Nuclear Electric Monopole Transitions in O^{16}, Ca^{40}, Ge^{72}, and Zr^{90}†

M. NESSIN, T. H. KRUSE, AND K. E. EKLUND
Columbia University, New York, New York

(Received August 29, 1961)

A study was made of electric monopole transitions from the 0^+ first excited states of certain nuclei. A search for 6.05-Mev O^{16} $E0$ conversion electrons gave an upper limit of 1×10^{-4} for the ratio of the probability for conversion electrons to that of pair emission.

Ca^{40} $E0$ conversion electrons were observed and a ratio of $W_{\text{conversion}}/W_{\text{pair}}=(6.94\pm0.20)\times10^{-3}$ was obtained. The momentum distributions of both components of pairs from the first excited state were observed. An upper limit of 6×10^{-3} was placed on the ratio of double gamma emission to pair emission. The energy of the first excited state was found to be 3.353±0.003 Mev by measurement of the conversion electron energy.

An excitation function for population of the first excited state of Ge^{72} by inelastic scattering of protons was observed between 2.3 and 6.2-Mev proton bombarding energy. The K and $(L+M+N+O)$ $E0$ conversion electrons were resolved and the ratio $W_K/W_{(L+M+N+O)}=8.70\pm0.06$ was obtained. The energy of the first excited state was found to be 0.690±0.001 Mev.

An excitation function for population of the first excited state of Zr^{90} by inelastic scattering of protons was obtained between 3.35 and 6.5 Mev proton bombarding energy. The momentum distributions of both components of pairs were observed. The energy of the first excited state was measured as 1.762±0.002 Mev. 2.38±0.08 was obtained for the ratio $W_{\text{conversion}}/W_{\text{pair}}$. The K and $(L+M+N+O)$ conversion electrons were resolved and the ratio $W_K/W_{(L+M+N+O)}=7.06\pm0.08$ was obtained.

All results are consistent within experimental errors with theoretical predictions.

INTRODUCTION

THE principal decay modes in nuclear electric monopole transitions ($\Delta I=0$, no change in parity) are internal conversion of electrons from the various atomic shells, nuclear pair emission (requiring at least $2m_ec^2$ transition energy), and double gamma emission. Single gamma emission is strictly forbidden if angular momentum is conserved in the transition.

The theory of $E0$ transitions for the atomic case was developed by Goeppert-Mayer[1]; treatments of the electronic decay probabilities and the pair component spectra have been given by Yukawa and Sakata[2] and by Thomas.[3] Theoretical discussions of the probability for nuclear double gamma emission have been given by many authors.[3–5]

Experimentally, the electronic decay modes are much stronger than the double gamma. The negatron and positron distributions in the 6.05-Mev pair decay of the first excited state of O^{16} have been shown to agree well with theory.[6] Studies of the relative atomic shell electron conversion probabilities have been made by Latyshev *et al.*,[7] by Yuassa *et al.*,[8] and by Lazar *et al.*[9] These results show a discrepancy with the presumably well-understood theoretical predictions for the ratios.

A study was made of the relative decay rates of the electronic decay modes in the cases of O^{16}, Ca^{40}, Ge^{72}, and Zr^{90}. Nuclei with 0^+ first excited states were chosen because of the absence of competing gamma cascades and because of the relatively high (p,n) thresholds in these magic nuclei. In addition, a search for double gamma emission was made in the case of Ca^{40}.

ELECTRON SPECTROMETER

All electron measurements in this experiment were performed with an intermediate-image iron-free pair spectrometer, the construction and performance of which are described elsewhere.[10,11] In order to decrease the background and to make it possible to observe the continuous distribution of electrons and positrons separately, the following modifications were made in the spectrometer.

(1) A pair of helical baffles was installed, one on each side of the adjustable annulus (iris). The baffle consists of a brass ring with attached plates spaced one inch apart. The plates are turned 17° with respect to the axis of the spectrometer so that particles of a particular charge sign can pass between the plates, while the counter-rotating particles of opposite charge are stopped by the plates.

(2) The split plastic scintillator pair detector was replaced by a single plastic crystal of truncated conical shape. The plastic scintillator is optically coupled to a long lucite light pipe, on the other end of which a 2-in. photomultiplier is attached.

To check the effectiveness of these baffles in stopping one of the components of the pairs, a spectrum from a Cs^{137} source was taken for each direction of the spectrometer current. The results show that the baffles stop more than 99.75% of the undesired component of the pairs at this energy.

† Work partially supported by the U. S. Atomic Energy Commission.

[1] M. Goeppert, Naturwissenschaften **17**, 932 (1929); and M. Goeppert-Mayer, Ann. Physik **9**, 273 (1931).

[2] H. Yukawa and S. Sakata, Proc. Phys.-Math. Soc. Japan **17**, 397 (1935).

[3] R. Thomas, Phys. Rev. **58**, 714 (1940).

[4] J. R. Oppenheimer and J. S. Schwinger, Phys. Rev. **56**, 1066 (1939).

[5] D. P. Grechukhin, Soviet Phys.—JETP Vol. 5, 846 (1957).

[6] V. K. Rasmussen, W. F. Hornyak, C. C. Lauritsen, and T. Lauritsen, Phys. Rev. **77**, 617 (1950).

[7] G. D. Latyshev, L. A. Sliv, I. F. Barchuk, and A. A. Baschilov, Isvest. Akad. Nauk **13**, 340 (1949).

[8] T. Yuasa, J. Laberrique-Frolow, and L. Feuvrais, J. phys. radium **18**, 498 (1957).

[9] N. H. Lazar, L. M. Langer, and W. G. Smith, **110**, 513 (1958).

[10] D. E. Alburger, Rev. Sci. Instr. **27**, 991 (1956).

[11] K. E. Eklund, thesis, Columbia University, **1960** (unpublished).

