$.

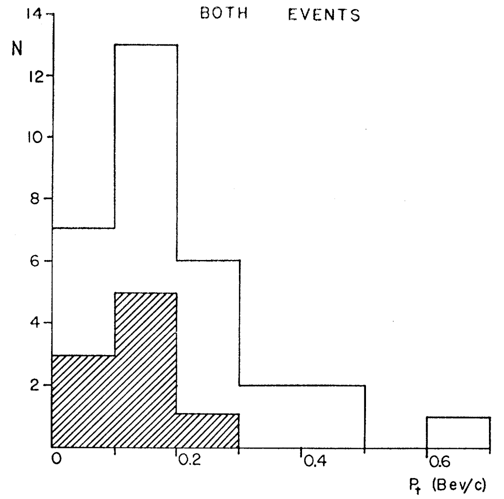


FIG. 4. Distribution of transverse momentum for the primary $(2+16p)$ and the secondary $(2+16p)$ events. The shaded area represents measurements when only lower limits were obtained.

long track length is required for energy determination, which means possible radiation loss of the electrons has to be taken into consideration; also the creation of bremsstrahlung pairs by one of both members of a pair close to the tracks will complicate the analysis, further.

As the energies of some of the photons are very high, we did not apply the above method, but instead we measured and plotted the lateral distribution of the electrons at distances of 2, 3, 4, 6, and 8 cascade units (1 cascade unit = 2.9 cm in emulsion) from the origin of the pairs. A correction was applied to eliminate background tracks from the secondary nuclear interactions and neighboring cascades. We estimated the energies of these four γ rays by making use of Pinkau's method[12] which is based on the theory of Nishimura and Kamata.[13] Energies of γ rays were checked independently by measuring the decrease in ionization (known as Chudakov effect)[14,15] of the track of the pair near the point of conversion. This decrease in ionization is due to the destructive interference of the electromagnetic fields of two particles with equal charges but opposite in sign. Ionization measurements are very reliable for photon energy greater than 100 Bev. Both the above methods gave the same energies for the γ rays, within their experimental errors. We then tried to match the γ rays by using the kinematical relation between the opening angle and the energy of pair γ rays, which is given by

$$\theta_{2\gamma}=m_{\pi^0}c^2(\sqrt{n}+1/\sqrt{n})/E_{\pi^0}, \tag{3}$$

where $n=E_{\gamma1}/E_{\gamma2}$, $m_{\pi^0}c^2$ = rest energy of π^0 meson, and E_{π^0} = energy of π^0 meson. Here only a rough estimate of n is necessary to determine E_{π^0}. From the above relation, we thus obtained the value of two π^0 mesons as 1.15×10^{12} and 3.5×10^{11} ev. Their transverse momenta were 0.60 Bev/c and 0.42 Bev/c, respectively. The energies of all other π^0 mesons were small as compared with these two values, as they did not originate comparably energetic cascades and their contribution at 4, 6, and 8 cascade units would be negligible. It is very interesting to know that a single π^0 meson could take as high an energy as up to 35% of the total energy of the primary particle. These two π^0 mesons can carry away an appreciable fraction of the total energy going into meson production. Ordinarily one would expect that soft cascades contribute only about $\frac{1}{2}$ of the energy of the charged shower particles.

