Measurement of Nuclear Transitions with 10^{-20}-sec Half-Lives and the Scattering Cross Sections of Unstable Particles by Proximity Scattering

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Proximity scattering uses the particular form of a two-particle final-state interaction in a three-particle final-state system, in which the two interacting particles just prior to their interaction are free. Proximity scattering is the reaction in which an incident particle (0) strikes a target nucleus (1), forming a composite nucleus (2) which then decomposes into two particles (3) and (4). Particle (4) at some time later decays into two particles (5) and (6), whereupon particle (5) then interacts with particle (3). The separability in space of (5) and (3) and their scattering are analyzed in terms of wave packets. The lifetime of particle (4) and the (5–3) scattering cross section is shown to be obtainable from the (5–3) energy and angular correlation. Illustrative numerical examples are given.

I. INTRODUCTION

THE direct measurement of very short lifetimes would help to obtain a better understanding of the theory of nuclear reactions. For example, a direct reaction[1] describes an interaction which takes place within approximately 10^{-22} sec, the time it takes an incident particle to traverse a target nucleus. The time delay, however, associated with the formation of a compound nucleus[2] in a heavy element can be as much as 10^{-16} sec. A direct measurement of the lifetime would discriminate between these two reactions not only qualitatively but quantitatively. It would also be very useful if we could measure the scattering cross section of unstable particles such as the neutron-neutron interaction. Proximity scattering may allow us to measure lifetimes on the order of 10^{-20} sec and measure the cross sections of unstable particles,[3] under the conditions in which the unstable particles just prior to their interaction are free.

In Sec. II we define proximity scattering and give the conditions for proximity scattering which is developed in this paper. Section III describes the behavior of the wave functions outside the range of nuclear forces for the motion of the final-state particles (5) and (3), neglecting (5–3) scattering. Section IV describes the (5–3) interaction and the separability in space of the motion of (5) and (3). In Sec. V the lifetime of the intermediary nucleus (4) and the (5–3) scattering cross section are obtained from the energy and angular correlation of (5) and (3) with numerical examples of this given in Sec. VI.

II. PROXIMITY SCATTERING

Proximity scattering is defined as the reaction in which an incident particle (0) strikes a target nucleus (1), forming a composite nucleus (2) which very shortly breaks up into particles (3) and (4) where (4) is in an excited state. Particle (4) at some time later decays into two particles (5) and (6) whereupon particle (5) then interacts with particle (3). The range of the interactions (3–4), (5–6), and (5–3) are nonoverlapping. We are interested in determining the lifetime of (4) in an excited state E_4, and/or the corresponding scattering cross section of (3) and (5).

A few typical reactions are discussed in this paper which contain the general principles of proximity scattering. The proximity scattering reactions are described in the (0–1) c.m. system and have the following conditions:

1. The masses of (3) and (5) are much less than the masses of (4) and (6). Under this assumption the c.m. of (3–4) and (5–6) is at the c.m. of (4) and (6), respectively, which is at the c.m. of (0–1). 2. Spin, charge, and relativistic effects are neglected. 3. The direction for the emission of (5) from (4) is weakly or noncorrelated with the direction for the emission of (3) from (2). 4. The distance from the (0–1) c.m. to the average position of the (5–3) interaction is much greater than the range of the nuclear forces of (3–4) or (5–6). 5. Particles (5) and (3) when they interact are well separated from particle (6). 6. The polar angle, defined by the interaction region from (0–1) c.m., is very small. 7. The maximum angle of scattering of (5) or (3) in

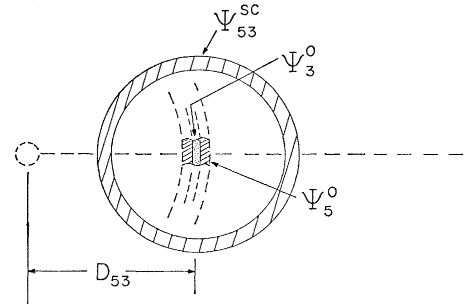

FIG. 1. The scattering of (5) and (3) in the (0–1) c.m. system (schematic). The wave packet Ψ_5^0 scatters on the wave packet Ψ_3^0 producing the scattered wave packet Ψ_{53}^{sc}.

[1] N. Austern, S. T. Butler, and H. McManus, Phys. Rev. **92**, 350 (1953); S. T. Butler, N. Austern, and C. Pearson, *ibid.* **112**, 1227 (1958); C. A. Levinson in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960), Part B, p. 670.

[2] N. Bohr, Nature **137**, 344 (1936); V. F. Weisskopf, Phys. Rev. **52**, 295 (1937); V. F. Weisskopf and D. H. Ewing, *ibid.* **57**, 472, 935 (1940); J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 340.

[3] R. Fox, Bull. Am. Phys. Soc. **6**, 56 (1961).

the (0–1) c.m. system due to the (5–3) interaction is very small.

The interaction is shown in Fig. 1. The incident waves (5) and (3) are in the form of wave packets due to the energy spread associated with the lifetime of their emission. The interaction of the incident wave packets of (5) and (3), $\Psi_5{}^0$ and $\Psi_3{}^0$, results in the scattered wave packet $\Psi_{53}{}^{sc}$.

III. THE MOTION OF (5) AND (3), NEGLECTING (5–3) SCATTERING

1. The Wave Function of (3)

Particle (3) may be the result of a direct reaction and may be peaked strongly in the forward direction. The wave function for (3), ϕ_3, is discussed in this section for the stripping reaction,[4] a typical direct reaction which can produce a strong forwardly peaked angular distribution. The reaction is

$$\overset{(0)}{[(3)+(A)]}+(1)\rightarrow(3)+\overset{(4)}{[(A)+(1)]},\qquad \text{(III-1)}$$

where (0) is a composite of particle (3) and a particle (A), and (4) is a composite of (1) and (A). The wave function of (3) outside the range of nuclear forces, under the stripping and Born approximations, is given by

$$\phi_3(\mathbf{r}_3)=\frac{-m_3}{2\pi\hbar^2}\int\frac{e^{ik_3|\mathbf{r}_3-\mathbf{r}_3'|}}{|\mathbf{r}_3-\mathbf{r}_3'|}\phi_4{}^*(\mathbf{r}_{1A})(V_{3A})\times e^{i\mathbf{k}_i\cdot\mathbf{r}_i}\phi_0(\mathbf{r}_{3A})d^3r_{1A}d^3r_3',\qquad \text{(III-2)}$$

where $\phi_3(\mathbf{r}_3)$ is the wave function of (3) when free, m_3 is the mass of (3), $\phi_4{}^*$ is the complex conjugate of the wave function of (1) when part of (A), V_{3A} is the nuclear potential between (3) and (A), k_3 is the wave number of (3), $e^{i\mathbf{k}_i\cdot\mathbf{r}_i}$ is the incident plane wave, and ϕ_0 is the wave function of (3) when part of (0).