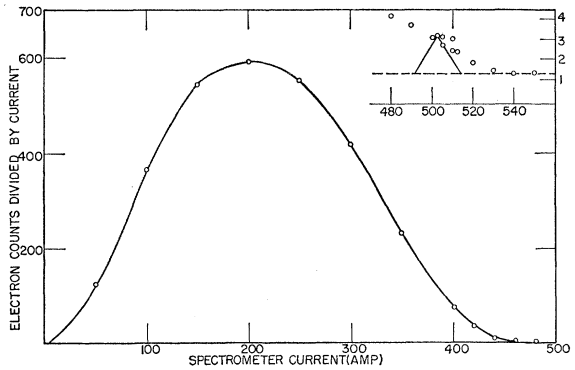

FIG. 1. Electron spectrum from O^{16}, corrected for resolution.

EXPERIMENTAL RESULTS AND ANALYSIS

Oxygen-16

A search was made for electric monopole conversion electrons from the 6.05-Mev first excited state of O^{16}. The reaction $F^{19}+p \rightarrow O^{16}+He^4$ ($Q=8.12$ Mev) populates the ground and excited states of O^{16}. The products of this reaction and relative population of excited states as a function of bombarding energy have been intensively studied.[6,12–15] In order to decrease the background, which was believed to be due to population of higher excited states, the run was repeated at various resonant bombarding energies: 1.88, 1.78, and 0.85 Mev.

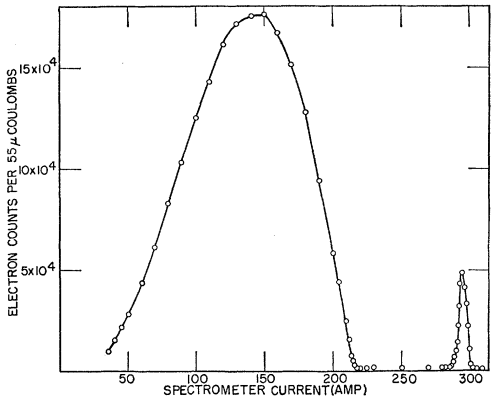

FIG. 2. Spectrum of continuous distribution of electrons from pairs and of conversion electrons from the 3.35-Mev excited state of Ca^{40}.

The best background-to-yield ratio was obtained with 1.78-Mev proton bombarding energy. A target of 0.4–0.5 mg/cm² was prepared by evaporation of BaF_2 onto a 15.4-mg/cm² silver backing. The spectrometer was adjusted to two percent resolution. The spectrum corrected for resolution (yield divided by spectrometer current) is shown in Fig. 1. The calculated spectrometer current at which the electric monopole conversion electrons should appear is 503 amperes. No peaking above the smooth distribution appears, and, therefore, only an upper limit of the ratio of conversion electron transition probability to pair transition probability can be estimated. The large background above the continuous distribution of negatrons from pairs may be internal and external conversion electrons, electrons from internal and external pairs, and Compton electrons from higher excited states of O^{16}, since these states are always populated whenever the first excited state is populated. The upper limit deduced from this spectrum for W_{e^-}/W_{pair} is 1×10^{-4}.

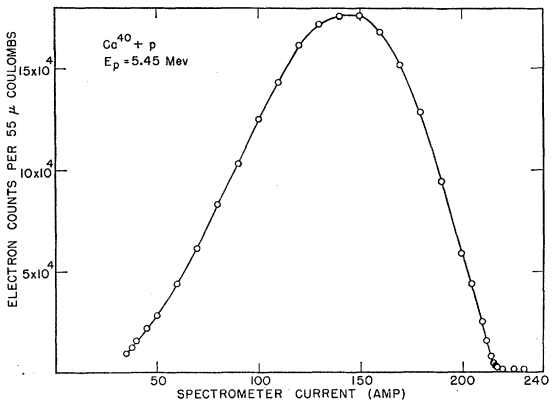

FIG. 3. Uncorrected spectrum of continuous distribution of electrons from pairs from the 3.35-Mev excited state of Ca^{40}.

Calcium-40

1. Conversion Electrons

It was found that, when a thick layer of metallic calcium was evaporated onto silver or aluminum foil, it was possible to strip it off. In this way a self-supporting target of metallic calcium approximately 0.8 mg/cm² thick was prepared.

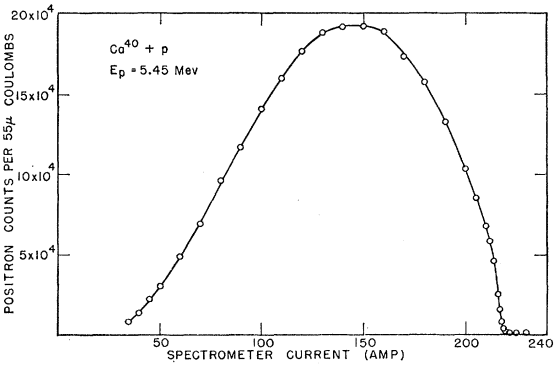

FIG. 4. Uncorrected spectrum of continuous distribution of positrons from pairs from the 3.35-Mev excited state of Ca^{40}.

[12] J. F. Streib, W. A. Fowler, and C. C. Lauritsen, Phys. Rev. **59**, 253 (1941).
[13] W. E. Bennett, T. W. Bonner, C. E. Mandeville, and B. E. Watt, Phys. Rev. **70**, 882 (1946).
[14] W. A. Fowler and C. C. Lauritsen, Phys. Rev. **56**, 840 (1939).
[15] S. Kojema, Proc. Imp. Acad. (Tokyo) **19**, 282 (1943).

The spectrum of pulses from the electrons at each setting of the magnetic field in the spectrometer was displayed on the 256-channel analyzer for the determination of the discriminator level in the scaler.

The momentum spectrum of electrons resulting from the proton bombardment of Ca^{40} at 5.45-Mev proton bombarding energy was taken approximately from 0.25-Mev to 3.6-Mev electron energy. The uncorrected spectrum of the electrons is shown by circles in Fig. 2. The peak at 295 amperes of spectrometer current is the electric monopole conversion electrons from the 3.35-Mev first excited state of Ca^{40}, and on the left is a continuous distribution of electrons from pairs from the same state of Ca^{40}.