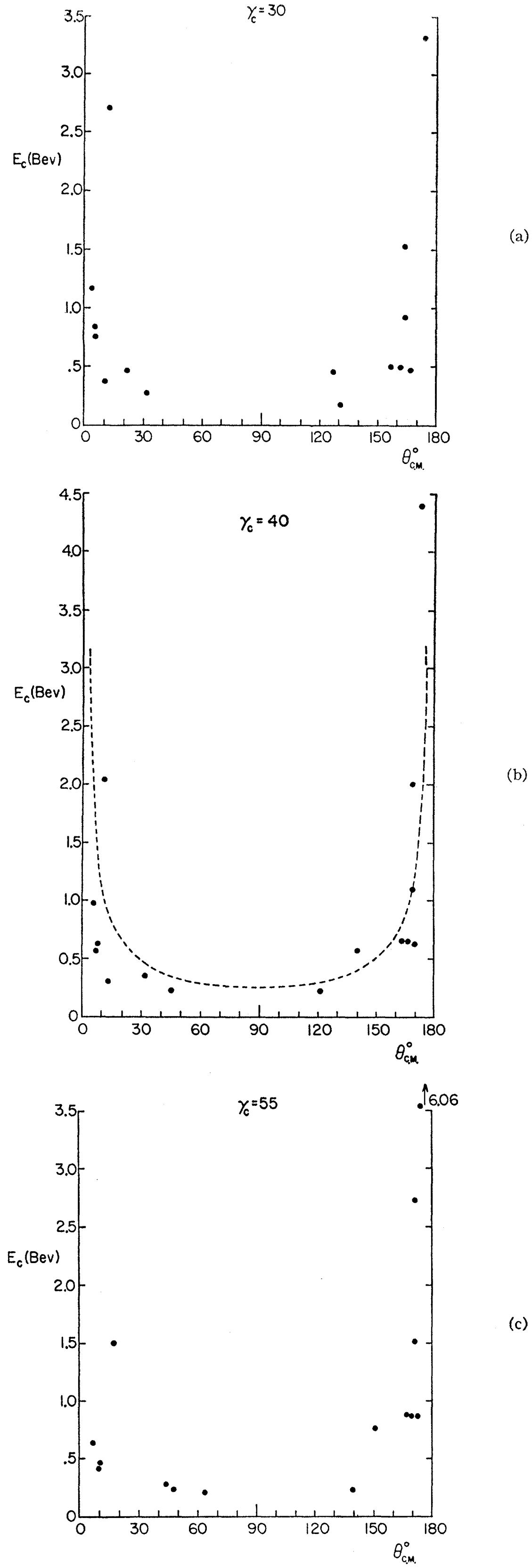
4. TRANSVERSE MOMENTUM

Nishimura[16] first pointed out that on the average the transverse momentum ($P_t=P\sin\theta$) of the secondary particles produced in relativistic interactions is constant (0.32 Bev/c). Recently this has been confirmed by many experimental results.[17,18] Fretter[19] has shown that the transverse momentum not only for pions but also for heavy mesons is also constant. Since P_t is Lorentz invariant, it will be the same in all reference frames moving in the direction of the primary particles. (See Fig. 4.) Our experimental results also indicate that the angles and energies of shower particles produced in nuclear interactions are correlated in such a way as to make the transverse momentum P_t approximately constant. This correlation can be seen in Figs. 5 and 6 for primary and secondary events. The values of P_t for all shower particles of the two events are given in Tables I and II. For some cases only a lower limit for P_t could be established. In Table III are shown the directly measured average values of P_t, for both events in the forward and the backward cones separately. For primary event $2+16p$, $\langle P_t\rangle$ in the forward cone seems slightly lower than in the backward cone. This may be due to the fact that most of the energies of the backward cone tracks could be actually measured, whereas in the

TABLE III. Average transverse momentum P_t.

Interaction	$\langle P_t\rangle$(forward cone) Bev/c	$\langle P_t\rangle$(backward cone) Bev/c
$2+16p$ (primary)	$\geqslant0.17$	0.24
$2+16p$ (secondary)	$\geqslant0.22$	$\geqslant0.16$

[12] K. Pinkau, Phil. Mag. **2**, 1389 (1957).
[13] J. Nishimura and K. Kamata, Progr. Theoret. Phys. **7**, 185 (1952); **5**, 889 (1950).
[14] D. H. Perkins, Phil. Mag. **46**, 1146 (1955).
[15] A. E. Cudakov, Compt. rend. **19**, 651 (1955).
[16] Z. Koba, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, New York, 1956), Vol. IV, p. 46.
[17] B. Edwards, J. Losty, D. H. Perkins, K. Pinkau, and J. Reynolds, Phil. Mag. **3**, 237 (1958).
[18] M. Schein, D. Haskin, E. Lohrmann, and M. Teucher, Phys. Rev. **116**, 1238 (1959) (other references are given here).
[19] L. Hansen and W. Fretter, Phys. Rev. **118**, 812 (1960).