For r_3 large, performing the integration in (III-2), we obtain

$$\phi_3(\mathbf{r}_3)=(-m_3/2\pi\hbar^2)(e^{ik_3r_3}/r_3)UF_i(\mathbf{q}_3)F_f{}^*(\mathbf{q}_1),\qquad \text{(III-3)}$$

with

$$F_i(\mathbf{q}_3)=\int e^{i\mathbf{q}_3\cdot\mathbf{r}}\phi_0(\mathbf{r})d^3r,\quad F_f(\mathbf{q}_1)=\int e^{-i\mathbf{q}_1\cdot\mathbf{r}}\phi_4(\mathbf{r})d^3r,$$

$$\mathbf{q}_3=\mathbf{k}_3+\mathbf{k}_1(m_3/m_6),\qquad \mathbf{q}_1=(-m_1/m_4)\mathbf{k}_3-\mathbf{k}_1,$$

$$U=E_0+\hbar^2q_3{}^2/2\mu_{3A},$$

where $\mathbf{k}_3$, $\mathbf{k}_1$ and m_0, m_1, m_3, and m_4 are the respective wave numbers and masses of the designated particles, μ_{3A} is the reduced mass of (3) and (A), and E_0 is the binding energy of (0).

$\phi_3(\mathbf{r}_3)$ in (III-3) is an outgoing wave with an angular distribution determined by the Fourier transforms of the wave functions of (0) and (4) over the wave numbers $\mathbf{q}_3$ and $\mathbf{q}_1$, respectively.

The angular distribution of $\phi_3(\mathbf{r}_3)$ is strongly peaked at zero degrees for zero orbital angular momentum transfer. There are now many examples of this with the experimental points closely fitting the theoretical curves.[5]

In many reactions (3) is more accurately considered to be the result of the decaying composite nucleus (2) rather than that of a direct reaction. Its angular distribution will be then more slowly varying with angle. The wave function for (3) is then similar to that of (5) as discussed in the next section.

2. The Wave Function of (5)

Particle (5) is emitted from the decaying state (4). The wave function for the motion of (5) is

$$\psi(r_5)=\sum_l\psi_l(r_5),\qquad \text{(III-4)}$$

with

$$\psi_l(r_5)=[\mu_l{}^{(+)}(r_5)/r_5]\sum_m F_{lm}Y_{lm}(\theta,\phi),\qquad \text{(III-5)}$$

where F_{lm} is the expansion coefficient of $\psi_l(r_5)$ on the spherical harmonic Y_{lm} and $\mu_l{}^{(+)}(r_5)$ is an outgoing wave given by the spherical Hankel function of the first kind of order l. In (III-5), r_5 is greater than the range of the (5–6) nuclear force. For r_5 large with respect to l/k_5,

$$\mu_l{}^{(+)}(r_5)\cong\exp[i(k_5r_5-\tfrac{1}{2}l\pi)],\qquad \text{(III-6)}$$

where k_5 is the wave number of the motion of (5). From (III-5) and (III-6) we observe that the wave function of (5) is a slowly varying function of angle for small orbital angular momenta.

IV. THE MOTION OF (5) AND (3), INCLUDING (5–3) SCATTERING

The wave functions for (5) and (3) describe outgoing spherical waves for r_3 and r_5 large with respect to l_3/k_3 and l_5/k_5, where l_3 and l_5 are the orbital angular momentum quantum numbers in the respective waves. We have the angular distribution of (3), $f(\theta_3)$, and the angular distribution of (5) with respect to the emission of (3), $g(\theta_{53},\phi_{53})$.

The wave packets $\Psi_3{}^0$ and $\Psi_5{}^0$ in the channel α de-

[4] R. Serber, Phys. Rev. **72**, 1008 (1947); S. T. Butler, *ibid.* **80**, 1095 (1950); S. T. Butler, *Nuclear Stripping Reactions* (John Wiley & Sons, Inc., New York, 1957); M. K. Banerjee in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960), Part B, p. 695.

[5] W. R. Cobb and D. B. Guthe, Phys. Rev. **107**, 181 (1957); C. K. Bockelman, C. M. Braams, C. P. Browne, W. W. Buechner, R. D. Sharp, and A. Sperduto, *ibid.* **107**, 176 (1957); C. K. Bockelman and W. W. Buechner, *ibid.* **107**, 1366 (1957); A. G. Rubin, *ibid.* **108**, 62 (1957); C. E. Dickerman, *ibid.* **109**, 443 (1958); W. F. Vogelsang and J. N. McGruer, *ibid.* **109**, 1663 (1958); H. A. Enge, E. J. Irwin, Jr., and D. H. Weaner, *ibid.* **115**, 949 (1959); E. W. Hamburger and A. G. Blair, *ibid.* **119**, 777 (1960).

scribing the motion of (3) and (5) are

$$\Psi_3{}^0=[f(\theta_3)/r_3]\int N_3(k_3)\exp(ik_3r_3)dk_3,$$
$$\Psi_5{}^0=[g(\theta_{53},\phi_{53})/r_5]\int N_5(k_5)\exp(ik_5r_5)dk_5, \quad \text{(IV-1)}$$

where $\Psi_3{}^0$ and $\Psi_5{}^0$ are outgoing spherical waves integrated over all momenta with the weighting functions N_3 and N_5, respectively. N_3 and N_5 are energy breadths associated with the lifetime of emission of (3) and (5), respectively. The wave function describing the motion of both (5) and (3), $\Psi_{53}{}^0$, is a product of $\Psi_5{}^0$ and $\Psi_3{}^0$ or

$$\Psi_{53}{}^0=[f(\theta_3)g(\theta_{53},\phi_{53})/r_5r_3]\Phi_{53}{}^0, \quad \text{(IV-2)}$$

with

$$\Phi_{53}{}^0=\Phi_5{}^0\Phi_3{}^0=\left[\int N_3(k_3)\exp(ik_3r_3)dk_3\right]\times\left[\int N_5(k_5)\exp(ik_5r_5)dk_5\right]. \quad \text{(IV-3)}$$

In the quantum mechanical description of a decaying state, a system with only an outgoing wave has a complex energy.[6] The wave function for (5) and (3) is given to first order by

$$\Phi^0(r)=C\exp[-\hbar^{-1}(E_0t-mv_0r)-(2\tau)^{-1}(t-rv_0{}^{-1})], \quad r<v_0t, \quad \text{(IV-4)}$$
$$\Phi^0=0 \quad r>v_0t,$$

and the intensity by

$$|\Phi^0(r)|^2=|C|^2\exp(r/v_0\tau)\exp(t/\tau), \quad r<v_0t,$$
$$|\Phi^0(r)|^2=0, \quad r>v_0t, \quad \text{(IV-5)}$$

where Φ^0 is the wave function describing the radial motion of (5) or (3), E_0 and v_0 are the average energy and velocity of particles (5) or (3), r is the radial coordinate of (5) or (3), τ is the lifetime of particles (2) or (4), and t is the time elapsed after the creation of particles (2) or (4).

