Velocity-Dependent Nuclear Potentials for Singlet-Even States*

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The usual models for the two-nucleon interaction involve a potential which includes a repulsive hard core at short distances and as a result, the treatment of the many-body problem is mathematically complicated. Peierls, Levinger, and others have suggested that ordinary perturbation theory may be useful if the two-body interaction can be described by a well-behaved potential. We attempt to provide such a description of the singlet-even states in terms of a potential of the form

$$-V_0J(r)+(\lambda/M)[p^2\omega(r)+\omega(r)p^2].$$

$J(r)$ and $\omega(r)$ are well-behaved functions of the relative separation and $\mathbf{p}$ is the operator for the relative momentum. The effective range and zero-energy scattering length along with the ^1S, ^1D, and ^1G phase shifts in the energy range 20 to 340 Mev are calculated and adjusted to fit the experimental values. We can fit the low energy parameters and the ^1S phase shifts very well. Agreement with the ^1D and ^1G phase shifts is plausible but less satisfactory. Suggestions for improving the fit are made. The treatment is nonrelativistic and Coulomb effects are ignored.

INTRODUCTION

SEVERAL authors[1,2] have treated the problem of a phenomenological description of the two-nucleon interaction with some success. For example, Gammel and Thaler[1] have used a repulsive core and a static, attractive Yukawa form outside the core to provide an accurate description of the two-body data. Such a potential also provides a reasonable description of the energy per particle of a many-body system.[3] Signell and Marshak and Hamada[2] have found similar potentials using approaches which are partly phenomenological and partly based on meson theory. Nevertheless, it is by no means well established that the hard core is the only possible representation of physical reality. It is apparently possible to achieve a description of the two-nucleon interaction in terms of a repulsive velocity-dependent part of the potential rather than an infinite repulsive core.[4,5] And, in fact, there is some theoretical evidence for the existence of such velocity-dependent terms in the potential.[6,7]

CALCULATION

To describe the interaction in singlet-even states of the system, we have chosen a potential of the form[8]

$$v(r,p)=-V_0J(r)+(\lambda/M)[p^2\omega(r)+\omega(r)p^2]. \quad (1)$$

Here, r is the magnitude of the relative separation and $\mathbf{p}$ is the operator for the relative momentum. In this preliminary work we have ignored Coulomb effects, which are well understood, and simply solved the Schrödinger equation with the potential (1) using an automatic numerical procedure. Phase shifts were then calculated in the usual way by comparing the asymptotic solution to the free wave solution. We have compared these results with the best fit (YLAM) to the available experimental data obtained by Breit *et al.*[9]

Using the potential (1), the Schrödinger equation becomes

$$[1+2\lambda\omega(r)]\left[R_l''+\frac{2}{r}R_l'\right]+2\lambda\omega'(r)R_l'(r)+\left\{k^2+\frac{MV_0}{\hbar^2}J(r)+\lambda\omega''(r)+\frac{2\lambda}{r}\omega'(r)-\frac{l(l+1)}{r^2}\right\}R_l(r)=0. \quad (2)$$

The fact that the potential is central allows the usual separation of the wave function into radial and angular parts and the equation for the radial part of the lth partial wave is given here. The wave number $k=(ME/\hbar^2)^{\frac{1}{2}}$ and the prime denotes differentiation with respect to r. The substitution $u_l(r)=r[1+2\lambda\omega(r)]^{\frac{1}{2}}R_l(r)$ removes the first derivatives of the wave function and yields

$$u_l''(r)+\left[k^2+W_{\text{eff}}(k,r)-\frac{l(l+1)}{r^2}\right]u_l(r)=0, \quad (3)$$

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[4] M. Razavy, O. Rojo, and J. S. Levinger, *Proceedings of the International Conference on Nuclear Structure, Kingston* (University of Toronto Press, Toronto, 1960), p. 128; A. Green (private communication).
[5] M. Razavy, G. Field, and J. S. Levinger, preceding paper [Phys. Rev. **125**, 269 (1962)].
[6] M. Sugawara and S. Okubo, Phys. Rev. **117**, 605, 611, (1960); S. Okubo and R. E. Marshak, Ann. Phys. **4**, 166, (1958).
[7] We have recently learned of the work of J. S. Bell at CERN who has shown that by a canonical transformation, the hard core can be replaced by a velocity-dependent potential.
[8] This form has the necessary invariance properties. The other obvious form for the quadratic velocity-dependence, $\mathbf{p}\cdot V(r)\mathbf{p}$ is essentially equivalent to this form.
[9] Breit, Hull, Lassila, and Pyatt, Phys. Rev. **120**, 2227, (1960).