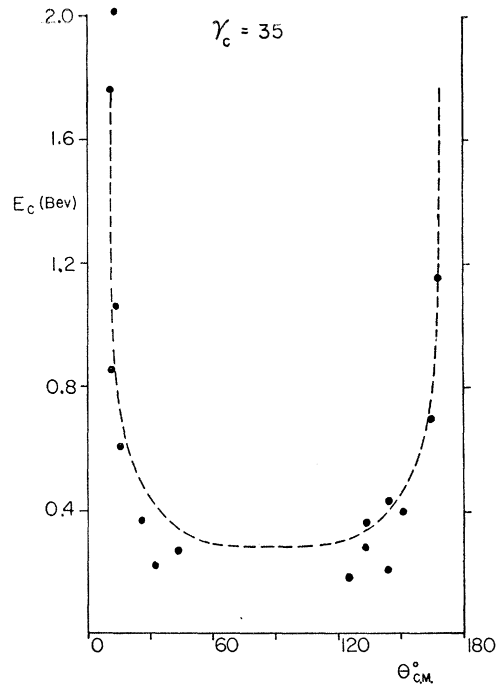


FIG. 6. Transformation to c.m. system of secondary event $2+16p$. $\gamma_c=35$, $E=2.1$ Tev. The dashed line corresponds to a constant value of the transverse momentum $P_t=1.8m_\pi c$.

forward cone, for 3 out of 7 tracks, only a lower limit could be established. As the shower particles in both these interactions are emitted equally backwards and forwards from the collision system, considerations of symmetry suggest that P_t should be the same for both forward and backward particles. If so, then the value of $\langle P_t\rangle$ in the forward cone cannot be increased more than a factor of 1.41; otherwise the energy contained in the secondary particles would be higher than the primary energy. In the case of the secondary event, the $\langle P_t\rangle$ of the forward cone is slightly higher than $\langle P_t\rangle$ for the backward cone. There are only two tracks in the forward cone with lower limits and if we once again assume that the $\langle P_t\rangle$ is equal for both the cones, then the $\langle P_t\rangle$ in the forward cone cannot be increased more than a factor of 1.38; otherwise their total energy would be higher than the primary energy determined from the angular distribution of shower particles. In the above discussion we have assumed that the tracks, with the lower limits in the energy in the backward cone of the primary event and in the forward cone of the secondary event, give their energy values from the scattering measurements as the true values of their energy. This point will be further discussed in Sec. 6.

In Fig. 4 are shown the values of P_t for both the events. The shaded area represents measurements when

FIG. 5. Transformation to c.m. system of primary event $2+16p$. In (a) $\gamma_c=30$, $E=1.8$ Tev. In (b) $\gamma_c=40$, $E=3.2$ Tev. The dashed line corresponds to a constant value of the transverse momentum $P_t=2m_\pi c$. In (c) $\gamma_c=55$, $E=6.0$ Tev.

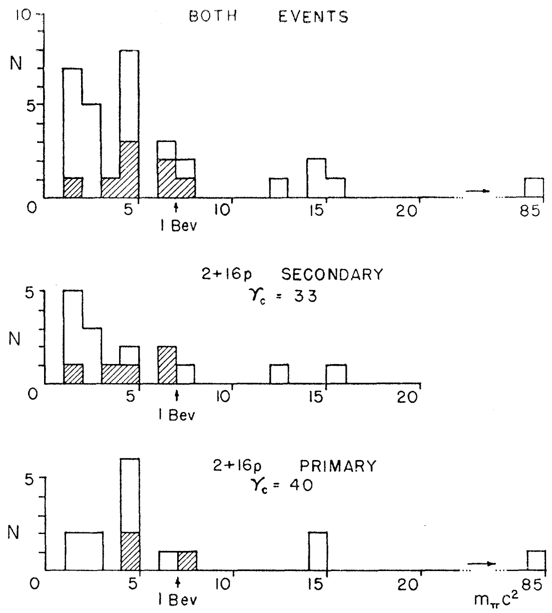


FIG. 7. Energy distribution in the c.m. system of the primary and the secondary events ($2+16p$). The abscissa is in units of the pion rest mass.

only lower limits were obtained. The peak value of the distribution is at about 0.2 Bev/c. No values of $P_t > 0.70$ were observed. The average value of P_t for both these events is $\sim$0.30 Bev/c, which is consistent with other authors.[16–19]

5. ENERGY AND ANGULAR DISTRIBUTION IN THE CENTER-OF-MASS SYSTEM

We considered the relativistic transformation[20] from the laboratory system into the center-of-mass system for both the primary and the secondary events, assuming each collision as a nucleon-nucleon collision. In this transformation we have also assumed that all the particles produced are π mesons.