Let t_{20} be the time of the creation of (2). The average value of the time for the creation (3) and (4), $\bar{t}_{40}$, is equal to $t_{20}+\tau_2/(\ln 2)$. Substituting in (IV-5), $(t-t_{20})$ for t_3 and $[t-t_{20}-\tau_2/(\ln 2)]$ for t_5 we obtain

$$|\Phi_3{}^0|^2=|C_3|^2\exp(r_3/v_{30}\tau_2)\exp[(t-t_{20})/\tau_2], \quad r_3<v_{30}(t-t_{20});\ t<t_{20}, \quad \text{(IV-6)}$$
$$|\Phi_3{}^0|^2=0, \quad r_3>v_{30}(t-t_{20});\ t>t_{20},$$

and

$$|\Phi_5{}^0|^2=|C_5|^2\exp(r_5/v_{50}\tau_4)\exp[(t-\bar{t}_{40})/\tau_4] \quad r_5<v_{50}(t-\bar{t}_{40});\ t>\bar{t}_{40}, \quad \text{(IV-7)}$$
$$|\Phi_5{}^0|^2=0, \quad r_5>v_{50}(t-\bar{t}_{40});\ t>\bar{t}_{40},$$

where $\bar{t}_{40}=t_{20}+\tau_2/(\ln 2)$.

[6] G. Breit and F. L. Yost, Phys. Rev. 48, 203 (1935); G. Breit, *ibid.* **58**, 506, 1068 (1940); G. Breit in *Handbuch der Physik* (Springer-Verlag, Berlin, 1959), Vol. 41, p. 1.

It is to be noticed in (IV-6) and (IV-7) that the intensity is exponentially dependent on the time; and the width of the packet, $|\Phi|^2$, is equal to $v_0\tau$.

There is little overlap of the wave packets $\Phi_3{}^0$ and $\Phi_5{}^0$ for τ_2 much shorter than τ_4 and for $t-\bar{t}_{40}$ small. Particles (3) and (5) are thus separable in space. The motion of the center of the packet, $\bar{r}$, is that of a particle of mass m and velocity v_0.

As the wave packets $\Phi_5{}^0$ and $\Phi_3{}^0$ overlap, (5) and (3) interact. The maximum interaction occurs at a time when the wave packets have maximum overlap. Let $\bar{r}$, the center of the packet, be the radial coordinate for which the particle has a 50% probability of being at $r>\bar{r}$ and a 50% probability for being at $r<\bar{r}$.

We make the approximation that the entire interaction occurs for $r_3\cong r_5\cong\bar{r}_3=\bar{r}_3=D_{53}$. The wave packet of (5) is well removed from (6), the (0–1) c.m. in proximity scattering, at the position of the (5–3) interaction, or

$$(v_{50}\tau_4/2)/D_{53}\ll 1. \quad \text{(IV-8)}$$

We have for (IV-2),

$$\Psi_{53}{}^0=f(\theta_3)g(\theta_{53},\phi_{53})\Phi_{53}{}^0/D_{53}{}^2. \quad \text{(IV-9)}$$

Let $\lambda\!\!\!^-$ be the wavelength corresponding to the relative wavelength in the (5–3) system, and l_m be the largest quantum number of importance in the (5–3) interaction. The polar angle $\theta_{lm}{}^+$ defined by the interaction region and the (0–1) c.m. is small for proximity scattering, and is given by

$$\theta_{lm}{}^+\cong\tfrac{3}{2}(l_m+1)\lambda\!\!\!^-/D_{53}\ll 1. \quad \text{(IV-10)}$$

The only important part then of $\Psi_{53}{}^0$ is θ_{53} near zero for the calculation of $\Psi_{53}{}^{sc}$, the scattered wave. $\Psi_{53}{}^{0\prime}$, equal to $\Psi_{53}{}^0$ ($\theta_{53}=0$), is now used to calculate $\Psi_{53}{}^{sc}$. Using (IV-9), $\Psi_{53}{}^{0\prime}$ is given by

$$\Psi_{53}{}^{0\prime}=[f(\theta_3)g(0)/D_{53}{}^2]\Phi_{53}{}^0. \quad \text{(IV-11)}$$

For a given orientation of the (5–3) c.m. momentum, $\hbar\mathbf{K}$, we have

$$\Psi_{53}{}^{0\prime}=\int\Psi_{53K}{}^{0\prime}d\Omega_K, \quad \text{(IV-12)}$$

$$\Psi_{53K}{}^{0\prime}=[f(\theta_K)g(0)/D_{53}{}^2]\Phi_{53}{}^0, \quad \text{(IV-13)}$$

where $\theta_K=\theta_3$.

We approximate the exponential packets in (IV-6) and (IV-7) with Gaussian packets where the Gaussian packets have the same widths as the exponential packets. The centers of these Gaussian packets for (5) and (3) are $\bar{r}_5$ and $\bar{r}_3$, respectively, the $\bar{r}_5$ and $\bar{r}_3$ in the exponential packets in (IV-6) and (IV-7). We then obtain for $\Phi_{53}{}^0$

$$\Phi_{53}{}^0=(2\pi\Delta k_3\Delta k_5)^{\frac{1}{2}}\exp[ik_{30}(r_3-\bar{r}_3)-\tfrac{1}{2}(r_3-\bar{r}_3)^2(\Delta k_3)^2]\times\exp[ik(r_5-\bar{r}_5)-\tfrac{1}{2}(r_5-\bar{r}_5)^2(\Delta k_5)^2], \quad \text{(IV-14)}$$

where $\Delta k_3=(v_{30}\tau_2)^{-1}$ and $\Delta k_5=(v_{50}\tau_4)^{-1}$.

