Langevin and de Nercy[16] and 10.15±0.006 Mev for Tobin.[17] De Nercy and Langevin[10] conclude from the angular distributions of these γ rays that the peak consists of two γ rays, one directly to the ground state and one to the first excited state of Mg^{24}. Relative intensities of these two γ rays are given as $\frac{3}{5}$ and $\frac{2}{5}$, respectively. This interpretation does not agree with the spectrum of Fig. 7, for which the detector resolution was good enough to separate the two γ rays if present in these intensities (as, for example, in the Si^{28} data).

Si

A $\frac{1}{8}$-in.-thick sample of powdered Si in a Mylar holder produced the spectrum of Fig. 8, for which 16-Mev electrons were used. Response functions have been drawn corresponding to two γ rays of energy 11.4±0.1 and 9.6 Mev. Both of these γ rays are too high in energy to come from $(\gamma,n\gamma)$ or $(\gamma,p\gamma)$ reactions in the Si isotopes. It is assumed that they are both from resonant excitation of Si^{28}. The 11.4-Mev γ-ray energy agrees well with the results of Tobin,[17] and the 9.6-Mev γ ray is assumed to be a transition to the first-excited state at 1.78 Mev. If it is assumed that the transitions are (0–1–0) and (0–1–2) and that the 9.4-Mev γ ray is also dipole, then angular distributions of these two resonant-scattered γ rays are $1+\cos^2\theta$ and $1+(1/13)\times\cos^2\theta$, respectively, and the ratio of decays to the first-excited state to decays to the ground state is 0.19±0.04. As in the case of Mg, this contradicts an experiment in which it is concluded from the angular distribution of these two unresolved γ rays that this ratio is 0.6.[18]

ACKNOWLEDGMENTS

It was a pleasure to work with Dr. H. W. Koch during the first part of this experiment. The author is indebted to him both for suggesting that the accelerator be used for an experiment of this type and for much help with the experimental setup; to R. Shafer and C. Jupiter, who provided valuable assistance in data-taking; to F. Fulton, who made several targets by pressing powdered B and Si in molds; and to Dr. S. Fultz, who made accelerator time available. The work was done under the auspices of the U. S. Atomic Energy Commission.

[16] M. Langevin and A. Bussière de Nercy, J. phys. radium **20**, 831 (1959).
[17] R. A. Tobin, Phys. Rev. **120**, 175 (1960).
[18] A. Bussière de Nercy, J. phys. radium **22**, 119 (1961).

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Magnetic Octupole Moments of Axially Symmetric Deformed Nuclei*†

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The magnetic octupole moments of twenty odd-A nuclei whose energy levels are reasonably well described by the axially-symmetric collective model in strong coupling have been calculated. With but two exceptions, the predicted moments are of the same magnitude as or larger than the moments measured for seven nuclei. The model parameters have been determined by fitting the measured values of the lower moments and transition probabilities. The assumption has been made that the orbital gyromagnetic ratio for the odd nucleon is that of a free particle. Then from the total gyromagnetic ratio of the particle, which was determined from the measured magnetic dipole moment and $M1$ transition probabilities, the spin gyromagnetic ratio, g_s, was determined. With but one exception g_s was found to have a value between the free particle and pure Dirac particle values.

INTRODUCTION

The magnetic dipole and electric quadrupole moments of most nuclei have been measured and reasonably well understood for some length of time. More recently, better measurements of hyperfine structure and reinterpretation of old data have yielded the magnetic octupole moments of seven nuclei.

The extreme single-particle model calculation of the octupole moments has been done by Schwartz[1] and predicts limits similar in nature to the Schmidt limits for the magnetic dipole moments. (See Figs. 1 and 2). The uniform or Margenau-Wigner (M-W) limits can also be used in connection with this model. One may note however, that although the Schmidt limits reasonably well describe the observed dipole moments, the Schwartz limits (or even the M-W limits) do not characterize the octupole moments well. With the exception of the two chlorine isotopes, all the measured

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[1] C. Schwartz, Phys. Rev. **97**, 380 (1955).

octupole moments lie rather far from the limit which characterizes their dipole moment.

Suekane and Yamaguchi[2] have attempted to attribute these deviations from the Schwartz limits to a contribution from a permanently deformed, spheroidal core which consists of all but the odd nucleon. This calculation is not self-consistent, however, since the Schwartz limits presuppose a spherically symmetric potential so that J, the particle's total angular moment, is a good quantum number. In the presence of an axially symmetric core, however, only m_J, the projection of J on the body symmetry axis, is a good quantum number[3] (in strong coupling).