The correlation between the energy and the angle is shown in Figs. 5 and 6 for primary and for secondary events, respectively. The distribution shows a sharp collimation of particles at small angles with the shower axis. The dashed line corresponds to a constant value of the transverse momentum and it is symmetrical with respect to the center.

The energies of primary and secondary events as calculated from Eqs. (1) and (2) have values 3.1 and 2.5 Tev (1 Tev$\equiv 10^{12}$ ev) which correspond to approximate values for γ_c of 40 and 35, respectively. Figure 5 (a) and (c) show the correlation between the angles and the energies in the c.m. system for $\gamma_c = 55$ and 30, which approximately corresponds to double and one-half energy of the primary event, respectively. The shape of the distributions indicates that the transformation is not very sensitive to the choice of γ_c at these high energies. With change of γ_c values, there is slight shift of all the particles about the dashed curve of constant P_t towards its right or left depending upon whether we increase or decrease γ_c value, respectively, but the general shape of the distribution remains the same. The same type of phenomenon is observed in the secondary event for $\gamma_c = 35$. Two tracks with energy about 1 and 0.65 Bev in the backward cone have lower limits in energy, and a small change in the value of their energy and angle might make the distribution a little more symmetrical. These small changes fall within the experimental errors. We may also point out that the tracks in the backward cone which have low energies in the laboratory system are very sensitive to the mass of the particles. Considering a particle heavier than the π meson in the backward cone can make a great change in the value of its energy and angle in the c.m. system.

In the primary event one charged particle in the backward cone has an energy of 4.5 Bev, which has a predominant influence on the average energy of the charged particles in the backward cone. The average energy for the forward and for the backward cones is 1.28 and 0.72 Bev, respectively. The energy value for the forward cone gives only the lower limit, as for several tracks in the forward cone only the lower limits of the energy in the laboratory system could be established. Also we have not taken into consideration the existence of neutral mesons. One of the π^0 mesons has an energy of about 12 Bev in the c.m. system, and if we consider the average energy of charged and neutral mesons, this one particle will have a considerable influence upon the average energy in the forward cone. In the secondary event ($\gamma_c = 35$), the average energy of the forward cone is 0.91 and of the backward cone is 0.45 Bev. There are two charged particles in the forward cone with energy equal to 2.1 and 1.75 Bev, which have more influence on the average energy of the forward cone. In the secondary event we have not considered any neutral π mesons, which, on account of their low energy, are hard to detect in the laboratory system.

The individual and the combined energy distribution of the mesons is given for both the events in Fig. 7. The shaded area shows the measurements for those events where only lower limits were obtained. The energy distribution of the mesons in the c.m. system shows a peak towards low energies, and has a remarkably long tail at high energies extending up to 12 Bev. The average energy for the primary event is 1.02 Bev and for the secondary event is 0.67 Bev, respectively. These values are the lower limits because of the lower limits for several particles in the laboratory system. If we take into account the neutral π mesons, too, it can be shown from the consideration of momentum balance (just as in Sec. 6) in the c.m. system of the primary event that this lower limit cannot be far from

[20] R. G. Glasser, D. M. Haskin, M. Schein, and J. J. Lord, Phys. Rev. **99**, 1555 (1955).

TABLE IV. Energy balance of the primary $2+16p$ event.

	Total energy lower limit (Bev)	Most probable value (Bev)
Shower particles of primary event $2+16p$	410	580
Soft cascades	1500	1500
Shower particles of secondary event $2+16p$	680	940
Total	2590	3020

the true value. The average energy of the mesons in the c.m. system of both the events is approximately 1.1 Bev.

This is a very interesting result as Fermi's[1] theory predicts a much higher average value (about 7 Bev). Heisenberg's[2] theory is also to some extent in disagreement with our results, but Landau's[3] theory can be adjusted to agree with our experimental value of average energy. We may point out here that the average energies are very sensitive to the existence of a few particles in the high-energy tail of the distribution. Also if there are heavy mesons among the secondaries, there would be a small effect on the high-energy particles in the core, while the effect on the particles in the backward cone would be large. But the present experimental evidence indicates that the fraction of the particles of non pionic mass among the shower particles is small and as the multiplicity of both these events is also small, we can safely say that not more than one or two of the particles in the backward cone could be K mesons.