We have in general

$$\mathbf{K}=\mathbf{k}_3+\mathbf{k}_5 \quad \text{and} \quad \mathbf{k}=(\mu/m_5)\mathbf{k}_5-(\mu/m_3)\mathbf{k}_3, \tag{IV-15}$$

where $\mathbf{k}$ and μ are the wave number and reduced mass of the (5–3) system, respectively. The vectors $\mathbf{K}$, $\mathbf{k}$, $\mathbf{k}_3$, and $\mathbf{k}_5$ are collinear in (IV-14). Substituting (IV-15) in (IV-14)

$$\Phi_{53}{}^0=(2\pi W)\left\{\int \exp[-(K-K_0)^2/2W^2]\exp(iKR)dK\right\} \times\left\{\int \exp[-(k-k_0)^2/2W^2]\exp(ikr)dk\right\}, \tag{IV-16}$$

where $W^2=(\Delta k_3)^2+(\Delta k_5)^2$. Substituting (IV-16) in (IV-13), we obtain

$$\Psi_{53K}{}^{0\prime}=f(\theta_K)g(0)2\pi W/D_{53}{}^2 \times\left\{\int \exp[-(K-K_0)^2/2W^2]\exp(iKR)dK\right\} \times\left\{\int \exp[-(k-k_0)^2/2W^2]\exp(ikr)dk\right\}. \tag{IV-17}$$

The asymptotic form of the wave function $\chi_{53}{}^T$, which is the wave function for the scattering of a plane wave in the (5–3) c.m. system, is

$$\chi_{53}{}^T=\exp[ik(\mathbf{K}/|\mathbf{K}|)\cdot\mathbf{r}] +H_k(\theta)[\exp(ikr)]/r, \tag{IV-18}$$

where $\exp[ik(\mathbf{K}/|\mathbf{K}|)\cdot\mathbf{r}]$ is an incident plane wave in the (5–3) system in the direction of the unit vector $\mathbf{K}/|\mathbf{K}|$; and $H_k(\theta)$ is the scattering amplitude. $\Psi_{53K}{}^{0\prime}$ given by (IV-17), is the unperturbed wave function of importance for (5–3) scattering in the region $r_3\cong r_5 \cong D_{53}$, for a given $\mathbf{K}$. Substituting the scattering amplitude $H_k(\theta)$ for the incident wave in the (5–3) system e^{ikr} in (IV-17), we obtain the scattered wave of (5) and (3), $\Psi_{53K}{}^{sc}$, given by

$$\Psi_{53K}{}^{sc}=f(\theta_K)g(0)2\pi W/D_{53}{}^2 \times\left\{\int \exp[-(K-K_0)^2/2W^2]\exp(iKR)dk\right\} \times\left\{\int \exp[-(k-k_0)^2/2W^2]H_k(\theta)[\exp(ikr)/r]dk\right\}. \tag{IV-19}$$

The relations between the momenta and angles in the (5–3) c.m. system and the (0–1) c.m. system are

$$(\mathbf{k}\cdot\mathbf{K})/(|\mathbf{k}||\mathbf{K}|)=\cos(\theta), \tag{IV-20}$$

$$\tan\theta_{5,3}{}^+=\sin\theta_{5,3}/ [\cos\theta_{5,3}+m_{5,3}{}^{-1}(\mu M T_{\text{c.m.}}T^{-1})^{\frac{1}{2}}], \tag{IV-21}$$

$$\theta_3=\pi-\theta_5, \tag{IV-22}$$

$$T=(m_5/M)T_{30}+(m_3/M)T_{50} -2[(\mu/M)T_{30}T_{50}]^{\frac{1}{2}}, \tag{IV-23}$$

$$T_{\text{c.m.}}=(m_3/M)T_{30}+(m_5/M)T_{50} +2[(\mu/M)T_{30}T_{50}]^{\frac{1}{2}}, \tag{IV-24}$$

$$\delta E=4(\mu T T_{\text{c.m.}}/M)^{\frac{1}{2}}, \tag{IV-25}$$

$$\cos\theta_{5,3}=(2/\delta E)[T_{5s,3s}-(m_{3,5}/M)T -(m_{5,3}/M)T_{\text{c.m.}}], \tag{IV-26}$$

$$\theta_{5m,3m}{}^+=\sin^{-1}[(m_{3,5}/m_{5,3})(T/T_{\text{c.m.}})]^{\frac{1}{2}}, \tag{IV-27}$$

where θ^+ is the angle in the (0–1) c.m. system corresponding to θ in the (5–3) c.m. system; T_5 and T_3 are the kinetic energies of (5) and (3) in the (0–1) c.m. system; T and $T_{\text{c.m.}}$ are the internal and the c.m. kinetic energies of the (5–3) system; μ and M are the reduced mass and the total mass of the (5–3) system; $T_{5s,3s}$ is the kinetic energy in the (0–1) system of a particle (5) or (3) with mass $m_{5,3}$ that has scattered through an angle $\theta_{5,3}$ in the (5–3) system; T_{50} and T_{30} are the kinetic energies of (5) and (3) for no proximity scattering; δE is the maximum energy change of (5) or (3) due to proximity scattering; and $\theta_{5m,3m}{}^+$ is the maximum angle of scattering of (5) or (3) respectively in the (0–1) c.m. system.

The condition in proximity scattering that the maximum angle of scattering be very small of (5) or (3) in the (0–1) c.m. system due to the (5–3) interaction gives

$$\theta_{sm}{}^+\ll 1, \tag{IV-28}$$

where $\theta_{sm}{}^+$ is equal to $\theta_{5m}{}^+$ or $\theta_{3m}{}^+$, whichever is greater.

The particles (5) and (3) which have proximity scattered are contained in the solid angle Ω_c centered on $\mathbf{K}$, given by

$$\Omega_c=2\pi[1-\cos\theta_c{}^+], \tag{IV-29}$$

with

$$\theta_c{}^+=[(\theta_{Im}{}^+)^2+(\theta_{sm}{}^+)^2]^{\frac{1}{2}}, \tag{IV-30}$$

where $\theta_{Im}{}^+$ is given by (IV-10) and $\theta_{sm}{}^+$, the greater of $\theta_{5m}{}^+$ or $\theta_{3m}{}^+$, is given by (IV-27).

V. LIFETIME τ_4 AND THE (5–3) SCATTERING CROSS SECTIONS σ

We desire now to use the conditions of proximity scattering and the relations of Sec. IV to obtain τ_4 and the (5–3) scattering cross section σ. All numerical relations in this section and in Sec. VI are in units such that energies are in Mev, lengths are in fermis, cross sections are in barns, and masses are in amu.

