For positive pions $\eta_{21}=2\eta_{22}$ on the basis of the total scattering cross sections in this energy range. η_{21} was taken from the values given by Frank *et al.*[20] The data are given as a function of the kinetic energy of the pion within the nucleus, measured in the laboratory frame of reference. In order to find this energy, the nuclear potential is included in the way specified by Frank *et al.*, and finally the energy was adjusted by adding or subtracting the Coulomb potential energy corresponding to a particle of unit charge at R_p for negative and positive pions, respectively. The values of η_{21} are listed in Table V.

PHYSICAL REVIEW VOLUME 125, NUMBER 2 JANUARY 15, 1962

Pion Production in Pion-Nucleon Collisions and the Pion-Pion Interaction*

EMMA PEREZ FERREIRA
Comisión Nacional de Energía Atómica, Buenos Aires, Argentina
(Received July 31, 1961)

The momentum distributions of the pions of all charge states produced in the interaction of 960-Mev π^- mesons with protons are calculated on the assumption of a primary $\pi-\pi$ interaction followed by a $\pi-N$ interaction with excitation of the (3,3) isobar state of the nucleon. The predicted spectra are in reasonable agreement with experimental data when the $\pi-\pi$ resonance energy is assumed to be about 700 Mev.

EXPERIMENTAL results[1] on single pion production in π^--p collisions at 960 Mev were found to be in reasonable agreement with the predictions of the isobar model[2] regarding both branching ratios and momentum distributions of the final pions in the two possible reactions:

$$\pi^-+p\rightarrow\pi^-+\pi^++n, \quad \text{(I)}$$

$$\pi^-+p\rightarrow\pi^-+\pi^0+p. \quad \text{(II)}$$

However, a similar experiment[3] performed at 1 Bev showed some discrepancies with the predicted momentum distribution of the final pions of reaction II. Selleri[4] and Landovitz and Marshall[5] have recently suggested that those discrepancies could be due to the rapid onset of a different process, in addition to pion-nucleon resonances, viz. a pion-pion resonance, which, for instance, is necessary in order to account for the nucleon electromagnetic structure, as well as for the low-energy pion-nucleon phase shifts.[6] These authors proposed therefore a model, according to which the production process should start with the interaction between the incoming pion and a pion of the cloud. Then, a final-state interaction would excite the nucleon to the (3,3) isobar state, with a further decay into a pion and a nucleon.

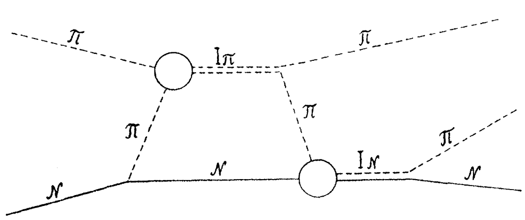

FIG. 1. Diagram for the pion production process. I_π stands for the $T=J=1$ di-pion state and I_N for the $T=J=\frac{3}{2}$ nucleon isobar.

Some attempts[4,7–10] have been made in order to justify through that predominant one-pion exchange (with or without final-state interaction), the behavior of the total pion-nucleon cross sections, in particular, the maximum in the π^--p interaction at 900 Mev and in π^+-p at 1400 Mev. Furthermore, several authors have investigated the possible influence of the pion-pion interaction in pion production through the analysis of the energy spectrum of the secondary nucleon in the laboratory frame of reference,[11,12] of charge state branching ratios[7,13] and by means of the more refined method suggested by Chew and Low[14] which was recently applied by Anderson *et al.*[15] to their experimental results. Neither the latter method nor the semi-phenomenological treatment have led to any definite

* This is part of a work submitted to the Faculty of Sciences of the University of Buenos Aires, in partial fulfillment of the requirements for the degree of Doctor in Physics.

[1] V. Alles-Borelli, S. Bergia, E. Perez Ferreira, and P. Waloschek, Nuovo cimento **14**, 211 (1959).

[2] R. B. Sternheimer and S. J. Lindenbaum, Phys. Rev. **109**, 1723 (1958).

[3] I. Derado and N. Schmitz, Phys. Rev. **118**, 309 (1960).

[4] F. Selleri, Nuovo cimento **16**, 775 (1960).

[5] L. Landovitz and L. Marshall, Phys. Rev. Letters **4**, 474 (1960).

[6] J. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters **5**, 386 (1960).

[7] P. Carruthers and H. A. Bethe, Phys. Rev. Letters **4**, 536 (1960).

[8] R. F. Peierls, Phys. Rev. Letters **5**, 166 (1960).

[9] F. J. Dyson, Phys. Rev. **99**, 1037 (1955).

[10] G. Takeda, Phys. Rev. **100**, 440 (1955).

[11] F. Bonsignori and F. Selleri, Nuovo cimento **15**, 465 (1960).

[12] I. Derado, Nuovo cimento **15**, 853 (1960).