It seems apparent, therefore, that magnetic octupole moments are not as well explained by the extreme single particle model as are the magnetic dipole moments. In view of the difficulties encountered in the measurement of magnetic octupole moments it is not too surprising that more experimental data is not available. However, since they are one of the static properties of all nuclei (with spin greater than or equal to $\frac{3}{2}$) a table of their experimental values would seem to be an essential tool to those working in the field

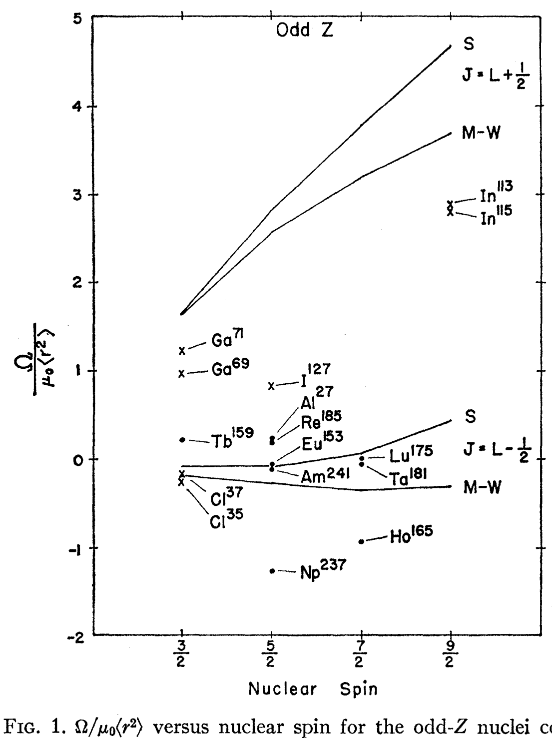

FIG. 1. $\Omega/\mu_0\langle r^2\rangle$ versus nuclear spin for the odd-Z nuclei considered. The experimental values are denoted by X's and the theoretical values by dots. S denotes the extreme single-particle or Schwartz limits and M-W the uniform or Margenau-Wigner limits. To reduce both experimental and theoretical values, $\langle r^2\rangle$ was taken as $\frac{3}{5}\mathcal{R}_0^2$. The theoretical values are as calculated in the text.

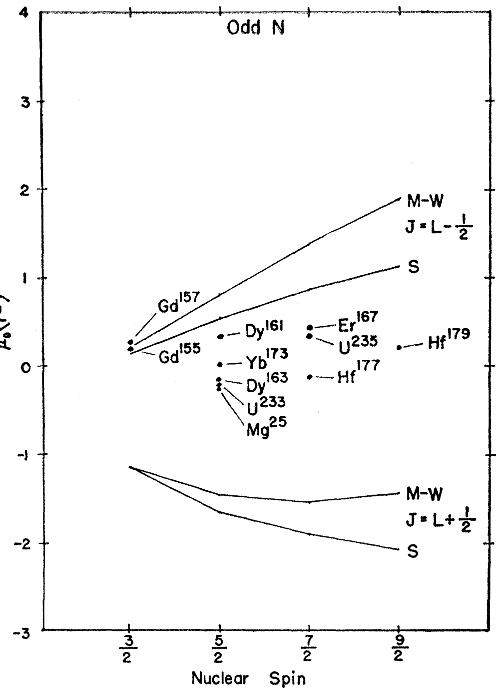

FIG. 2. $\Omega/\mu_0\langle r^2\rangle$ versus nuclear spin for the odd-N nuclei considered. S denotes the extreme single particle or Schwartz limits and M-W the uniform or Margenau-Wigner limits. To reduce the theoretical values as calculated in the text, $\langle r^2\rangle$ was taken as $\frac{3}{5}\mathcal{R}_0^2$.

of nuclear groundstates. The experimentalist might well be encouraged to undertake their measurement if he could be reasonably assured that the values he seeks would be of the same order of magnitude as those already measured so that at least no greater difficulties would be incurred in their measurement than were encountered in earlier work.

There exists a large group of nuclei within the so-called deformed region,[4] many of whose ground state properties are reasonably well described by the axially-symmetric collective model in strong coupling. Hence this would seem an appropriate time to present a consistent calculation of their magnetic octupole moments; this is the aim of this paper.

The paper is divided into three principal sections. In the first section the generalized electric and magnetic multipole operators are introduced. In Sec. II the eigenvectors are presented and the expectation values of the necessary operators of Sec. I are given. In Sec. III the results of the calculation are discussed and a predictive table of octupole moments is presented.

SECTION I

Generalized Multipole Moment Operators

In the axially-symmetric collective model two coordinate systems are at one's disposal. One is fixed

[2] S. Suekane and Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) 17, 443 (1957).

[3] A. K. Kerman, *Nuclear Reactions* (Interscience Publishers, Inc., New York, 1959), Vol. I.

[4] $A\approx 25$, $150\lesssim A\lesssim 180$, $A\gtrsim 220$.

in the core (the body system); its axes are the principal inertial axes of the nucleus. The other is the laboratory system. It is generally convenient to have the laboratory operator, whose expectation value is to be calculated, in terms of its body system components. In what follows, all Euler angle rotations will be those which effect the transformation from the laboratory to the body system.