Let $\mathcal{C}_{53}{}^T$, be the coincidence rate of (3) emitted in the solid angle Ω_3 centered at θ_3, and (5) emitted in the solid angle Ω_5 centered at (θ_5,ϕ_5). $\mathcal{C}_{53}{}^T$ is equal to $\mathcal{C}_{53}{}^0$ plus $\mathcal{C}_{53}{}^{sc}$, where $\mathcal{C}_{53}{}^0$ is the coincidence rate if there were no proximity scattering, and $\mathcal{C}_{53}{}^{sc}$ is the contribution to $\mathcal{C}_{53}{}^T$ due to proximity scattering.

The particles (5) and (3) which have proximity scattered, occur within the solid angle Ω_c. Ω_c is given by (IV-30) and under the conditions of proximity scattering is small. $\mathcal{C}_{53}^T$ peaks at θ_{53} equal to zero as θ_{53} is varied. The width of this peak is θ_w, where θ_w is comparable to θ_c when Ω_3 and Ω_5 is comparable to, or less than, Ω_c. This is shown in Fig. 2. The extrapolation of $\mathcal{C}_{53}^T$ for $\theta_{53}>\theta_w$ to θ_{53} equal to zero gives $\mathcal{C}_{53}^0$. The subtraction of $\mathcal{C}_{53}^0$ from $\mathcal{C}_{53}^T$ gives $\mathcal{C}_{53}^{sc}$. The maximum of $\mathcal{C}_{53}^{sc}$, $\mathcal{C}_{53m}^{sc}$, occurs for θ_{53} equal to zero.

The solid angle of (3), Ω_3, is fixed at a particular value of θ_3 and a given energy of excitation of (6), E_6. This is equivalent to T_3 plus T_5 equal to C_6 where C_6 is a constant, for m_3, $m_5 \ll m_4$, m_6.

Particle (4) at an excitation energy E_4 has a lifetime τ_4 for the emission of (5), leaving (6) at an excitation energy E_6. The excitation energy E_4 corresponds to the kinetic energy T_{30} equal to C_4. The corresponding emitted energy of (5) is T_{50} equal to (C_6-C_4).

We analyze the coincidence correlation in the solid angle Ω_R, centered at (θ_3,ϕ_3). The polar angle of Ω_R, θ_R, is much greater than θ_w.

We have the average intensities of (3) and (5) over Ω_R, $\bar{f}_0$, $\bar{g}_0$, $\bar{f}_{sc}$, and $\bar{g}_{sc}$: $\bar{f}_0$, the average intensity per solid angle per Mev of (3) for the energy range $T_{30} \pm(\Delta E/2)$; $\bar{g}_0$, the average intensity of (5) in the energy range $T_{50}\pm(\Delta E/2)$; $\bar{f}_{sc}$, the average intensity of (3) for the energy interval $\delta\epsilon$ in the energy range $T_{3L} = T_{30}+\delta\epsilon_1$ to $T_{3U}=T_{30}+\delta E-\delta\epsilon_2$; and $\bar{g}_{sc}$, the average intensity of (5) for the energy interval $\delta\epsilon$ in the energy range $T_{5L}=T_{50}-\delta E+\delta\epsilon_2$ to $T_{5U}=T_{50}-\delta\epsilon_1$. The maximum energy change of (5) or (3) in proximity scattering, δE, is given by (IV-26).

$\mathcal{C}_{53}^T$ detects (3) and (5) in the energy interval $\delta\epsilon$ in the energy range T_{3L} to T_{3U} and T_{5L} to T_{5U}. Let F be the fraction of the proximity scattered particles in this energy range having had initial energies T_{30} and T_{50}. For S wave (5–3) scattering, we have for F

$$F=\tfrac{1}{2}[\cos\theta_1-\cos\theta_2], \qquad \text{(V-1)}$$

where

$$\cos\theta_1=(2/\delta E)[T_{5L}-(m_3/M)T-(m_5/M)T_{\text{c.m.}}], \qquad \text{(V-2)}$$

$$\cos\theta_2=(2/\delta E)[T_{3L}-(m_5/M)T-(m_3/M)T_{\text{c.m.}}], \qquad \text{(V-3)}$$

R, the ratio in Ω_R of the average number of coincidences due to proximity scattering to the total number of particles of (5) and (3) in the energy range $T_{30} \pm(\Delta E/2)$ and $T_{50}\pm(\Delta E/2)$ and in the solid angles Ω_3 and Ω_5, is

$$R=\left[\Omega_R^{-1}\int^{\Omega_R}\mathcal{C}_{53}^{sc}2\pi d(\cos\theta_{53})\right]\Big/ \bar{f}_0\bar{g}_0\Omega_3\Omega_5(\Delta E)^2 = GF(4\pi/\Omega_R), \qquad \text{(V-4)}$$

or

$$\mathcal{C}_{53m}^{sc}=[4\pi G\bar{f}_0\bar{g}_0\Omega_3\Omega_5F(\Delta E)^2]/\Omega_w, \qquad \text{(V-5)}$$

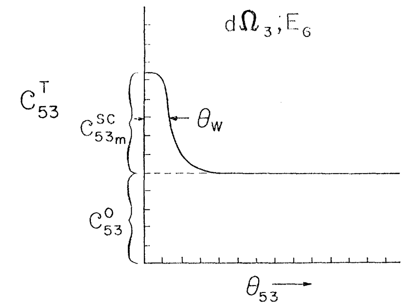

FIG. 2. The coincidence count rate, $\mathcal{C}_{53}^T$, as a function of θ_{53} for a given solid angle of emission of (3), $d\mathbf{\Omega}_3$, and an excitation energy of (6), E_6. The net rise in $\mathcal{C}_{53}^T$ for $\theta_{53}<\Delta\theta_w$ is $\mathcal{C}_{53}^{sc}$, the result of (5–3) scattering.