6. ENERGY BALANCE AND INELASTICITY

The total energy given out by the primary collision can be estimated by summing up the energies of all the shower particles and also adding to it the total energy of the soft cascades produced in the primary collision. But as some of the shower particles have lower limits in energy, we can only know the total energy approximately. This is shown in Table IV. The first column contains the energy of the shower particles by scattering measurements and it has been based on the approximation that the lower limit is the true value of the energy of the shower particle in the primary as well as in the secondary event. The estimate of the soft cascade was based on four high-energy electromagnetic cascades and we did not include the energy of the other low-energy cascades arising from low-energy π^0 mesons. The lower limit on the energy of the secondary jet was also derived from scattering measurements and we have added about 50% more energy for π^0 mesons. Thus the total dissipated energy of the primary event is calculated as best as possible and is shown in the first column, which is equal to 2.59 Tev. The most probable values of the primary event are obtained on the basis of symmetry arguments between the forward and the backward cones as discussed in Sec. 4. For the shower particles of the primary as well as of the secondary event, the energies were raised by a factor of 1.40 and 1.38, respectively, and the values are given in the second column of Table IV. If we assume that no major fraction of the energy escaped detection, then we get for the primary event the most probable energy value of 3.0 Tev, which is approximately the same value as estimated from its angular distribution. Thus if the energy of the primary event is correct, then definitely the energy of the secondary event as calculated from the angular distribution is not correct. One can expect this, as stated in Sec. 3, since the energy values determined from the angular distribution by means of Eqs. (1) and (2) only give a rough estimate of the primary energy and for an individual case it can fluctuate greatly. Thus in the secondary event $2+16p$, the energy is overestimated at least by a factor of two. This is further supported from the fact that if we consider the transverse momentum of shower particles to be constant and has a value equal to 0.30 Bev/c, then the total energy for all the charged particles of the secondary event is equal to about 700 Bev. There were neither any high-energy electromagnetic cascades nor any high-energy tertiary interaction observed in the secondary event. If we add about 50% of the energy for the neutral particles, then the total energy of the secondary event is about 1050 Bev, which is very close to the most probable energy value of the secondary event in Table IV. Of course, one cannot rule out the possibility that the energy of the primary event as determined from the angular distribution could also be wrong.

In the study of high-energy nuclear interactions, one of the important quantities of great interest is the inelasticity parameter η of the collision. η is defined as the fraction of the total available kinetic energy of the colliding particles, before the collision, which is subsequently used for the production of mesons and other particles. As was pointed out earlier, most probably the secondary event $2+16p$ was produced by the primary particle of the first shower, which, after making the primary interaction, continues after the collision. The inelasticity of the primary event in the laboratory system is then given approximately by

$$\eta = 2080/3020 = 0.68.$$

To estimate limits of error for η, we used the lower limit of energy in Table IV, which gives the upper limit for $\eta \sim 0.80$. A lower limit for η is obtained by considering an uncertainity of about a factor of 2 for the determination of energies by means of the angular distribution of the shower particles. This will give a lower limit of about $\eta \sim 0.40$. Thus the final value is

$$\eta = 0.68_{-0.28}^{+0.12}.$$

But if we consider that the energy of the secondary event is correctly given by Eqs. (1) and (2), i.e., 2.5×10^{12} ev and that the primary energy is under-

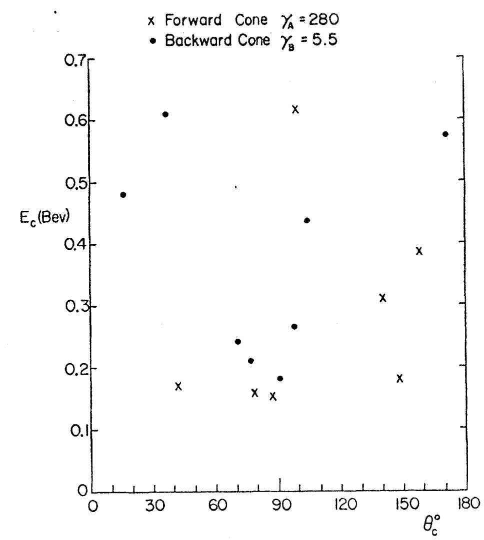


FIG. 8. Transformation of primary event $2+16p$ using the two-center model.

estimated by a factor of 2 then the value of inelasticity η of the primary event is $\sim$0.34.