From time to time the target was checked by taking the yield at 295 amperes. Activity, with beam stopped, was taken at each spectrometer current setting. The measured spectrum was then corrected for (a) background, (b) activity, and (c) spectrometer resolution. (a) Constant background, equal to the flat portion of the spectrum above the end of the continuous distribution, was subtracted. This constituted about 0.9%

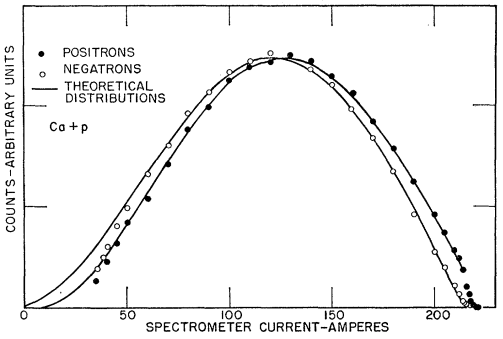

FIG. 5. Superimposed corrected spectra of electrons and positrons from the 3.35-Mev excited state of Ca^{40}.

of the yield at the peak of the continuous distribution. (b) The activity was normalized to the time of the run and subtracted. (c) The remaining yield at each spectrometer current setting was divided by the current.

For the comparison of the transition probability of the conversion electrons to the transition probability of the pairs, the theoretical distribution curve was normalized to fit best the experimental points; then the areas under the continuous distribution of pair electrons and conversion electrons were compared. The experimental ratio of the probabilities of all conversion electrons to pairs from the first excited state of Ca^{40} was calculated to be $(6.94\pm0.20)\times10^{-3}$.

2. *Measurement of Momentum Distribution of Pair Components*

In the measurement of the momentum distribution of components of pairs, a self-supporting metallic target 1 mg/cm² thick was used. A flat portion of the excitation function (5.45 Mev) was chosen and yield periodically

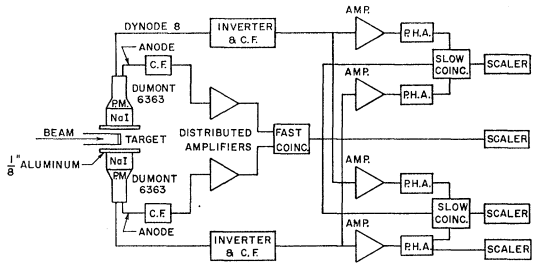

FIG. 6. Block diagram of the experimental arrangement in Ca^{40} double-gamma search.

was checked at 295 amperes spectrometer current. The electron and positron spectra were taken to 310 amperes of spectrometer current, and at each point the spectrum of the pulses was displayed on a 256-channel analyzer for setting of the discriminator. The activity from the target, with the proton beam stopped, was taken at each point.

The uncorrected continuous distributions of electrons and positrons from the first excited state of Ca^{40} are shown in Figs. 3 and 4, respectively. To get a true momentum distribution of electron and positron components of $E0$ pairs, the measured spectra were corrected for background, activity, and spectrometer resolution. The background corrections equal to the flat portions of the spectra above the ends of the continuous distributions of electrons and positrons constituted 0.9% and 0.4% of the peak height, respectively. The corrected spectra of the momentum distributions of electrons and positrons are shown in Fig. 5. The solid curves represent the theoretical distributions, with the same normalization, calculated using the Thomas expression[3] for the electric monopole pair transition probability.

3. *Search for Double Gamma Transition*

A search was made for double gamma emission as the theoretically possible competing mode of decay in the case of electric monopole transition in Ca^{40}.

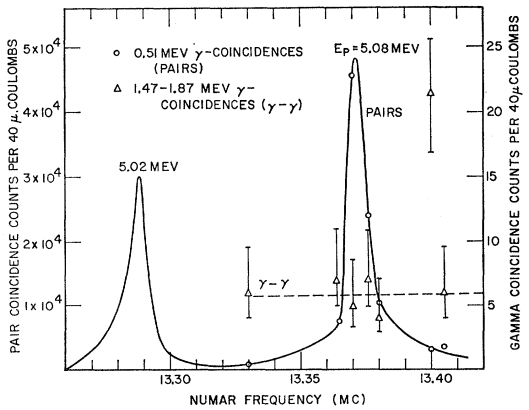

FIG. 7. Gamma-gamma coincidence spectrum from Ca^{40}. The solid line is the excitation function for $Ca^{40}(p,p')Ca^{40*}$, $E_p=5.0-5.1$ Mev.