with

$$\Omega_w\equiv\left[\int^{\Omega_R}\mathcal{C}_{53}^{sc}2\pi d(\cos\theta_{53})\right]\Big/\mathcal{C}_{53m}^{sc}\equiv I_{sc}/\mathcal{C}_{53m}^{sc}, \qquad \text{(V-6)}$$

where G is the ratio of the number of (5–3) particles that have proximity scattered with initial energies T_{50} and T_{30}, to the original number of (5–3) particles of energy T_{50} and T_{30}, for the case of Ω_R equal to 4π. The primary contribution to the integral in (V-6) occurs within the angle $\theta_w \ll \theta_R$, and is thus insensitive to the exact value of Ω_R. The (5–3) scattering occurs at a radius in the (0–1) c.m. system, $\bar{r}_3=\bar{r}_5=D_{53}$, where $D_{53}\gg ƛ$. The average interaction radius D_{53} is

$$D_{53}=v_{50}v_{30}\tau_4/v(\ln 2), \qquad \text{(V-7)}$$

$$D_{53}=13.2(\mu T_{30}T_{50}/m_5m_3T\Gamma_4^2)^{\frac{1}{2}}, \qquad \text{(V-8)}$$

where Γ_4 is equal to $\hbar/\tau_4$. We have for $ƛ$

$$ƛ=4.55/(T\mu)^{\frac{1}{2}}. \qquad \text{(V-9)}$$

D_{53} is correctly given by (V-7) for $R_{56}\cong R_{34}$ and (3) and (5) neutral, where R_{56} and R_{34} are the ranges of the (5–6) and (3–4) nuclear interactions, respectively. Equation (V-7) should be increased by an amount of order R_{56} if (5) is charged or decreased by an amount of order R_{34} for (3) charged. Since however we have the conditions for proximity scattering $R_{56}/D_{53}\ll 1$ and $R_{34}/D_{53}\ll 1$, Eq. (V-7) is correct for (5) and (3) neutral or charged. The function G is equal to σ, the total scattering cross section in the (5–3) c.m. system divided by $4\pi D_{53}^2$. Using (V-8), we obtain

$$G=0.0457(\Gamma_4^2T\sigma m_3m_5/T_{30}T_{50}\mu). \qquad \text{(V-10)}$$

The ratio of the coincidence count rate due to proximity scattering at θ_{53} equal to zero, $\mathcal{C}_{53m}^{sc}$, to the coincidence count rate if there were no proximity scattering, $\mathcal{C}_{53}^{sc}$, is, using (V-5),

$$\mathcal{C}_{53m}^{sc}/\mathcal{C}_{53}^0=(4\pi GF/\Omega_w)(\bar{f}_0\bar{g}_0/\bar{f}_{sc}\bar{g}_{sc})(\Delta E/\delta\epsilon)^2, \qquad \text{(V-11)}$$

or

$$\Gamma_4^2\sigma = 1.74(I_{sc}T_{30}T_{50}\mu/FTm_3m_5\mathcal{C}_{53}^0) \times(\bar{f}_{sc}\bar{g}_{sc}/\bar{f}_0\bar{g}_0)(\delta\epsilon/\Delta E)^2. \quad \text{(V-12)}$$

Ω_w is approximately equal to Ω_c for Ω_3 and Ω_5 small compared to Ω_c. For Ω_3 and Ω_5 much larger than Ω_c, Ω_w is much larger than Ω_c thereby decreasing $\mathcal{C}_{53m}^{sc}/\mathcal{C}_{53}^0$.

Using (V-12) we can obtain Γ_4 and σ when the conditions for proximity scattering hold as discussed in this paper. They are

$$m_3,m_5 \ll m_4,m_6, \quad \text{(V-13)}$$

$$(v_{50}\tau_4/2)/D_{53} = 0.347[(m_3T_5/m_5T_3)^{\frac{1}{2}}-1] \ll 1, \quad \text{(V-14)}$$

$$\theta_{Im}^+ \cong [3/2(l_m+1)\lambda\!\!\!^-]/D_{53} = 0.521(l_m+1) \times\Gamma_4(T_5T_3)^{-\frac{1}{2}}(m_3m_5)^{\frac{1}{2}}\mu^{-1} \ll 1, \quad \text{(V-15)}$$

$$\theta_{5m,3m}^+ = \sin^{-1}[(m_{3,5}/m_{5,3})(T/T_{\text{c.m.}})]^{\frac{1}{2}} \ll 1, \quad \text{(V-16)}$$

$$R/D_{53} = (R/13.2)(m_5m_3T\Gamma_4^2/\mu T_5T_3) \ll 1, \quad \text{(V-17)}$$

where

$$R \cong (1.4m_{4,6}^{\frac{1}{3}}+\rho). \quad \text{(V-18)}$$

R is the effective radius of the (3–4) or (5–6) nuclear interaction with $m_{4,6}$ the mass of (4) or (6) in a.m.u. and ρ equal to zero for (3) or (5) a proton or neutron and ρ equal to 1.2 fermis for (3) or (5) an alpha particle. We can also obtain σ and τ_4 when neither is known. Let us assume we can represent σ by using an effective-range theory,

$$\sigma(v) = 4\pi/\{k^2+[(r_0k/2)-(1/a)]^2\}, \quad \text{(V-19)}$$

where a is the scattering length and r_0 is the effective range. Using (V-19) we have the function J defined by

$$J \equiv [\hbar(\ln 2)]^2(GT_5T_3)^{-1}(m_3m_5/4\mu^2), \quad \text{(V-20)}$$

equal to

$$J = \tau_4^2[1+(r_0/2\lambda\!\!\!^- - \lambda\!\!\!^-/a)]^2. \quad \text{(V-21)}$$

J is a function of the lifetime τ_4, the effective range r_0, and the scattering length a.

In particular for large $\lambda\!\!\!^-$

$$J = \tau_4^2[1+(\lambda\!\!\!^{-2}/a^2)] \quad \text{for} \quad |\lambda\!\!\!^-/a] \gg r_0/2\lambda\!\!\!^-. \quad \text{(V-22)}$$

$\lambda\!\!\!^-$ is varied by varying C_6 but keeping E_4 and C_6-C_4 constant. The function J plotted against $\lambda\!\!\!^{-2}$ gives a straight line for $\lambda\!\!\!^-/a \gg r_0/2$. The extrapolated intercept of this line to $\lambda\!\!\!^{-2}=0$ is the lifetime of (4) squared. The slope of the line is equal to the square of the ratio of the lifetime over the scattering length.

Relations (V-11) and (V-12) are satisfactory only if $\bar{f}_0$, $\bar{g}_0$, $\bar{f}_{sc}$, and $\bar{g}_{sc}$ are slowly varying in the solid angle $\mathbf{\Omega}_R$. Accurate averages of these intensities can be then obtained. Under certain conditions however it may be desired to have $\mathbf{\Omega}_R$ in a region where f_0 may be rapidly varying. This is the case when f_0 is the result of a stripping reaction.