Magnetic Multipoles

The generalized l,mth magnetic multipole moment of an assemblage of current density, $\mathbf{j}(\mathbf{r})$, is given by Schwartz[1] in a classical calculation as

$$M_{l,m}=\frac{i}{l+1}\left[\frac{4\pi}{2l+1}\right]^{\frac{1}{2}}\int[\mathbf{L}^*r^lY_{l,m}^*(\theta,\phi)]\cdot\mathbf{j}(\mathbf{r})d\tau, \quad \text{(I-1)}$$

where $\mathbf{L}=-i\mathbf{r}\times\nabla$ is the laboratory system angular momentum operator, $\mathbf{r}$ the position vector of the current density, $\mathbf{j}(\mathbf{r})$.[5]

The current density is divided into a part due to the extra (odd) nucleon and a part due to the core.

In the case of the odd nucleon, $\mathbf{j}(\mathbf{r})$ is replaced by the usual quantum mechanical expression and one finds[1] for the single-particle operator

$$(M_{l,m})_{\mathrm{sp}}=\mu_0\left[\frac{4\pi}{2l+1}\right]^{\frac{1}{2}}[\nabla(r^lY_{l,m}^*)]\cdot\left[\frac{2g_L}{l+1}\mathbf{L}+g_s\mathbf{S}\right], \text{(I-2)}$$

where μ_0 is a nuclear magneton, g_L and g_s the orbital and spin gyromagnetic ratios respectively, and $\mathbf{L}$ and $\mathbf{S}$ the orbital and spin angular momentum operators.

It is usual in applying Eq. (I-2) to replace g_L and g_s by their free-particle values. While it seems reasonable that g_L should take on the free-particle value, it is by no means obvious that this should also be true for g_s.[6,7] Thus the quantity, $[2g_L/(l+1)]\mathbf{L}+g_s\mathbf{S}$ of Eq. (I-2) is replaced by $g(l)\mathbf{J}$, where $\mathbf{J}$ is the total angular momentum operator for the particle and $g(l)$ is the "total" gyromagnetic ratio. Then $g(l)$ is regarded as an empirical fitting parameter from which g_s is determined assuming g_L has the free-particle value. Note that although the particle's "total" gyromagnetic ratio is a function of l, the "octupole" ($l=3$) gyromagnetic ratio and indeed all higher ratios are determined from the "dipole" ($l=1$) gyromagnetic ratio under the above assumption.

The right side of Eq. (I-2) is still in terms of the laboratory system components. The transformation to body system components is most easily effected once the scalar product is written in terms of the pseudo-spherical components of the operators which are transformed by using the three-dimensional representation of the rotation group, $\mathbf{D}$.[8] Then

$$(M_{l,m})_{sp}=(-1)^m[4\pi l]^{\frac{1}{2}}r^{l-1}g(l)\mu_0 \times\sum_{\nu,\sigma}C(l-1,1,l;\sigma-\nu,\nu)D_{-m,\sigma}{}^lY_{l-1,\sigma-\nu}J_\nu, \quad \text{(I-3)}$$

where C is a Clebsch-Gordan coefficient.

The core currents are assumed to arise solely out of the collective motion of the core as seen from the laboratory system. Within this model, the velocity currents, $\mathbf{v}(\mathbf{r})$, are described by irrotational flow under the assumption that the gradient of the mass density, ρ_m, is normal to the velocity vector and that ρ_m does not depend explicitly on time. Further, the surface of the core is assumed to be described by

$$\mathcal{R}(\theta,\phi)=\mathcal{R}_0[1+\sum_{\lambda,\mu}\alpha_{\lambda,\mu}Y_{\lambda,\mu}], \quad \text{(I-4a)}$$

in the laboratory system, and by

$$\mathcal{R}(\theta,\phi)=\mathcal{R}_0[1+\sum_{\lambda,\mu}a_{\lambda,\mu}Y_{\lambda,\mu}], \quad \text{(I-4b)}$$

in the body system. In both Eqs. (I-4a,b) $\mathcal{R}_0$ is the radius of a sphere of the same volume as the deformed nucleus. In what follows consideration is limited to surfaces of quadrupole deformation only ($\lambda=2$) and the first subscript of the expansion parameters is dropped.

Then, to first order

$$\mathbf{v}(\mathbf{r})=\tfrac{1}{2}\sum_\mu\dot{\alpha}_\mu\nabla(r^2Y_{2,\mu}). \quad \text{(I-5)}$$

The core current density, $\mathbf{j}(\mathbf{r})$, is then given by

$$\mathbf{j}(\mathbf{r})=(\rho_e/c)\mathbf{v}(\mathbf{r}), \quad \text{(I-6)}$$

where ρ_e is the electric charge density. After integration, one has to first order[9]

$$(M_{l,m})_{\mathrm{core}}=(-1)^{m+1}\frac{15i\langle\rho_e\rangle}{c}\mathcal{R}_0{}^{l+4}\left[\frac{l(2l+1)}{l+1}\right]^{\frac{1}{2}} \times F(l)\sum_{\mu=-2}^{2}\alpha_{\mu-m}^*\dot{\alpha}_\mu C(2l2;\mu,-m), \quad \text{(I-7a)}$$

where

$$F(l)=\sum_jC(l1j;00)C(22j;00)W(lj11;1l) \times W(22lj;12), \quad \text{(I-7b)}$$

with W denoting a Racah coefficient.