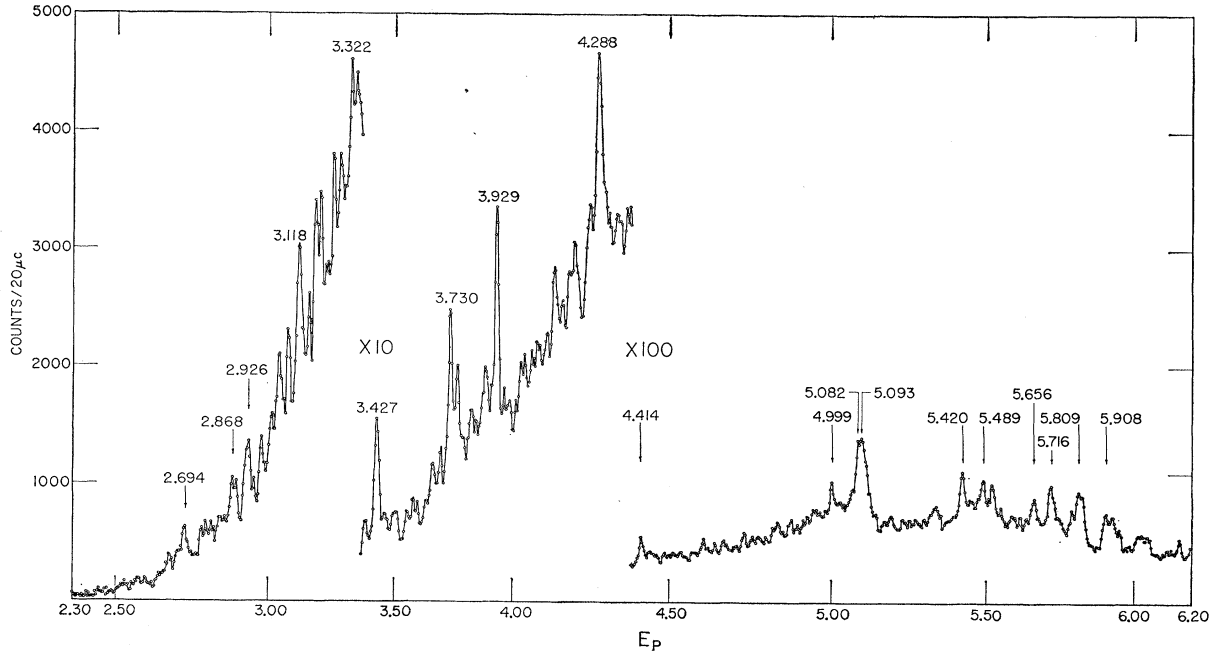

FIG. 8. Excitation function for population of the first excited state of Ge^{72} by proton bombardment.

The target was evaporated metallic calcium, approximately 0.3 mg/cm² thick, on tantalum backing Two sodium iodide crystals, one 3 in.×3 in. and one 3 in. ×2 in., mounted on Dumont type 6363 photomultiplier tubes, were used as the gamma-ray detectors and were placed at 180° to each other. The experimental arrangement and block diagram of the electronics is shown in Fig. 6. The pulses from the anodes of the photomultipliers were amplified by Hewlett-Packard type-*A* distributed amplifiers and fed into a fast coincidence circuit of the Garwin type.[16] The output of the fast coincidence circuit then fed into two sets of slow triple coincidence circuits. Pulses from the eighth dynode of the photomultipliers were inverted and used (after amplification) to feed single-channel analyzers of the slow triple coincidence circuits. The single-channel pulse-height analyzers of the first slow triple coincidence circuit were set to select the pulses corresponding to photopeak of annihilation quanta, and the output of this circuit was then taken proportional to pair yield. In the other triple coincidence circuit, the single channels were set to select pulses of energy between 1.47 and 1.87 Mev. The output of this circuit should then include double gamma yield. The scheme was to run over known resonances[17] in the production of the first excited state of Ca^{40} and to look for resonant behavior of coincident gamma yield. The isolated resonances in the excitation function are at 5.02- and 5.08-Mev proton bombarding energies. After the resonances were located, long runs were taken going over the second (large) resonance. The single channels were checked after each point. The result of this run is plotted in Fig. 7. The spectrum of the gamma-gamma coincidences has a very small rate (about 6 counts per 40 μcoulombs) and a flat distribution over the resonance. The rate of the accidental coincidences taken with 60 ft of RG-114 U cable is one count per 20 μcoulombs. The residual gamma-gamma coincidences cannot be explained by annihilation of positron in flight (for which the annihilation quanta have enough energy to satisfy the slow coincidence requirement), because such coincidences should peak at the resonance for population of the first excited state of Ca^{40} which shows up in the yield of pairs. Taking into account the geometry and efficiency of the crystals, the upper limit for the ratio of gamma-gamma transition probability to pair transition probability from the first excited state of Ca^{40} is 6×10^{-3}.

Germanium-72

1. Excitation Function

A thin target was prepared by evaporation of natural germanium on 4.7-mg/cm² silver backing. By weighing, the target was estimated to be about 0.25 mg/cm² thick. It was bombarded by protons from the Van de Graaff generator, and electric monopole conversion electrons from the first excited state of Ge^{72} were detected in the intermediate image electron spectrometer. The background was estimated by regularly observing the yield with the spectrometer set just above the conversion line. The spectrometer was adjusted to approximately 7% resolution, which corresponds to approximately 10% transmission.

The number of conversion electrons, corrected for

[16] R. L. Garwin, Rev. Sci. Instr. **21**, 569 (1950).
[17] R. D. Bent and T. H. Kruse, Phys. Rev. **109**, 1240 (1958).

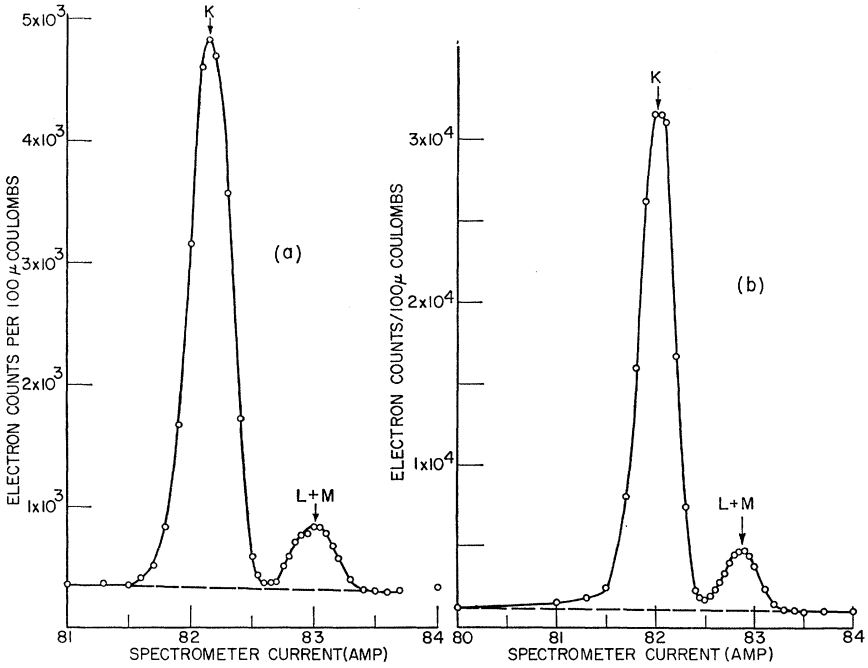

FIG. 9. (a) Spectrum of K and $L+M+N+O$ conversion electrons from the 0.69-Mev excited state of Ge^{72} (0.2-mg/cm² target). (b) Spectrum of K and $L+M+N+O$ conversion electrons from the 0.69-Mev excited state of Ge^{72} (1-mg/cm² target).

background, as a function of the proton bombarding energy is shown in Fig. 8. The excitation function was taken in steps of approximately 4 kev from 2.3 to 6.2 Mev bombarding energy.