Thus the value of inelasticity η of the secondary event is $\sim$0.3 and it is based on scattering measurements of the secondary particles, adding 50% energy for the π^0 mesons.

7. SHAPE OF THE ANGULAR DISTRIBUTION AND PREDICTION OF THE TWO-CENTER MODEL

The shape of the angular distribution determines the dispersion which is given by the relation

$$\sigma = \langle (x-\bar{x})^2 \rangle^{\frac{1}{2}}, \tag{6}$$

where

$$x = \log_{10} \tan\theta_L. \tag{7}$$

The dispersion of this distribution is a measure of the degree of anisotropy and for an isotropic distribution in the symmetry system, σ is equal to 0.39. σ increase with increasing collimation of the shower particles in the direction of the shower axis and for $\sigma > 0.60$ the distribution is strongly anisotropic. Recently[8,21,22] it has been proved that for events with $\sigma > 0.60$, the differential angular distribution in the nucleon-nucleon as well as in the nucleon-nucleus collision is not normal, but may show two separate maxima, corresponding to a "two center" model. There have been theoretical predictions for the "two center" model by many authors.[4–6] A special feature of this theory is that it shows two distinctive maxima in the angular distribution with a sufficient deficit of particles at 90° in the c.m. system. From the point of view of the "two center" model the angular distribution is a superposition of two separate Gaussian distributions. Therefore, we can expect the appearance of the separate maxima only if there is a sufficient separation of the partial distributions corresponding to a sufficiently high dispersion of the resulting distribution. But when the distance between the two centers is small, the two maxima may overlap each other and thus smear away the fine structure corresponding to individual centers, with the result that we cannot distinguish the shower particles belonging to one center from the ones belonging to the other. The values of σ for the primary and for the secondary events are 0.95 and 0.68, so according to the criterion for the "two center" model, the angular distribution of both these events should show two-center structure.

The Duller-Walker[23] plot for primary and secondary events is shown in Fig. 3 (a) and (b), respectively, where $\log\{F(\theta)/[1-F(\theta)]\}$ has been plotted against $\log \tan\theta$, and $F(\theta)$ is the fraction of total number of particles having an angle of emission less than θ in the laboratory system. The slope of this distribution is $\sim$1 indicating that the distribution of shower particles is anisotropic. In both cases the continuous curves passing through the corresponding points of the shower particles show the characteristic humps which indicate that the particles are emitted in the c.m. system of the collision not by a single center but by two centers. In order to check this point further, F plots of both the events are broken into two groups, the forward group A and the backward group B (the wide and the narrow cone) in either case. The values of γ_A and γ_B are 280 and 5.5, and 150 and 8 for the primary and for the secondary event, respectively. The transformation (just as in Sec. 5) shown in Figs. 8 and 9 for primary and for secondary events indicates that the emission of particles from the assumed centers could probably be considered reasonably isotropic within the statistical errors. The

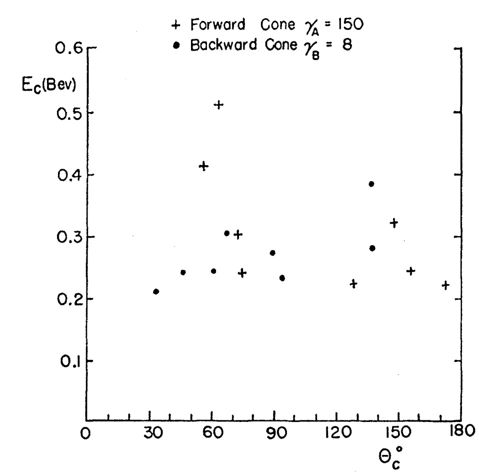


FIG. 9. Transformation of the secondary event $2+16p$ using the two-center model.

[21] J. Bartke, P. Ciok, J. Gierula, R. Holynski, M. Miesowicz, and T. Saniewska, Nuovo cimento **18** (1960).

[22] J. Gierula, M. Miesowicz, and P. Zielinski, Nuovo cimento **18**, 102 (1960).