TABLE I. ^1S phase shifts.[a]

E_L (Mev)	$\delta_0(V_I)$	$\delta_0(V_{II})$	Breit	GT
20	0.889	0.897	0.856	0.859
60	0.568	0.585	0.584	0.568
100	0.389	0.398	0.380	0.379
140	0.255	0.258	0.240	0.236
180	0.144	0.143	0.136	0.120
220	0.049	0.043	0.048	0.020
260	−0.037	−0.045	−0.033	−0.069
300	−0.114	−0.125	−0.120	−0.146
340	−0.185	−0.198	−0.195	···

[a] The ^1S phase shifts in radians, calculated from the potentials of (7) and (8), are given as functions of laboratory energy and compared with the values quoted by Breit[9] for the experimental data and with the values calculated by Gammel and Thaler[1] for their static potential with hard core.

where the velocity-dependence is expressed in the effective potential

$$W_{\text{eff}}(k,r)=\left[\frac{1}{1+2\lambda\omega(r)}\right]\times\left[W_0J(r)-2\lambda k^2\omega(r)+\frac{\lambda^2\omega'^2(r)}{1+2\lambda\omega(r)}\right], \quad (4)$$

with $W_0=MV_0/\hbar^2$.

Equation (3) has been solved numerically for several choices of the functions $J(r)$ and $\omega(r)$ subject to the boundary condition that $R_l(r)$ be finite at $r=0$. If $\omega(r)$ is of short range, it will be seen that the true wave functions $R_l(r)$ and the function $u_l(r)/r$ have the same asymptotic form so that it is permissible to calculate phase shifts using the latter function.

Two potentials will be mentioned which give reasonable fits to the experimental data. For the first, we take both $J(r)$ and $\omega(r)$ to be exponentials. In the second, we have applied the criterion of simplicity to the Schrödinger equation in the form (3). Choosing

$$W_0J(r)=[1+2\lambda\omega(r)]\times[W_0J_1(r)-\lambda^2\omega'^2(r)/[1+2\lambda\omega(r)]^2], \quad (5)$$

the effective potential assumes the simple form

$$W_{\text{eff}}(k,r)=-2\lambda k^2\omega(r)/[1+2\lambda\omega(r)]+W_0J_1(r). \quad (6)$$

TABLE II. ^1D phase shifts.[a]

E_L (Mev)	$\delta_2(V_I)$	$\delta_2(V_{II})$	Breit	GT
20	0.031	0.011	0.015	0.007
60	0.102	0.055	0.045	0.047
100	0.132	0.089	0.072	0.096
140	0.142	0.107	0.097	0.141
180	0.140	0.114	0.120	0.181
220	0.130	0.112	0.140	0.213
260	0.116	0.104	0.160	0.239
300	0.099	0.091	0.175	0.263
340	0.080	0.075	0.184	···

[a] The ^1D phase shifts in radians, calculated from the potentials of (7) and (8), are given as functions of laboratory energy E_L and compared with the values quoted by Breit[9] for the experimental data and with the values calculated by Gammel and Thaler[1] for their static potential with hard core.

TABLE III. ^1G phase shifts.[a]

E_L (Mev)	$\delta_4(V_I)$	$\delta_4(V_{II})$	Breit	GT
186.8	0.05	0.03	0.012	0.02
332	0.07	0.05	0.017	0.05

[a] The ^1G phase shifts estimated by Born approximation for the potentials of (7) and (8) are compared with the values quoted by Breit[9] and by Gamel and Thaler.[1]

The effective range and scattering length can be used to determine $W_{\text{eff}}(0,r)=W_0J_1(r)$ completely (for any assumed shape) following the treatment of Blatt and Jackson.[10] This reduces the number of degrees of freedom and the Breit phase shifts may then be used to determine λ and the range parameter of $\omega(r)$.

^1S and ^1D phase shifts are used to fix the form of the potential. ^1G phase shifts are then calculated by Born approximation.

DISCUSSION

The two most successful potentials, to date, are

$$V_I:\begin{cases}V_0J(r)=55\exp(-r),\\ \lambda\omega(r)=\ 5\exp(-3.6r),\end{cases} \quad (7)$$

and

$$V_{II}:\begin{cases}(\hbar^2W_0/M)J_1(r)=V_0J_1(r)=112.5\exp(-1.4r),\\ \lambda\omega(r)=\ \ 5\ \ \exp(-3.6r);\end{cases} \quad (8)$$

where r is in fermis and energies are in Mev.