2. *Resolution of K and L Conversion Electrons*

The energy separation between K and L conversion electrons in germanium is 9.7 kev, which corresponds to 0.9% separation in momentum. Using a source of Bi^{207} one millimeter in diameter, the spectrometer was adjusted to 0.55% resolution. With a collimator 0.85 mm in diameter, a slight additional adjustment gave less than 0.50% resolution.

Two targets of natural germanium were prepared by evaporation of metallic powder onto a silver backing 4.7 mg/cm² thick. One target was 0.2 mg/cm² thick and the other 1.0 mg/cm² thick. In order to avoid straggling effect in the backing, the target was placed with the backing facing the beam so that the conversion electrons would go only through the thickness of the germanium itself. The uncorrected spectrum of K and $(L+M+N+O)$ $E0$ conversion electrons from the thin target is shown in Fig. 9a. Uncorrected data from the thick target are plotted in Fig. 9b.

The second line in the spectra shown in Figs. 9a and 9b consists of L, M, N, and O shell electrons. Therefore, only a ratio of K conversion electrons to $(L+M+N+O)$ conversion electrons could be determined from these spectra. After subtraction of the flat portion of the background and correction for the spectrometer resolution, the ratio of the areas under the curves in 9a was calculated. In the case of the thicker target (Fig. 9b) a decomposition of lines was performed as follows: The low-energy tail of the $(L+M+N+O)$ line was extrapolated according to the shape of the low-energy tail of K line, since the $(M+N+O)$ contribution does not change appreciably the very low energy tail of the L line.

The ratio of $K/(L+M+N+O)$ $E0$ conversion electrons obtained from the thin target is 8.72±0.06. The ratio obtained from the thick target is 8.69±0.08. The average value for the ratio of K $E0$ conversion electrons from the first excited state of Ge^{72} to the $(L+M+N+O)$ conversion electrons is 8.70±0.06.

Zirconium-90

1. *Excitation Function*

A target of natural zirconium of approximately 0.25 mg/cm² was prepared by evaporation, using tungsten filaments as described in Oak Ridge National Laboratory Report ORNL-2237 (1957). The excitation function for population of the first excited state of Zr^{90} was measured by detecting $E0$ conversion electrons in the electron spectrometer with the same resolution and transmission as for Ge^{72}. The background was estimated in the same way as in the case of germanium by taking the yield with the spectrometer set just above the conversion line. The excitation function is shown in Fig. 10. The bombarding energy was varied from 3.35 to 6.5 Mev by steps of approximately 4 kev.

2. *Measurement of Momentum Distribution of Pair Components*

A 2-mg/cm² hammered target of natural metallic zirconium was used in measurement of the continuous

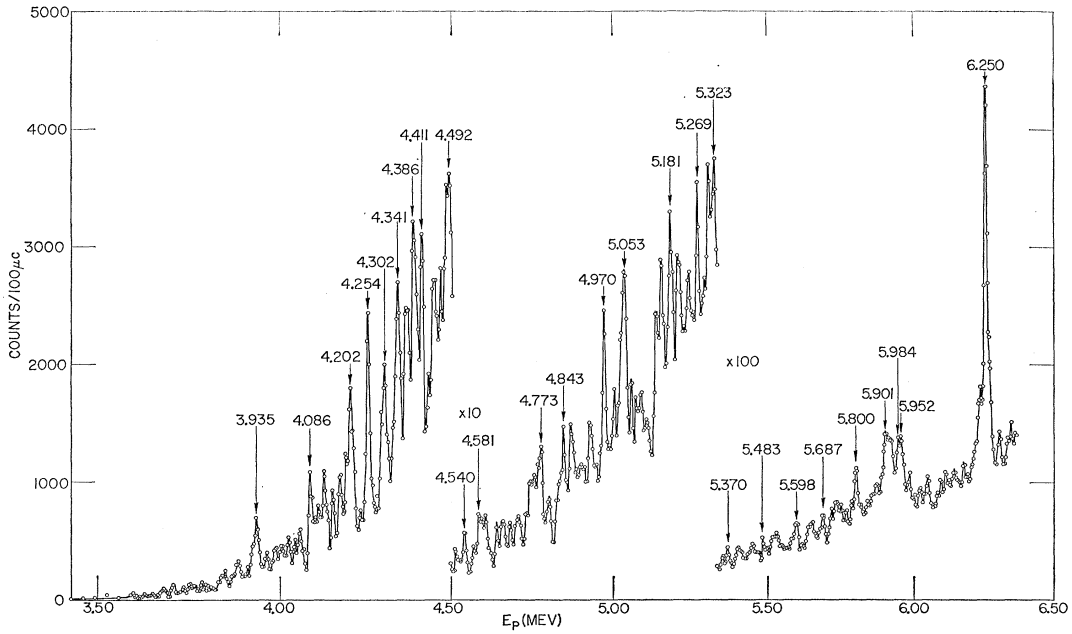

FIG. 10. Excitation function for population of the first excited state of Zr^{90} by proton bombardment.

distribution of both components of the pairs. The spectrum of the pulses at each point was displayed on a 256-channel analyzer for setting of the discriminator. In the low-energy region of the continuous spectrum of pair components, the background was a serious problem, amounting to 15–20% of the total yield. To estimate the background along the continuous distribution, where there was no direct means to measure it, the spectrum of the pulses was printed every 8 amperes. The yield from the target with the proton beam stopped was also taken at each point.