The laboratory expansion parameters are related to the corresponding body system parameters by

$$\alpha_\mu=\sum_\nu D_{\mu,\nu}{}^{2*}(\theta_i)a_\nu. \quad \text{(I-8)}$$

Further, since the surface is real $a_\nu=a_\nu^*$. Then, the time derivatives of the α_μ are given in terms of the time derivatives of the a_ν and those of the Euler angles. The former are ignored ($\dot{a}_\nu\approx0$) and the latter are

[5] The phases of all spherical harmonics utilized in this work are those as given in M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[6] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).

[7] R. D. Evans, *The Atomic Nucleus* (McGraw-Hill Book Company, Inc., New York, 1955).

[8] The rotational functions, $D_{m,m'}{}^l(\theta_i)$ used in the present work are related to those of reference 5 by

$$[D_{m,m'}{}^l(\theta_i)]_{\text{present work}}=[(-1)^{m'-m}D_{m,m'}{}^{l*}(\theta_i)]_{\text{Rose}}.$$

[9] Compare Eq. (1-7a) and Eq. (A2) of reference 2 for the case $l=3$, $m=0$. One will note that at this point the cited work and the present calculation are in complete agreement. There is a multiplicative numerical difference in the final results, however.

expressed in terms of the body system components of the core angular momentum operator and the principal moments of inertia.[10]

Thus, one has

$$\sum_\mu \alpha_{\mu-m}^* \dot{\alpha}_\mu C(2l2;\mu,-m) = -i \sum_{\sigma,\nu,\nu'} \Big[[a_{\nu'} a_\nu D_{-m,\nu'-\sigma}{}^l \times C(2l2;\sigma,\nu'-\sigma) \sum_{k=1}^{3} (M_k)_{\nu\sigma} \frac{\hbar R_k}{\mathscr{J}_k} \Big], \quad \text{(I-9)}$$

where $(M_k)_{\nu\sigma} = \langle 2\nu | M_k | 2\sigma \rangle$ in the representation in which $\mathbf{M}^2$ and M_3 are diagonal and where the body system Cartesian components of $\mathbf{M}$ satisfy the commutation rules $\mathbf{M}\times\mathbf{M} = -i\mathbf{M}$. The R_k are the body system Cartesian components of the core angular momentum operator and the $\mathscr{J}_k$ are the principal moments of inertia.

The moments of inertia calculated from the hydrodynamical model are[11]

$$\mathscr{J}_1 = \mathscr{J}_2 = \tfrac{3}{2} \langle \rho_m \rangle \mathscr{R}_0{}^5 \beta^2,$$
$$\mathscr{J}_3 = 0,$$

where $\beta = a_0$ and axial symmetry is assumed.

Then one has for the core operator

$$(M_{l,m})_c = (-1)^{m+1} \mu_0 (20\sqrt{3}) g_c \times \mathscr{R}_0{}^{l-1} A(l) \sum_{\nu=-1}^{1} D_{-m,\nu}{}^l R_\nu, \quad \text{(I-10)}$$

where

$$g_c = \frac{\langle \rho_e / e \rangle}{\langle \rho_m / m \rangle}, \quad \text{(I-11)}$$

which is expected to be of the order of Z/A of the core. Also,

$$A(l) = \left[\frac{l(2l+1)}{l+1} \right]^{\frac{1}{2}} C(2l2;1,-1) F(l).$$

R_ν are the pseudo-spherical components of the body system core angular momentum operator with $R_0 = 0$.[12]

The total angular momentum operator for the system (core plus particle) is $\mathbf{I} = \mathbf{R} + \mathbf{J}$. Thus, the generalized l,mth magnetic multipole operator for the system is given by

$$M_{l,m} = (-1)^{m+1} \mu_0 g_c (20\sqrt{3}) \mathscr{R}_0{}^{l-1} A(l) \sum_\nu D_{-m,\nu}{}^l (I_\nu - J_\nu) + (-1)^m \mu_0 g(l) [4\pi l]^{\frac{1}{2}} r^{l-1} \sum_{\nu,\sigma} C(l-1,1,l;\sigma-\nu,\nu) \times D_{-m,\sigma}{}^l Y_{l-1,\sigma-\nu} J_\nu. \quad \text{(I-12)}$$

[10] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **26**, No. 14 (1952).

[11] Note that $\mathscr{J}_3 = 0$ presents no difficulties here since the coefficient of R_3 is zero when axial symmetry is assumed.

[12] This is empirically observed [see reference 3; also R. K. Osborn and E. D. Klema, Phys. Rev. **100**, 822 (1955)], but here the necessity of $R_0 = 0$ is dictated by the fact that the R_3 component does not appear in the cartesian counterpart of Eq. (I-10).