Let f_0 be very large and equal to $\bar{f}_0'$ for $0<\theta_3<\Delta\theta$ and small for $\theta_3>\Delta\theta$, where $\Delta\theta$ is small. We make $\Omega_3=\Omega_5=\Omega_D$, where Ω_D is sufficiently large to contain all the proximity scatterings. Then

$$\theta_D = [\theta_c^2+(\Delta\theta)^2]^{\frac{1}{2}}, \quad \text{(V-23)}$$

$$\Omega_D = 2\pi(1-\cos\theta_D), \quad \text{(V-24)}$$

and

$$\Omega_\Delta = 2\pi(1-\cos\Delta\theta). \quad \text{(V-25)}$$

We then have for $\mathcal{C}_{53m}^{sc}/\mathcal{C}_{53}^0$,

$$\mathcal{C}_{53m}^{sc}/\mathcal{C}_{53}^0 = (4\pi GF\Omega_\Delta/\Omega_D^2) \times(\bar{f}_0'\bar{g}_0/\bar{f}_{sc}\bar{g}_{sc})(\Delta E/\delta\epsilon)^2. \quad \text{(V-26)}$$

Using (V-26) in conjunction with (V-10), (V-24), and (V-25) we can obtain Γ_4 and σ for the condition f_0 peaked strongly in the forward direction.

VI. ILLUSTRATIVE NUMERICAL EXAMPLES

We introduce a few numerical examples in this section to illustrate the magnitude of some of the expressions previously derived. The relevant equation numbers of Secs. IV and V are placed alongside the numerical values.

Example 1a: f_0, f_{sc}, g_0, and g_{sc} are slowly varying with angle in the solid angle Ω_R. A resonant state exists with a width Γ_4 which emits the unstable particle (5) with energy T_{50}. Particle (6) is left in its ground state resulting in C_6 equal to C_{6g}. There are no other resonances of (5) in the energy range T_{30} to C_{6g}. We desire to obtain the (5–3) scattering cross section where (5) and (3) are unstable particles. Let

$$\left.\begin{array}{l} (3)\to\text{neutron}, \\ (5)\to\text{neutron}, \\ m_4,\, m_6>10, \\ T_{30}=C_4=0.170, \\ T_{50}=C_6-C_4=0.400, \\ \Delta E=\Gamma_4=0.040, \\ \delta\epsilon=0.130, \\ T_{3L}\to T_{3U}=T_{5L}\to T_{5U}=0.220\to 0.350. \end{array}\right\} \quad \text{(VI-1)}$$

The above values are typical and the characteristics of the resonant state, T_{50} and Γ_4 are approximately that of ${}_8O^{16}$ (reference 7).

The fact that there are no resonant states in the energy range T_{30} to C_{6g} states that the only neutrons in the energy range $T_{30}\to T_{50}$ are T_{30} and T_{50}, and $\bar{f}_{sc}=\bar{g}_{sc}=0$. In any experiment there is always some background, however, so thus we use in our example,

$$\bar{f}_{sc}=\bar{g}_{sc}=0.03\bar{f}_0=0.03\bar{g}_0. \quad \text{(VI-2)}$$

[7] *Neutron Cross Sections*, Second edition, compiled by D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958).

Further we use Ω_3 and Ω_5 smaller than Ω_c, which results in Ω_w of the order of Ω_c. We assume we find from I_{sc}, (V-6),

$$\Omega_w = \Omega_c. \tag{VI-3}$$

Using (VI-1) we have

$$\left.\begin{array}{llll} \mu=0.50, & & M=2.0, & \\ T=0.024 & \text{(IV-23)}, & T_{\text{c.m.}}=0.546 & \text{(IV-24)}, \\ \lambda\!\!\!^-=41 & \text{(V-9)}, & R\cong 3 & \text{(V-18)}, \\ D_{53}=380 & \text{(V-8)}. & & \end{array}\right\} \tag{VI-4}$$

The conditions of proximity become

$$\left.\begin{array}{rl} m_3,m_5 \ll m_4,m_6, & \\ (v_{50}\tau_4/2)/D_{53}=0.18\ll 1 & \text{(V-14)}, \\ \theta_{Im}=0.16\ll 1 & \text{(V-15)}, \\ \theta_m{}^+=\theta_{5m}{}^+=\theta_{3m}{}^+=0.22\ll 1 & \text{(V-16)}, \\ R/D_{53}=0.0076\ll 1 & \text{(V-17)}. \end{array}\right\} \tag{VI-5}$$

We have further the relations

$$\left.\begin{array}{rl} \delta E=0.23 & \text{(IV-25)}, \\ \theta_c=0.27=15° & \text{(IV-30)}, \\ \Omega_c=0.23 & \text{(IV-29)}, \\ \cos\theta_1=\cos\theta_2=0.56 & \text{(V-2, V-3)}, \\ F=0.56 & \text{(V-1)}, \end{array}\right\} \tag{VI-6}$$

and

$$G=5.2\times 10^{-6}\sigma_{nn}(T) \quad \text{(V-10)}. \tag{VI-7}$$

We can represent the neutron-neutron scattering cross section in the energy range of interest by an effective range approximation given by (V-19). Here, a is the singlet neutron-neutron scattering length a_s, and r_0 is the singlet neutron-neutron effective range r_{0_s}. Approximate values for a_s and r_{0_s} are[8]

$$a_s=-29 \quad \text{and} \quad r_{0_s}=2.7. \tag{VI-8}$$

Using the values of (VI-1)

$$k^2=5.8\times 10^{-4}\times(10^{26}\ \text{cm}^{-2}). \tag{VI-9}$$

Substituting (VI-9) and (VI-8) in (V-19)

$$\sigma_{nn}=\sigma_{nn}'\equiv 69. \tag{VI-10}$$

We then obtain for $\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0$, from (V-11)

$$\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0=11(\sigma_{nn}/\sigma_{nn}'). \tag{VI-11}$$

It appears in our example from (VI-11) that the coincidence rate at θ_{53} equal to zero is about ten times the coincidence rate of the nonproximity scattered contribution.

Example 1b: f_0, f_{sc}, g_0, and g_{sc} are slowly varying with angle in $\mathbf{\Omega}_R$. A group of overlapping resonant states exist with a total energy width of ΔE at an average energy T_{50}. Particle (6) is left in its ground state. There are no other particle (5) resonant states in the energy range T_{30} to C_{6g}. We desire the average width Γ_4, or the lifetime, τ_4, of the overlapping states.