The uncorrected spectrum of the continuous distribution of electrons from pairs and of conversion electrons from the first excited state of Zr^{90} is shown in Fig. 11. In Fig. 12 is shown the continuous distribution of positrons from pairs from the same excited state of zirconium.

The pulse-height spectra of positrons were plotted for several currents, from which the unfocused part of the spectrum was estimated, and the curve of the background as a function of the spectrometer current was drawn. This curve was then used to subtract the background. The activity which was taken at each point was normalized to running time and subtracted also. Then the spectrum was corrected for the spectrometer resolution. The corrected continuous distribution of positrons from the first excited state of Zr^{90} is plotted in Fig. 13. The solid curve is theoretically calculated using the Thomas expression. The points at 40 amperes and lower are not reliable because of very high background.

A very high activity and conversion electrons from other transitions in Zr^{90} and from other isotopes made it impossible to find any method to estimate properly the background in the continuous distribution of electrons. The yield of electrons corrected for activity, for the background which appears just above the continuous distribution, and finally for resolution, is plotted in Fig. 14. Only points on the high-momentum side of the continuous distribution are properly corrected for the background. They coincide fairly well with the theoretical distribution, shown by the solid

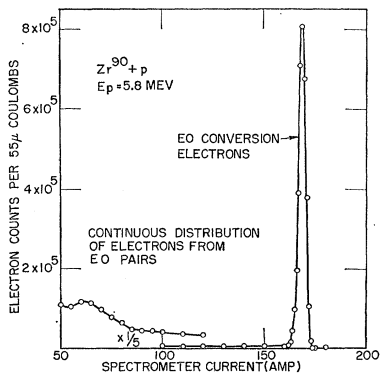

FIG. 11. Uncorrected spectrum of continuous distribution of electrons from pairs and of conversion electrons from the 1.76-Mev excited state of Zr^{90}.

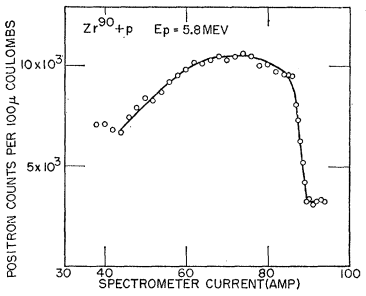

FIG. 12. Uncorrected spectrum of continuous distribution of positrons from pairs from the 1.76-Mev excited state of Zr^{90}.

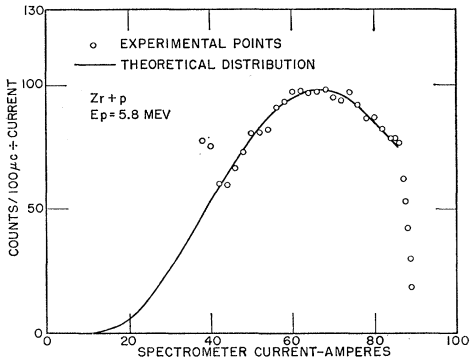

FIG. 13. Spectrum of continuous distribution of positrons from the 1.76-Mev excited state of Zr^{90} corrected for background, activity, and resolution.

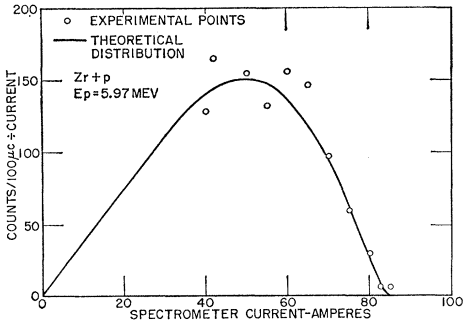

FIG. 14. Spectrum of continuous distribution of electrons from the 1.76-Mev excited state of Zr^{90} corrected for background, activity, and resolution.

curve, normalized to the theoretical curve used to fit the e^+ distribution.

In order to estimate the branching ratio of conversion electrons to pairs, the theoretical distribution (Thomas formula) was fitted at the high-yield part of the continuous distribution of positrons (48–84 amperes) and the low-momentum part of the distribution was then extrapolated according to the theoretical distribution. This procedure can be justified as follows: First, the rest of the positron distribution fairly well coincides with theory; second, the upper end of the electron distribution also coincides with the theoretical distribution with the same normalization. Then the area under the continuous distribution was compared with the area under the conversion electron spectrum corrected for background and resolution. The ratio of $W_{\text{conversion}}/W_{\text{pair}}$ thus obtained is 2.38 ± 0.08. The error in the ratio due to the uncertainty in the fitting of the curve is estimated as 0.02 and the rest is entirely due to the uncertainty in subtraction of the background.

3. *Resolution of K and (L+M+N+O) Conversion Electrons in* Zr^{90}

Two targets of natural zirconium were prepared by rolling a foil, then hammering, and finally etching in hydrofluoric acid. The spectrum shown in Fig. 15a is uncorrected data taken with a 1-mg/cm² target and about 0.45% resolution; Fig. 15b is the uncorrected spectrum taken with a 2-mg/cm² target and about 0.50% resolution. The K and $(L+M+N+O)$ conversion electrons from the first excited state of Zr^{90} were not as well resolved as in the case of Ge^{72}. In order to decompose them, the shape of the $(L+M+N+O)$ line was determined by adding two lines (N and O contributions are negligible) having the shape of the K line and separated 2.1 kev, having a 3:1 height ratio, and