Electric Multipoles

In the body coordinate system, the l,mth electric multipole is[1]

$$E_{l,m} = \left[\frac{4\pi}{2l+1} \right]^{\frac{1}{2}} \int \rho_e r^l Y_{l,m}{}^* d\tau. \quad \text{(I-13)}$$

The only operator we shall be interested in here is $E_{2,m}$. The laboratory operator is then given by

$$(E_{l,m})_{\text{lab}} = \sum_\nu D_{m,\nu}{}^l (E_{l,\nu})_{\text{body}}. \quad \text{(I-14)}$$

In all cases of interest it was found that the contribution to the matrix elements of the single particle part of this operator is less than 10%. Thus, it is preferable to approximate the $E2$ operator by

$$E_{2,m} \approx (E_{2,m})_{\text{core}} = \tfrac{1}{2} e D_{m,0}{}^2(\theta_i) Q_0, \quad \text{(I-15)}$$

where to first order, Q_0 is the intrinsic quadrupole moment given by

$$Q_0 = 3[5\pi]^{-\frac{1}{2}} Z \mathscr{R}_0{}^2 \beta. \quad \text{(I-16)}$$

The advantage of this approximation is that the deformation parameter, β, is uniquely determined (within the approximation) by the spectroscopic quadrupole moment eQ which is the expectation value of $2E_{2,0}$.

The following definitions of the static operators were used:

$$\mu = M_{1,0},$$
$$eQ = 2E_{2,0},$$
$$\Omega = -M_{3,0}.$$

SECTION II

Eigenvectors

The collective Hamiltonian is

$$H_{\text{particle}} + H_{\text{core}} + \text{RPC},$$

where RPC is the so-called rotation particle coupling. In the strong-coupling approximation (which is all that is considered here) the total angular momentum of the system is a good quantum number and the RPC contribution is considered to be a small perturbation to the total energy. Then the eigenvectors of the collective system are products of the eigenvectors of the core and those of the single particle.

The "core" Hamiltonian is that of a rotor; its eigenfunctions then are $D_{M,K}{}^I(\theta)$; where I is the total angular momentum of the system, M the projection of I on the laboratory z axis, and K the projection of I on the body symmetry axis. Except for the cases $K = \frac{1}{2}$, the ground state is given by $I = K$.

The particle Hamiltonian has been treated by Nilsson[13] as a deformed harmonic oscillator whose potential has the same deformation as that of the core.

[13] S. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

Also, small admixtures of spin-orbit and L^2 coupling are included so that in the limit of no deformation the intrinsic particle levels go over to the shell model levels. With N denoting the principal oscillator quantum number and considering only matrix elements diagonal in N, the particle eigenfunctions may be denoted as $\chi_{m_J}{}^N$; where m_J is the projection of J on the body symmetry axis and since the core rotates normal to this axis is equal to K.

The properly symmetrized approximate eigenfunctions for the system are[3]

$$|N,I,M\rangle=\left[\frac{2I+1}{16\pi^2}\right]^{\frac{1}{2}}\{\chi_K{}^N D_{M,K}{}^I + (-1)^{I+N-\frac{1}{2}}\chi_{-K}{}^N D_{M,-K}{}^I\}. \quad \text{(II-1)}$$

The eigenfunctions of the particle are expanded in terms of the eigenfunctions of the spherically symmetric harmonic oscillator, symbolically as:

$$\chi_K{}^N=\textstyle\sum_{L,\sigma} A_{L,K-\sigma}|N,\,L,\,K-\sigma,\,\sigma\rangle, \quad \text{(II-2)}$$

with

$$A_{L,\lambda}=A_{L,\lambda}{}^*=A_{L,-\lambda}, \quad \text{(II-3a)}$$

and

$$(-1)^L=(-1)^N;\quad 0\leq L\leq N. \quad \text{(II-3b)}$$

The expansion coefficients are functions of the deformation parameter, β, and are tabulated by Nilsson.[13]

Application of the Operators

In all the following, consideration is limited to those cases where $K\neq\frac{1}{2}$. The $K=\frac{1}{2}$ bands can in certain instances have a ground state with $I\neq K$; I is then large enough to support an octupole moment. However, in no case of interest was it necessary to include this slight complication.

The expectation value of the magnetic dipole operator is then

$$\mu=\frac{2I}{2I+1}\mu_0[g_c+Ig(1)]. \quad \text{(II-4)}$$

The "total" particle gyromagnetic ratio $g(l)$ was calculated to be

$$g(l)=\frac{1}{I}\sum_{L,\sigma} A_{L,I-\sigma}{}^2\left[\sigma g_s+\frac{2g_L}{l+1}(I-\sigma)\right]. \quad \text{(II-5)}$$

The quadrupole moment is found to be

$$Q=\frac{(2I)(2I-1)}{(2I+1)(2I+2)}Q_0, \quad \text{(II-6)}$$

where Q_0 is given by Eq. (I-16).