We use in this example the same conditions as for example 1a, (VI-1), (VI-2), and (VI-3), except we have now

$$\Delta E=0.080. \tag{VI-12}$$

We have the same conditions for proximity scattering as (VI-5), and relations (VI-4) and (VI-6) are the same. Using (VI-10) and letting Γ_4' equal 0.040 we obtain for $\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0$ from (V-11)

$$\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0=45(\Gamma_4/\Gamma_4')^2. \tag{VI-13}$$

It is seen again from (VI-13) that $\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0$ is very large. For Γ_4 on the order of 0.040 we have a lifetime of τ_4 equal to 1.6×10^{-21} sec. $\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0$ should be experimentally measurable to less than 1%. The lifetime, τ_4, then should be measurable to $\sim 10^{-17}$ sec.

Example 2a: f_0 is peaked strongly in the angular interval $0<\theta_3<\Delta\theta$ and is small for $\theta_3<\Delta\theta$. f_{sc}, g_0, and g_{sc} are slowly varying with angle in the solid angle Ω_R where Ω_R is centered at θ_3 equal to zero. A resonant state exists with a width Γ_4 which emits the unstable particle (5) with energy T_{50}. Particle (6) is left in its ground state, resulting in C_6 equal to C_{6g}. There are no other resonant states of (5) in the energy range T_{30} to C_{6g}. We desire the (5–3) scattering cross section where (5) and (3) are unstable particles.

We use again (VI-1). $\bar{f}'$ is very large due to the strong forward peaking of (3). Let

$$\left.\begin{array}{l} \bar{g}_{sc}=0.03\bar{g}_0, \\ \bar{f}_{sc}=0.003\bar{f}_0', \\ \Delta\theta=15°. \end{array}\right\} \tag{VI-14}$$

We have similarly the relations (VI-4), (VI-5), and (VI-6). We have further

$$\left.\begin{array}{rl} \theta_D=21° & \text{(V-23)}, \\ \Omega_D=0.41 & \text{(V-24)}, \\ \Omega_\Delta=0.21 & \text{(V-25)}. \end{array}\right\} \tag{VI-15}$$

and σ_{nn}' is equal to 69.

We then obtain, using (V-26),

$$\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0=32(\sigma_{nn}/\sigma_{nn}'). \tag{VI-16}$$

Example 2b: f_0 is peaked strongly in the angular interval $0<\theta_3<\Delta\theta$ and is small for $\theta_3>\Delta\theta$. f_{sc}, g_0, and g_{sc} are slowly varying with angle in the solid angle, Ω_R with Ω_R centered at θ_3 equal to zero. A number of states exist in the energy interval with a total energy width ΔE at an average energy, T_{50}. Particle (6) is left at an excitation energy E_6. A number of these states are reached by a zero orbital momentum transfer of the incident particle with the emission of (5). There are no

[8] H. P. Noyes, in *Proceedings of the 1960 Annual International Conference on High-Energy Nuclear Physics at Rochester* (Interscience Publishers, Inc., New York, 1960).

such states for $C_6/2<T_5<C_6$. We desire to obtain the average width, Γ_4, of the overlapping states.

We let (5) be a proton and (3) be a neutron in this example. We take

$$\begin{aligned}&(3)\rightarrow \text{neutron},\\&(5)\rightarrow \text{proton},\\&m_4,m_6\gtrsim 10,\\&T_{30}=C_4=1.70,\\&T_{50}=(C_6-C_4)=4.00,\\&\Delta E=0.80,\\&\delta\epsilon=1.30,\\&\Delta\theta=15°,\\&T_{3L}\rightarrow T_{3U}=T_{5L}\rightarrow T_{5U}=2.20\rightarrow 3.50,\\&[\Gamma_4\lesssim 0.40],\\&\bar{f}_{sc}=\bar{g}_{sc}=\bar{g}_0=0.02\bar{f}_0'.\end{aligned}\qquad\text{(VI-17)}$$

The relations (VI-4), (VI-5), and (VI-6) hold, where however all energies are multiplied by 10, distances are divided by $\sqrt{10}$, and dimensionless quantities remain the same, except for R which is the same and R/D_{53} which is multiplied by $\sqrt{10}$. The neutron-proton scattering cross section,[7] at T equal to 0.24 of our example, is 6.3. Let Γ_4' be equal to 0.40. We then have for $\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0$ from (V-26)

$$\mathcal{C}_{53m}{}^{sc}/\mathcal{C}_{53}{}^0=0.53(\Gamma_4/\Gamma_4')^2. \qquad \text{(VI-18)}$$

The effect of proximity scattering can be quite large as illustrated in the examples of this section.

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Scattering of Neutrons by Alpha Particles*

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A precision n-α scattering experiment has been carried out in the energy interval 2–3 Mev. The angular distribution of neutrons elastically scattered by alpha particles has been measured at angles smaller than those considered in similar experiments. The phase-shift analysis of the data has led to the following results: (a) No D waves are detectable. (b) The phase shifts for the interaction in the $L=0$ and $L=1$, $J=\frac{3}{2}$ state agree very well with the values quoted in the literature. (c) The phase shift arising from the interaction in the $L=1$, $J=\frac{1}{2}$ state turns out to be much smaller than expected; it has been found $\delta_{\frac{1}{2}}{}^1=4°$ at 2.37 Mev and $\delta_{\frac{1}{2}}{}^1=8°$ at 2.87 Mev, instead of $\sim$20° in this energy interval. It is emphasized that such an unusual experimental result should be carefully considered in theoretical investigations concerning the spin-orbit potential, and in experimental researches using helium as a polarization analyzer.

LEVINTOV *et al.*[1] have claimed that the n-α interaction in the $P_{\frac{1}{2}}$ state is not well defined experimentally; the value of the $\delta_1{}^1$ phase shift,[2] evaluated from a left-right experiment at the incident neutron energy $E=2.45$ Mev is not consistent with the value obtained from phase shift analysis[3] of the angular distribution of neutrons elastically scattered by alpha particles. A somewhat similar result has been formerly obtained also by Seagrave[3] at $E=2.61$ Mev independently of polarization measurements; this situation has been later stressed by Pisent and Villi[4] in the framework of a phenomenological effective-range approach.

A precision n-α scattering experiment has been performed in order to investigate the energy dependence of $\delta_1{}^1$; the accurate knowledge of this dependence is

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[2] The phase shift for the n-α interaction in the state of orbital angular momentum L and total momentum $J=L\mp\frac{1}{2}$ will be indicated as $\delta_{2J}{}^L$ ($\delta_1{}^0\equiv\delta_0$).

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[4] G. Pisent and C. Villi, Nuovo cimento **11**, 300 (1959); see also P. G. Burke, *Nuclear Forces and the Few-Nucleon Problem* (Pergamon Press, New York, 1960), Vol. II, p. 413.