[13] B. A. Muir, E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters **6**, 192 (1961).

[14] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

[15] J. A. Anderson, V. X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters **6**, 365 (1961).

conclusion about the role of a $T=J=1$ resonant pion-pion interaction.

There is still one more characteristic feature in the experimental results on single pion production, namely, the well-defined momentum distribution of the final pions in the center-of-mass system, which ought to be explained by any tentative model. In this paper we intend to find out which would be the predicted spectra, under the assumptions of the abovementioned model,[4,5] involving a primary pion-pion resonant interaction with final-state pion-nucleon interaction. We assume, therefore, the pion production process as taking place through a diagram like that of Fig. 1.

For a kinematical calculation, the following steps should be, thus, taken into account:

$$\pi+N\rightarrow N'+I_\pi, \tag{1}$$

$$I_\pi\rightarrow\pi'+\pi', \tag{2}$$

$$\pi'+N'\rightarrow I_N, \tag{3}$$

$$I_N\rightarrow\pi''+N''. \tag{4}$$

As a consequence of this process, the final pion momentum distribution must be similar to that predicted by the isobar model. As a matter of fact, one of the pions from the I_π decay will excite the nucleon to its (3,3) isobar state. Since the latter has a rather well-defined mass, the possible momentum values for the other pion should be confined to a correspondingly well-defined interval, producing a maximum in the spectrum, similar to the recoil-pion peak of the Lindenbaum and Sternheimer model. On the other hand, the momentum distribution of the pions originating in step 4 will result in a much broader peak since it implies a Lorentz transformation from the I_N rest system in to the c.m. frame, also in analogy with what happens in the isobar model.

The main difficulty arising in our problem is due to the fact that the I_N will be produced by pions coming from the di-pion decay, having, thus, momenta distributed over a wide range. In principle, the broader the pion-pion resonance, i.e., the greater the interval of possible I_π masses, the more complicate seems to be the decay-pion momentum distribution in the c.m. system of the original pion and nucleon. The pion-pion resonance half-width has been estimated as about 200–300 Mev. We have, therefore, calculated the c.m. spectra of pions decaying from the I_π for an incident kinetic energy of 960 Mev, for two different assumptions: (a) a resonant di-pion state with a half-width of 250 Mev, centered, according to Frazer and Fulco,[16] around 600 Mev, and (b) a sharp resonance at 600 Mev. It turns out that both decay spectra do not differ substantially. This can be understood from the fact that the probability of exciting a di-pion with a certain mass must be proportional to the total pion-pion cross-section

[16] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959).

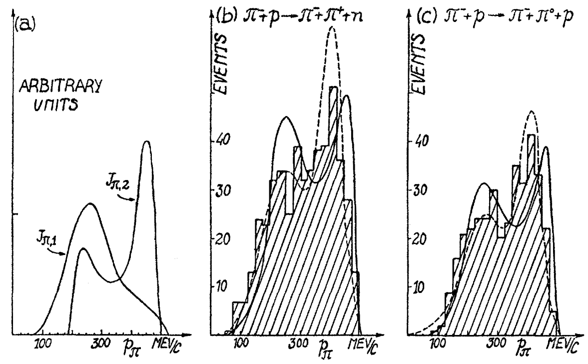

FIG. 2. Center of mass momentum spectra for pions from $\pi^--p\rightarrow\pi+\pi+N$, for an incident pion energy $T_{\rm inc}=960$ Mev and an assumed value of 600 Mev for the pion-pion resonance energy. (a) Predicted curves $J_{\pi,1}$ and $J_{\pi,2}$; (b) all pions from reaction I; (c) all pions from reaction II. In (b) and (c) the histogram represents the data of Alles-Borelli *et al.* (reference 1). The solid curves were obtained as described in this paper. The dashed curves give the result of the isobar model.

at the appropriate pion energy. Consequently, the contribution of the di-pion mass corresponding to the resonance energy will be dominant in both cases. Moreover, the Lorentz transformation of the decay-pion momentum into the c.m. system will reduce the existing differences, in particular, in the case where one assumes, as we do, that the di-pion decays isotropically in its own rest system.

Therefore, one could take a single value for the I_π mass as a reasonable approximation. In this case, since we have a fixed total energy W_c, the spectator nucleon in step 1 will have a fixed momentum. Therefore, for each angle between the nucleon and one of the pions in the c.m. system, one will obtain a fixed total energy for this pion-nucleon system. The transformation into its c.m. system will give the possible I_N mass. Accordingly, the probability of exciting an isobar with mass M_{I_N}, will be given by:

$$P_{\frac{3}{2}}(M_{I_N})dM_{I_N}=A\sigma_{\frac{3}{2}}(M_{I_N})N(M_{I_N})dM_{I_N}, \tag{f1}$$

where $\sigma_{\frac{3}{2}}(M_{I_N})$ is the total π^+-p cross section at the appropriate π^+ energy, $N(M_{I_N})$ is the probability for a pion to come out with the proper angle in the c.m. system to yield the corresponding M_{I_N}. A is a constant.