The second quantity which is necessary to determine $g(1)$ and g_c is

$$\delta=\left[\frac{T(E2;I\to I-1)}{T(M1;I\to I-1)}\right]^{\frac{1}{2}}=\Delta E(I\to I-1)\frac{Q_0}{g(1)-g_c}\times 2.41\times 10^{-3}\left[\frac{3}{5(2I-2)(2I+2)}\right]^{\frac{1}{2}}, \quad \text{(II-7)}$$

where $\Delta E(I\to I-1)$ is the I, $I-1$ energy separation in kev and Q_0 is in barns. The sign of δ is defined by Alder *et al.*[14] such that

$$\text{sign}\delta=\text{sign}\left(\frac{g(1)-g_c}{Q_0}\right). \quad \text{(II-8)}$$

Finally, the magnetic octupole moment is given by

$$\Omega=-\frac{(2I)(2I-1)(2I-2)}{(2I+1)(2I+2)(2I+3)}\times\mu_0\left[\frac{6}{7}g_c\mathfrak{R}_0{}^2+g(3)\frac{\hbar}{m\omega_0}G(I,N,\beta)\right], \quad \text{(II-9)}$$

where $[\hbar/m\omega_0]^{\frac{1}{2}}$ is an intrinsic oscillator length and $G(I,N,\beta)$ is a rather complicated function of I, N, and β. The dependence on β is via the expansion coefficient, $A_{L,\lambda}$, of Eq. (II-2).

$$\begin{aligned}G(I,N,\beta)=\sum_{L,\sigma}\Bigg\{&A_{L,I-\sigma}{}^2\frac{3(2N+3)}{2(2L-1)(2L+3)}[(3I-2\sigma)L(L+1)-(I-\sigma)^2(5I-2\sigma)-(I-\sigma)]\\&+A_{L+2,I-\sigma}A_{L,I-\sigma}\frac{3(5I-2\sigma)}{(2L+3)}\left[\frac{[(L+1)^2-(I-\sigma)^2][(L+2)^2-(I-\sigma)^2](N-L)(N+L+3)}{(2L+1)(2L+5)}\right]^{\frac{1}{2}}\Bigg\}\\&+\sum_L\Bigg\{A_{L,I-\frac{1}{2}}A_{L,I+\frac{1}{2}}\frac{3I(2N+3)}{(2L-1)(2L+3)}[(2L+1)^2-4I^2]^{\frac{1}{2}}\\&+A_{L,I-\frac{1}{2}}A_{L+2,I+\frac{1}{2}}\frac{3}{4(2L+3)}\left[\frac{(N-L)(N+L+3)[(2L+3)^2-4I^2](2L+2I+1)(2L+2I+5)}{(2L+1)(2L+5)}\right]^{\frac{1}{2}}\end{aligned}$$

[14] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. 28, 432 (1956).

$$-A_{L,I+\frac{1}{2}}A_{L+2,I-\frac{1}{2}}\frac{3}{4(2L+3)}\left[\frac{(N-L)(N+L+3)[(2L+3)^2-4I^2](2L-2I+1)(2L-2I+5)}{(2L+1)(2L+5)}\right]^{\frac{1}{2}}\Bigg\}$$

$$+(-1)^{N+1}\delta_{I,\frac{3}{2}}10\sum_L\left\{\frac{6L(L+1)(2N+3)}{(2L-1)(2L+3)}[(L-1)(L+2)]^{\frac{1}{2}}A_{L,1}A_{L,2}\right.$$

$$+\frac{3(2N+3)L(L+1)}{4(2L-1)(2L+3)}A_{L,1}^2-\frac{3(L+2)}{2(2L+3)}\left[\frac{(L-1)L(L+1)(L+3)(N-L)(N+L+3)}{(2L+1)(2L+5)}\right]^{\frac{1}{2}}A_{L+2,1}A_{L,2}$$

$$-\frac{3(L+1)}{2(2L+3)}\left[\frac{L(L+2)(L+3)(L+4)(N-L)(N+L+3)}{(2L+1)(2L+5)}\right]^{\frac{1}{2}}A_{L+2,2}A_{L,1}$$

$$\left.+\frac{3}{2(2L+3)}\left[\frac{L(L+1)(L+2)(L+3)(N-L)(N+L+3)}{(2L+1)(2L+5)}\right]^{\frac{1}{2}}A_{L+2,1}A_{L,1}\right\}.$$

SECTION III

Results

Fitting Scheme

The various parameters which enter into the expression for Ω, Eq. (II-9) were determined as follows:

(i) The particular Nilsson ground state for each nucleus has been in most cases picked by Nilsson and Mottelson[15] from considerations of beta decay and other related data, as well as from pairwise-counting of the energy levels (according to the Pauli principle). Their choices were used when given. In the cases of nuclei normally outside the range of this model the ground state was determined essentially by the latter method.

(ii) The core radius, $\mathcal{R}_0$, was taken throughout as $1.2\times10^{-13}A_{\text{core}}^{\frac{1}{3}}$ cm.

(iii) The intrinsic oscillator length, $[\hbar/m\omega_0]^{\frac{1}{2}}$, was taken as approximately $A^{1/6}\times10^{-13}$ cm. This value follows from the semi-empirical binding energy formula with $\hbar\omega_0=41\ A^{-\frac{1}{3}}$ Mev.