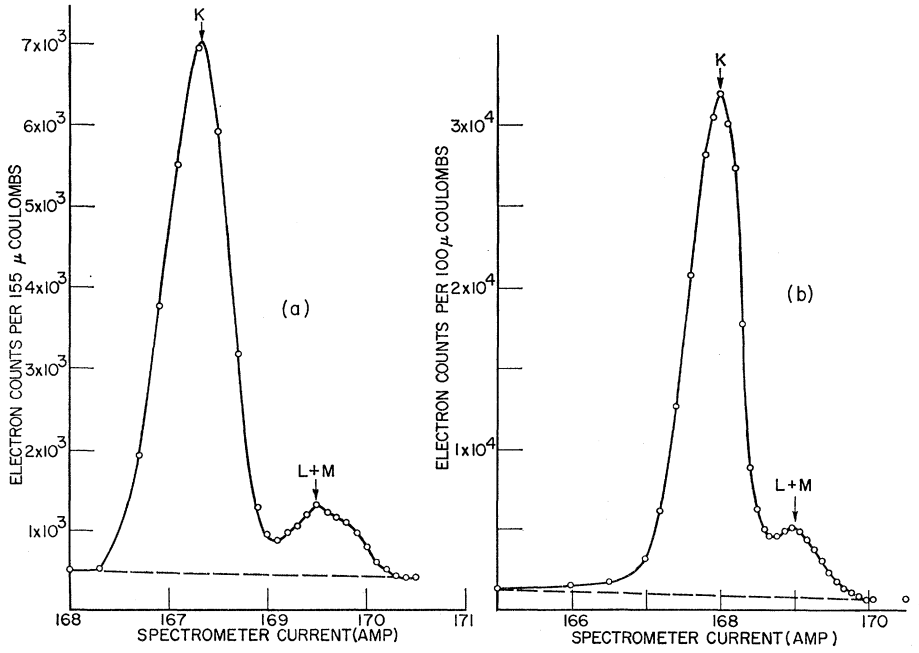

FIG. 15. (a) Spectrum of K and $L+M+N+O$ conversion electrons from the 1.76-Mev excited state of Zr^{90} (1-mg/cm² target). (b) Spectrum of K and $L+M+N+O$ conversion electrons from the 1.76-Mev excited state of Zr^{90} (2-mg/cm² target).

TABLE I. Measurements of the energies of the first excited states.

Element	Current at the peak of the K conversion electron in amperes
Bi^{207}	65.00±0.01
	106.90±0.02
Ca^{40}	293.60±0.05
Ge^{72}	82.05±0.03
Zr^{90}	168.18±0.03

normalized to the experimental $(L+M+N+O)$ line height. This gave the shape of the low-energy side of the $(L+M+N+O)$ line, which was not much different from the K line shape.

The spectra were corrected for the background, as shown in Figs. 15a and 15b, then were decomposed, corrected for resolution, and the areas under the curves used to obtain the ratio of $K/(L+M+N+O)$ $E0$ conversion electrons. The result for both the thin and the thick target is 7.06±0.10. The average ratio of the probability of the transition by K-shell electron to that by L, M, N, and O is then 7.06±0.08.

DISCUSSION

The ratio W_K/W_{pair} in $0^+ \rightarrow 0^+$ transition is very sensitive to the transition energy, because the probability for transition by pair emission increases with the transition energy much faster than the probability for conversion electron emission. The ratio is also approximately proportional to Z^3, so that pair emission is the dominant mode of decay at low Z and high transition energy, while conversion electron emission is the dominant mode of decay at high Z and low transition energy.

In order to be able to compare the experimental results for these branching ratios with the corresponding theoretical ratios, more precise knowledge of the energies of these excited states was needed for the above-mentioned theoretical calculations. Therefore, the energies of the first excited states of Ca^{40} and Zr^{90} were measured by measurement of the energies of $E0$ conversion electrons; the energy of the first excited state of Ge^{72} was also measured.

The spectrometer was calibrated with a Bi^{207} source of 1 mm in diameter. The Bi^{207} source has two strong K conversion lines at 0.5696 and 1.0639 Mev transition energies or 2838±0.4 and 4657.9±1.0 gauss-centimeters, respectively, and the spectrometer was calibrated with one Bi^{207} source.

Then the spectra of the K conversion electrons from the first excited states of Ca^{40}, Ge^{72}, and Zr^{90} were taken in steps of 0.1-ampere spectrometer current. To the energies of the conversion electrons calculated from these spectra, the binding energies and the average loss of energies of electrons in going through the target material were added. The results are shown in Table I.

In order to compare the experimental results for the branching ratio of conversion electrons to pairs in $E0$ transitions with the theoretical prediction, the calculated ratio using the Thomas expression must be corrected for the effect of the finite dimension of nuclear charge distribution and for the effect of atomic electron screening.

Calcium-40

The calculated ratio of the K-shell conversion electrons to pairs from the first excited state in Ca^{40} is $W_K/W_{e^-e^+}=7.10\times10^{-3}$, using the Thomas expression. The effect of finite nuclear size on the ratio is very small.[18] The finite size effect on probability of conversion electrons is less than 0.5%[19] and on the pair emission it is also very small.[20] The emission probability for the K-shell conversion electron for $Z=20$ is depressed by a factor of 0.91 due to attenuation of the bound state electron wave function in the region of the nucleus.[19] The effect of screening on the continuum electron wave function is negligible except for very low energy.[20] The probability for pair emission is enhanced by factor 1.02, estimated from the calculations of Reitz.[20] Then the predicted ratio $W_K/W_{e^-e^+}$, corrected for screening effect, is equal to 6.33×10^{-3}.