The c.m. energy spectrum of pions (of all charge states) which originate from pion-nucleon collisions, will thus, be given by:

$$\frac{d\sigma}{dT_\pi}=B\int_{M_1}^{M_2}\sigma_{\frac{3}{2}}(M_{I_N})N(M_{I_N})G_\pi dM_{I_N}+C\sigma_{\frac{3}{2}}(M_{I_N})N(M_{I_N})\frac{dM_{I_N}}{dT_\pi}, \tag{f2}$$

where M_1, M_2, B, C, and G_π have the same meaning as in reference 2, i.e., M_1 and M_2 are the limits of all the kinematically possible M_{I_N} values, B and C are constants which merely serve for normalizing both terms, and G_π is a factor giving the energy spectrum of the pions arising from the decay of an isobar of mass M_{I_N} moving with a velocity β in the c.m. system. The first term of (f2) represents the spectrum of the pions arising from the I_N decay (decay-pion), while the second one represents that of the pions arising from the I_π decay and not suffering a secondary interaction (recoil-pion).

Figure 2(a) shows the $J_{\pi,1}$ and $J_{\pi,2}$ spectra, corresponding to the decay pions and recoil pions for an incident energy $T_{\pi,\text{inc}}=960$ Mev and for the selected value $M_{I_\pi}=600$ Mev. In (b) and (c) the same curves, added together and normalized to the total number of events, are compared with the experimental results obtained by Alles-Borelli *et al.*[1] With a broken line we have represented also the curves predicted by the isobar model. It turns out that the spectra predicted by the model we are analyzing, qualitatively agree with the characteristics of the experimental distributions. Nevertheless, the high-energy peak is shifted with respect to the experimental one and the low-energy maximum has been exaggeratedly increased.

We repeated the described calculations for different values of M_{I_π} between 500 and 750 Mev and found the best quantitative agreement with the experimental results, at $M_{I_\pi}=700$ Mev. The corresponding curves are shown in Fig. 3.

Therefore, if the assumptions of the model are acceptable, this would indicate a value $t_R=25$ for the square of the pion-pion resonance energy (in units of m_π^2). This value is rather high compared with the one obtained by Frazer and Fulco from the nucleon electromagnetic structure, but agrees better with the value of 22 found by Bowcock *et al.*[6] on a latter analysis of the nucleon electromagnetic structure and the low-energy pion-nucleon phase shifts, as well as with that of 28 obtained by Erwin *et al.*,[17] from the analysis of inelastic $\pi^- - p$ collisions at 1.89 Bev.

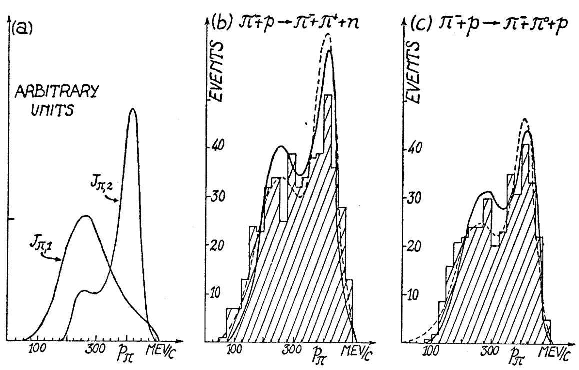

FIG. 3. Center-of-mass momentum spectra for pions from $\pi^- + p \to \pi + \pi + N$, for an incident pion energy of 960 Mev and an assumed value of 700 Mev for the pion-pion resonance energy.

On the other hand, we calculated the predicted spectra for an incident kinetic energy of 1 Bev, assuming again $M_{I_\pi}=700$ Mev, and tried to fit them to the experimental results of Derado and Schmitz.[3] The agreement is as good as it is for 960 Mev, only for the pions from reaction I. For the case II we find the same discrepancies that appeared between experimental results and the curve predicted by the isobar model. This fact does not constitute a proof against the model, but merely indicates the necessity of distinguishing between different charge states of the particles in the intermediate stage of the process, by means of an isotopic spin analysis. One could then assign different probabilities for the excitation of the nucleon isobar to the different pairs of particles. This would require the introduction of another factor in expression (f1) which, in turn, should be written as a sum of terms representing the contribution of each pair of particles to the total probability. Nevertheless, within the framework of our simple treatment, the additional assumptions required for that analysis would increase the number of parameters to be adjusted by the experimental results. More refined calculations in this sense, are now in progress.

ACKNOWLEDGMENTS

I am indebted to Professor J. G. Roederer for his continuous interest in the work. Thanks are due to Professor J. J. Giambiagi, Dr. J. Fulco, and Dr. D. Bes for stimulating discussions and to Miss A. Díaz Romero for valuable help in the computation work.

[17] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961).