(iv) Since Coulomb excitation data is generally more reliable than hyperfine structure measurements, the value of β was obtained wherever possible from the measured $E2$ reduced matrix elements which are proportional to Q_0^2. The sign of Q_0 was then assigned on the basis of the measured value of Q or by other specific means as indicated. When Q_0 was not available, β was determined from Q. Most of the values of Q_0 were taken from reference 14. As a check on the correctness of the values of β and $\hbar\omega_0$ for those nuclei with well measured, excited, intrinsic particle levels the experimental and theoretical differences in the energy levels were compared. In all cases the results of these comparisons were quite acceptable.

(v) Once the Nilsson level was identified and the deformation parameter known, the $A_{L,\lambda}$ determined $G(I,N,\beta)$.

(vi) From the measured dipole moment, Eq. (II-4), and the measured or deduced value of δ, Eq. (II-7), g_c and $g(1)$ were determined. In only one case was the sign of δ actually available. Hence in general two sets of values of g_c and $g(1)$ were found. That set was chosen for which g_c was closest to Z/A. The values of g_c and $g(1)$ found here agree with those of reference 14. When δ was not available g_c was assigned the value Z/A except as indicated and $g(1)$ was determined from μ. As a check on the reasonability of the value of $g(1)$, g_s was determined from Eq. (II-5). It was expected that g_s should have a value less than that of a free particle, but of the same sign.

Discussion of the Results

Figures 1 and 2 display the experimental values of the seven isotopes and the theoretical values for the deformed nuclei. To reduce both the experimental and theoretical values and thereby compare them with the Schwartz limits, $\langle r^2\rangle$ was taken as $\frac{3}{5}\mathcal{R}_0^2$ with $\mathcal{R}_0=1.2\times10^{-13}A^{\frac{1}{3}}$ cm. If indeed $\mathcal{R}_0$ is only as large as $1.5\times10^{-13}A^{\frac{1}{3}}$ cm, then, with the exception of the two chlorine isotopes, all the measured moments lie closer to the opposite single-particle limit than to the one which characterizes their dipole moment.

Figure 3 displays the deduced values of g_s. The nucleus Ga^{71} seems greatly out of place. If one allows g_c to take on a value greater than 1.0, then not only does g_s fall within the "correct" range, but also the theoretical value of Ω is much closer to the experimental value. In the case of Np^{237}, g_s was arbitrarily assigned the free-particle value since the magnetic dipole moment is only roughly known [$\pm6\pm2.5$ nm]. The general feature to be noted is that the majority of the odd-Z nuclei have a g_s value lying closer to the Dirac than to the free-particle limit, while for the odd-N nuclei the situation is just reversed.

Table I displays the experimental and theoretical values of the octupole movements of those nuclei considered. No theoretical value is given for I^{127} since

[15] B. Mottelson and S. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter 1, No. 8 (1959).

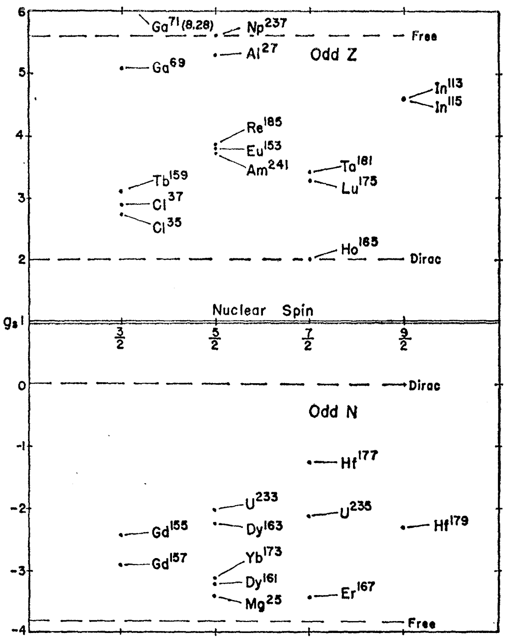

FIG. 3. The spin gyromagnetic ratio, g_s, for the odd nucleon versus the nuclear spin. The dashed lines indicate the free-particle and Dirac values for g_s.

the character of the Nilsson level that should be the ground state (if the collective model were valid for this nucleus) is such that Ω is essentially indeterminant except as to sign. In the case of Er^{167}, the sign of both μ and Q are experimentally in doubt. In order that $[633, \frac{7}{2}+]$ be the ground state Q must be positive. Further, the sign of g_s is only correct for negative μ and only of the right magnitude for positive Q. These choices of sign have also been noted by Osborn and Klema.[16]

For Np^{237}, the magnetic dipole moment is only very roughly known; $\mu = \pm 6 \pm 2.5$ nm. Further Q has not been measured, and only Q_0 is known. Since there are no $\frac{5}{2}+$ levels in the vicinity of 93 protons for a negative deformation parameter Q was taken as positive and $Q_{\text{theo}} \approx +3.2$ barns. Because of the uncertainty in μ, g_s was arbitrarily assigned the free-particle value. Then g_c turns out to be larger than Z/A, but the theoretical value of $\mu (+3.4$ nm) is nearly within the experimental uncertainty.