The experimental result for $W_{\text{conversion}}/W_{e^-e^+}$ for Ca^{40} contains, besides K conversion electrons, L, M, and N conversion electrons also, which could not be resolved in this experiment. In order to compare the experimental result for $W_{\text{conversion}}/W_{e^-e^+}$ with the theoretical prediction, $W_{L_I}/W_{e^-e^+}$, $W_{M_I}/W_{e^-e^+}$, and $W_{N_I}/W_{e^-e^+}$ (only s electrons contribute substantially) must be added to the calculated $W_K/W_{e^-e^+}$ ratio. The calculated ratio W_K/W_{L_I} (uncorrected for screening) is 7.80.[21] The screening[19] depressed W_K by a factor 0.91 and W_{L_I} by 0.62; thus the ratio W_K/W_{L_I} is increased by a factor 1.47. The calculated ratio W_{L_I}/W_{M_I} (uncorrected for screening) is 3; there are no available corrections for M_I electron wave functions. The N-shell contribution can be neglected, because, even uncorrected for screening, it is about 1.5% of K-shell contribution, and the screening effect on the N_I electron wave function must be much larger then on the L_{III} electron wave function, which is 0.44. Thus the N_I electron contribution is less than 0.5% of the K-electron contribution and, therefore, is neglected. To obtain limits for the theoretical ratio of $W_{\text{conversion}}/W_{e^-e^+}$, two limiting screening effects on M_I electron wave function can be assumed: (a) The effect of screening on the M_I electron wave function is greater than on the L_{III} electron wave function; and (b) The effect of screening

[18] S. D. Drell and M. E. Rose, Prog. Theoret. Phys. (Kyoto) 7, 125 (1952); and Oak Ridge National Laboratory Report ORNL-792, 1950 (unpublished).
[19] H. Brysk and M. Rose, Oak Ridge National Laboratory Report ORNL-1830, 1955 (unpublished).
[20] J. R. Reitz, Phys. Rev. 77, 10 (1950); M. E. Rose and D. L. Holmes, Oak Ridge National Laboratory Report ORNL-1022, 1951 (unpublished).
[21] E. L. Church and J. Weneser, Phys. Rev. 103, 1035 (1956).

TABLE II. Summary on data on $E0$ transition probabilities in the nuclei of O^{16}, Ca^{40}, Ge^{72}, and Zr^{90}.

		Theoretical branching ratios					Experimental branching ratios		
			"Point	Including screening					
	Energy (Mev)	Thomas W_K/W_{pair}	nucleus" W_K/W_{L_I}	W_K/W_{pair}	W_K/W_{L_I}	$W_{K+L_I+M_I}/W_{\text{pair}}$	$W_{\text{conv}}/W_{\text{pair}}$	$W_K/W_{(L+M+N+O)}$	W_K/W_{pair}
O^{16}	6.051 ±0.005	3.9×10^{-5}					$<1\times10^{-4}$		
Ca^{40}	3.353 ±0.003	7.10×10^{-3}	7.80	6.33×10^{-3}	11.47	$>6.88\times10^{-3}$ $<7.01\times10^{-3}$	$(6.94\pm0.20)\times10^{-3}$		
Ge^{72}	0.690 ±0.001		7.50		10.24			8.70±0.06	
Zr^{90}	1.762 ±0.002	2.34	7.20	2.14	9.47		2.38±0.08	7.06±0.08	2.08±0.08

is so large that the contribution of M_I conversion electrons in the case of Ca^{40} is negligible and, therefore, can be disregarded. Then the calculated ratio of $W_K/(W_{L_I}+W_{M_I})$ for Ca^{40} corrected for atomic screening effect is $9.27<W_K/(W_{L_I}+W_{M_I})<11.47$. Thus the calculated ratio of the emission probability for $E0$ conversion electrons from the first excited state of Ca^{40} to that for $E0$ pairs is $6.88\times10^{-3}<W_{\text{conversion}}/W_{e^-e^+}<7.01\times10^{-3}$. The experimental result is $(6.94\pm0.20)\times10^{-3}$.

Germanium-72

The calculated ratio for W_K/W_{L_I}, without screening taken into account, is 7.50; if the screening correction is applied to the K- and L_I-shell electrons, the ratio becomes $W_K/W_{L_I}=10.24$. The comparison of the experimental ratio $W_K/W_{(L_I+M_I+N_I+O_I)}=8.70\pm0.06$ with the theoretical W_K/W_{L_I} gives for $W_{L_I}/W_{(M_I+N_I+O_I)}$ 5.64 ± 0.25, which is almost twice as large as calculated without screening for $W_{L_I}/W_{M_I}(3.2)$. If we assume that the contributions of N_I and O_I are negligible, then the screening factor for the M_I conversion electrons must be 0.37. Since there must be small contribution from N_I and O_I electrons, the screening effect on M_I electron wave function therefore must be even larger. Thus, as expected, the screening effect on M_I conversion electrons is greater than on L_{III} conversion electron, for which the screening factor is 0.52.

Zirconium-90

The experimental result obtained for $W_{\text{conversion}}/W_{e^-e^+}$ is 2.38 ± 0.08, and for $W_K/W_{(L+M+N+O)}=7.06\pm0.08$. Subtracting the $(L+M+N+O)$ contribution from $W_{\text{conversion}}/W_{e^-e^+}$, an experimental value for $W_K/W_{e^-e^+}$ of 2.08 ± 0.08 is obtained. The calculated ratio $W_K/W_{e^-e^+}$, using the Thomas expression and the transition energy as measured in this experiment (1.762 ± 0.002 Mev), uncorrected for screening, is 2.34. The probability for K-shell conversion electrons due to screening is depressed by a factor of 0.955, and the probability for pair emission is increased (due to screening effect) by a factor 1.045; thus the calculated ratio of the probability of K conversion electrons to the probability of pair emission is 2.14. This is 2.8% higher then the experimental result, which is 2.08 ± 0.08.

All measured and calculated results pertaining to the first excited states of O^{16}, Ca^{40}, Ge^{72}, and Zr^{90} are tabulated in Table II.

The agreement between the experimental results and theoretical calculations is very satisfactory. The small differences (which are inside the experimental errors) can be due either to experimental errors (including systematic errors) or to uncertainty in the theoretical estimation of the various corrections. Some of the latter, such as the finite size effect and the screening effect on continuum wave functions for conversion electrons (which are estimated to be very small) are not included.