From Table I it can be seen that there is little reason to prefer one potential over the other on the basis of the ^1S phase shifts alone. (In fact, it has been found that these data are very easy to fit with a velocity-dependent potential.) The results shown in Table II for the ^1D phase shifts are less satisfactory. The decrease in the values of the calculated phase shifts which begins at about 140 Mev is interpreted as implying that the repulsive (velocity-dependent) part of the potential has been chosen with too long a range. The use of a functional form for $\omega(r)$ which cuts off more sharply, such as a Gaussian, will probably correct this behavior. The ^1G phase shifts of Table III are discouragingly large as compared to the Breit values. It has been shown[11] rather clearly, however, that for higher angular momentum states (interactions primarily at distances greater than about 1.6f) the one-pion-exchange-potential (OPEP) is the principal interaction and provides an accurate description of the experimental data. Therefore, one can expect that a potential such

[10] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18, (1949).
[11] For example, G. Breit and M. H. Hull, Nuclear Phys. **15**, 216 (1960).

as V_{II} must be joined on the OPEP at a distance of (say) 1.5f.[5]

The potential V_{II} seems to provide as accurate a description of the singlet-even scattering data as a "hard-core" model. It remains to be seen whether such an approach will also be successful for other spin states of the system.

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Anomalous Neutron Scattering of Ar^{36}†

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The thermal neutron scattering of argon is unusual in that it shows a large amount of incoherent scattering, even though it is even-even and practically monoisotopic. The incoherence has been ascribed by Henshaw to a large scattering cross section for Ar^{36}. Measurements with the BNL fast chopper for a gas sample enriched to 63% Ar^{36} have shown that the total cross section from 0.1 ev to 6 kev varies in such a way as to reveal the presence of a negative energy level. The parameters $\Gamma_n^0=82$ ev, $E_0=-9.8$ kev, and $\Gamma_\gamma=1.85$ ev have been deduced for this level. The consequences of this anomalous scattering for neutron studies of atomic motions in a liquid are discussed.

I. INTRODUCTION

THE element argon shows unusual behavior in its thermal neutron scattering properties in that, although it is even-even and practically monoisotopic, it displays a large amount of incoherent scattering. Henshaw[1] has shown that this incoherence results from the presence of the small (0.34%) Ar^{36} component, which has a remarkably high cross section.

It has been suggested that it may be possible to prepare a completely incoherent scatterer by mixing argon isotopes in the proper amounts. In principal, if $\sum_i f_i\alpha_i=0$, where f_i are the isotopic abundance factors, and α_i the corresponding scattering amplitudes, then complete incoherent scattering is obtained. Such a scatterer would permit the use of subthermal neutrons to investigate the atomic motions in a liquid, since for the completely incoherent case the experimental data are amenable to a rather simple interpretation in terms of a self-correlation function describing the motion of the atom.

The potential or "hard sphere" scattering amplitude is defined to be positive. A negative scattering amplitude is obtained only when a nearby resonance is present at an energy above the thermal region. In this case the negative resonance amplitude can overwhelm the hard-sphere component.

It is the purpose of this paper to show that the high thermal scattering amplitude of Ar^{36} is due to the presence of a resonance at negative energy (i.e., below the neutron binding energy). Hence, the Ar^{36} scattering length is positive and of the same sign as that for elemental argon.[2] This implies that both Ar^{36} and Ar^{40} have the same sign (+) for their scattering length. Complete incoherence, therefore, for argon cannot be achieved.

II. EXPERIMENT

The total neutron cross section of a sample of argon gas enriched to 62.7% of Ar^{36} was determined by transmission methods with the Brookhaven fast chopper and time-of-flight apparatus operating at the BNL graphite research reactor. The gas sample was enclosed at a pressure of 1800 lb/in.^{2} in a specially designed iron sample holder which fits into the sample slot of the fast chopper entrance stator.[3] These holders were previously used in transmission measurements on xenon and krypton isotopes.[4] The argon sample was prepared by the

TABLE I. Analysis of argon sample.

Isotope	Atomic %
Ar^{36}	62.7±1.0
Ar^{40}	33.8±1.0
Ar^{38}	2.2±0.2
N^{14}	~1.0

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