TABLE I. The theoretical and experimental values of the magnetic octupole moment, Ω; the deduced value of the single-particle spin gyromagnetic ratio, g_s; and the nuclear ground states. The method for the calculation of Ω is given in the text and the experimental values were taken from the Nuclear Data Cards.[n] The ground states are labeled $(N-)$ meaning Nilsson[13] level or by the asymptotic labeling of Mottelson and Nilsson.[15]

Nucleus	$\Omega_{\text{exptl.}}$	$\Omega_{\text{theoret.}}$	g_s	Ground state	Comments
Mg^{25}		−0.016	−3.40	$[202, \frac{5}{2}+]$	a
Al^{27}		0.019	5.30	$[202, \frac{5}{2}+]$	a
Cl^{35}	−0.020	−0.020	2.75	$N-8$	b
Cl^{37}	−0.015	−0.015	2.90	$N-8$	c
Ga^{69}	0.14	0.121	5.10	$N-19$	a,d
Ga^{71}	0.18	0.196	8.28	$N-19$	d,e
In^{113}	0.57	0.250	4.60	$N-18$	a
In^{115}	0.56	0.250	4.60	$N-18$	a
I^{127}	±0.18	probably+			f
Eu^{153}		−0.020	3.80	$[413, \frac{5}{2}+]$	g
Gd^{155}		0.044	−2.42	$[521, \frac{3}{2}-]$	g
Gd^{157}		0.054	−2.90	$[521, \frac{3}{2}-]$	g
Tb^{159}		0.054	3.13	$[411, \frac{3}{2}+]$	g,h
Dy^{161}		0.086	−3.20	$[642, \frac{5}{2}+]$	a,i
Dy^{163}		−0.034	−2.24	$[523, \frac{5}{2}-]$	a,j
Ho^{165}		−0.235	2.01	$[523, \frac{7}{2}-]$	g
Er^{167}		0.112	−3.42	$[633, \frac{7}{2}+]$	k
Yb^{173}		0.003	−3.11	$[512, \frac{5}{2}-]$	g
Lu^{175}		0.007	3.29	$[404, \frac{7}{2}+]$	g
Hf^{177}		−0.032	−1.24	$[514, \frac{7}{2}-]$	g
Hf^{179}		0.061	−2.30	$[624, \frac{9}{2}+]$	g,l
Ta^{181}		−0.001	3.41	$[404, \frac{7}{2}+]$	g
Re^{185}		0.064	3.84	$[402, \frac{5}{2}+]$	g
U^{233}		−0.061	−2.01	$[633, \frac{5}{2}+]$	g
U^{235}		0.121	−2.10	$[743, \frac{7}{2}-]$	g
Np^{237}		−0.409	5.59	$[642, \frac{5}{2}+]$	m
Am^{241}		−0.031	3.72	$[523, \frac{5}{2}-]$	a

[a] g_c was taken as Z/A and β was determined from Q.
[b] $g_c = 0.41$, $Z/A = 0.47$; β was determined from Q.
[c] $g_c = 0.05$, $Z/A = 0.44$; β was determined from Q. The difference between the g_c required for Cl^{37} and that for Cl^{35} is not understood.
[d] This ground state is an exception to the pairwise filling of levels which would predict $N-16$. However, that level is such that g_s would be negative.
[e] g_c was arbitrarily taken as 1.0. A larger value would make g_s more reasonable, and $\Omega_{\text{theoret.}}$ would be closer to $\Omega_{\text{exptl.}}$
[f] Pairwise level counting predicts a groundstate of $N-27$, and an exception to this rule predicts $N-31$. Neither level yields a reasonable value of g_s with an exact fit to μ.
[g] All the parameters were determined from Coulomb excitation data.
[h] The sign of Q has not been measured, but here signg_s = sign$Q$, so Q was taken as positive.
[i] The sign of μ has not been experimentally determined, but here signg_s = signμ so μ was taken as negative.
[j] The sign of μ has not been measured, but here the sign of g_s is opposite that of μ. Thus μ was taken as positive.
[k] The signs of μ and Q are not known. For reasons discussed in the text μ was taken to be negative and Q positive.
[l] Q has not been measured for Hf^{179}. However, the first excited intrinsic level of Hf^{179} is a $7/2-$ level which lies directly above the ground state only for positive Q. $Q_{\text{theoret.}} \approx +3.8$ barns.
[m] For reasons discussed in the text, g_s was arbitrarily assigned the free particle value; g_c was determined from Coulomb excitation data.
[n] *Nuclear Data Cards* (National Research Council-National Science Foundation, Washington D. C.).

CONCLUSION

The foregoing calculation demonstrates that the octupole moments of nuclei within the collective region may be expected to have values whose magnitudes are of the same order as the magnitudes of those already measured. This suggests that no greater difficulties would be encountered in their measurement than have been incurred before. It would seem worthwhile therefore that their measurement be undertaken since they provide information concerning the current, and hence the velocity distributions within the nucleus.

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[16] R. K. Osborn and E. D. Klema, Nuclear Phys. 3, 571 (1957).