[23] N. Duller and W. Walker, Phys. Rev. **93**, 215 (1954).

average energy of the charged particles would then be $\bar{E}_p=0.35$ and $\bar{E}_{sec}=0.30$ Bev, respectively. It has been pointed out[6] that for the two-center system the average longitudinal momentum $\langle P_l\rangle$ of both the branches A and B in their own reference frames should be of the same order of magnitude as that of transverse momentum itself and we have found that for two branches of the primary event $\langle P_t\rangle\approx 0.24$ and 0.17 while $\langle P_l\rangle \approx 0.23$ and 0.18 for the forward and the backward cones, respectively. Similarly in the case of the secondary event the average longitudinal momentum is approximately the same as that of the transverse momentum for each branch separately. The analysis of these events is consistent with the "two-fireball" model.

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Spin and Parity of the ω Meson*

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The spin and parity of the ω meson have been determined to be 1^- by a quantitative comparison of the density of points on the Dalitz plot with the predicted density of points. The simplest matrix element for the $J=1^-$ meson (which predicts maximum density of points at the center of the Dalitz plot and zero density on the boundary) fits the data remarkably well. On the other hand, the simplest matrix elements for 1^+ and 0^- mesons (both of which predict zero density at the center of the plot) do not fit at all. A quantitative treatment for higher J values has not been attempted. However, the simplest matrix elements for $J=2^+$ and 2^- also vanish at the center of the Dalitz plot, and are again inconsistent with the data.

In a previous paper[1,2] we presented data suggesting a spin assignment $J=1^-$ (vector meson).

The density of points on our Dalitz plot was depopulated near the boundary, and densely populated near the middle. This density was compared with the three possible simple matrix elements for $J\leqslant 1$. Their qualitative features are summarized in Table I, which is slightly more detailed than the version that we presented earlier. The meaning of the angular momenta $\mathbf{l}$ and $\mathbf{L}$ is as follows: The matrix element is analyzed in terms of a single pion plus a dipion. The pions of the dipion are assigned a momentum $\mathbf{q}$, and an angular momentum $\mathbf{L}$ (in the dipion rest frame). Then another pair of variables, $\mathbf{p}$ and $\mathbf{l}$, describe the remaining pion in the ω rest frame.[3] A $T=0$ state of three pions must be antisymmetric in all pairs; hence all three of the competing matrix elements must vanish where any two pions "touch" in momentum space (i.e., $q=0$). This corresponds to regions b, d, and f, on the Dalitz plots in Fig. 1, D and E.

The three competing matrix elements have the following characteristics:

(a) The axial matrix element (1^- meson) involves cross products of momentum vectors and vanishes whenever all momenta are collinear. This occurs on the boundary of the Dalitz plot.

(b) The scalar matrix element (0^- meson) vanishes whenever two pions have the same energy, i.e., along the straight lines that divide the Dalitz plot.

(c) The vector matrix element (1^+ meson) vanishes whenever all three pions have the same energy, i.e., at the center of the plot. In contrast to the scalar matrix element, it does not vanish at points a, c, and e, where $|p|=0$ (the finite population here is contributed by the $l=0$ partial wave). Values of $p=1$ and 2 (units of $m_\pi c$) are indicated in Fig. 1. E.

We concluded that the data suggest that ω is a 1^- meson. This paper presents quantitative support for

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[1] B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters **7**, 178 (1961).

[2] The collaborating groups of Johns Hopkins and Duke Universities have informed us of evidence for the ω meson in the reaction $\pi^++d\rightarrow p+p+\pi^++\pi^-+\pi^0$ at 1.23-Bev/c incident momentum. Further evidence for the ω meson has been found by Xuong and Lynch in the reaction $\bar{p}+p\rightarrow 3\pi^++3\pi^-+\pi^0$. In their paper [N. H. Xuong and G. R. Lynch, Phys. Rev. Letters **7**, 327 (1961)] they report $M_\omega=780$ and $\Gamma/2<18$.

[3] C. N. Yang (private communication) points out that the matrix elements of Table I must be formed of linear combinations of different values of L and l. Furthermore, L and l are meaningful only in the limit of nonrelativistic pions. In this limit, the following linear combinations are required: 1^+ meson, $l=2$ and 0, $L=1$; 0^- meson, $l=3$ and 1, $L=3$ and 1; where $l=1$, $L=1$ is the minimum complexity necessary for the 1^- meson. These additions are included in Table I.