Mach's Principle and Invariance under Transformation of Units*

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(Received October 19, 1961)

A gravitational theory compatible with Mach's principle was published recently by Brans and Dicke. It is characterized by a gravitational field of the Jordan type, tensor plus scalar field. It is shown here that a coordinate-dependent transformation of the units of measure can be used to throw the theory into a form for which the gravitational field appears in the conventional form, as a metric tensor, such that the Einstein field equation is satisfied. The scalar field appears then as a "matter field" in the theory. The invariance of physical laws under coordinate-dependent transformations of units is discussed.

IN a recent paper,[1] a modified relativistic theory of gravitation, closely related to Jordan's theory,[2] was developed, compatible with Mach's principle. It was indicated that the resulting formalism was but one particular representation of the theory, based upon a particular definition of the units of mass, length, and time.

The purposes of this note are, first to discuss very briefly the invariance of physical laws under units transformations,[3] and second to give another representation of the above theory, completely equivalent to it and derived from it by a simple transformation of units.

The first representation of the theory[1] could be characterized concisely as a relativity theory for which the gravitational field is described by a metric tensor and a scalar, but for which the equations of motion of matter in a given field are identical with those of General Relativity, not being explicitly dependent upon the scalar field. Because of the inclusion of the auxiliary scalar, as part of the gravitational field, the theory is both formally and in its physical interpretation different from General Relativity. It will be shown that this is only apparent, and that a simple redefinition of units causes the scalar to appear in the theory as a nongravitational field, Einstein's field equations being satisfied.

INVARIANCE UNDER TRANSFORMATIONS OF UNITS

Everyone, including the college freshman, is familiar with the usefulness of dimensional considerations in formulating physical laws. Dimensional analysis is essentially an elementary group theoretic technique applied to the equations of physics. It is evident that the particular values of the units of mass, length, and time employed are arbitrary and that the laws of physics must be invariant under a transformation of units. (The units and dimensions employed need not be three in number, nor need they be limited to the traditional mass, length, and time.)

The invariance which we wish to consider here is broader than the elementary consideration described above. Imagine, if you will, that you are told by a space traveller that a hydrogen atom on Siruis has the same diameter as one on the earth. A few moments' thought will convince you that the statement is either a definition or else meaningless. It is evident that two rods side by side, stationary with respect to each other, can be intercompared and equality established in the sense of an approximate congruence between them. However, this cannot be done for perpendicular rods, for rods moving relatively, or for rods with either a space- or time-like separation. Their intercomparison for purposes of establishing equality cannot be made until rules of correspondence are established.

Generally, there may be more than one feasible way of establishing the equality of units at different space-time points. It is evident then, that the equations of motion of matter must be invariant under a general coordinate-dependent transformation of units. It should be emphasized that the coordinate system is to be held fixed under a units transformation, whereas under a general coordinate transformation the system of physical units is held fixed but coordinates are varied. Thus, under a general transformation of units, the labeling of the space-time coincidence between two particles (coordinates) is invariant, whereas the scalar curvature and other purely geometrical scalars, invariant under coordinate transformations, are generally not invariant under a transformation of units.

It may be noted, for example, that a units transformation can be used to redefine the Riemannian geometry of general relativity in such a way that the resulting geometry is flat. (See Appendix I.)

We are not concerned here with the problem of the general transformation of units, but rather with one of more limited scope, the transformation of the formalism discussed in reference 1 under a limited class of units transformations. The transformation to be considered is a simple position-dependent scale factor applied to units of length, time, and reciprocal mass.

The velocity of light is invariant under such a trans-

* This work was supported by the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] C. Brans and R. H. Dicke, Phys. Rev. **124** (1961).

[2] P. Jordan, *Schwerkraft und Weltall* (Friedrick Vieweg und Sohn, Braunschweig, 1955).

[3] Alternative systems of units seem to have been first introduced into relativity by E. A. Milne [*Kinematic Relativity* (Oxford University Press, New York, 1948)]. Systematic discussions of units transformations were introduced by A. G. Walker [Proc. Roy. Soc. Edinburgh **62**, 164 (1946)] and by G. C. McVittie [Proc. Roy. Soc. Edinburgh **62**, 147 (1945)]. More recently a discussion of conformal units transformations has been given by H. Nariai and Y. Ueno [Progr. Theoret. Phys. (Kyoto) **24**, 593 (1960)].

formation and the local-Lorentz invariance of the theory is preserved. It should be noted that this is not a matter of necessity but rather of convenience.

The unit of action, hence Planck's constant, is also invariant under the transformation. This is a matter of convenience in the sense that the form assumed by the equations of a quantum-mechanical formalism are familiar.

Under this transformation, all three quantities, time, length, and reciprocal mass transform in the same way. Hence we may, if we wish, assign the same dimension, say time, to the three quantities. As mentioned above, it is necessary to interpret the generalized coordinates of a point as fixed labels, invariant under a transformation of units. Then with the assumption that

$$ds=(g_{ij}dx^idx^j)^{\frac{1}{2}} \tag{1}$$

has the dimensions of time, g_{ij} transforms like a time squared. If the size of the unit of time is scaled by a dimensionless factor $\lambda^{-\frac{1}{2}}$, an arbitrary function of coordinates, a time interval (i.e., the number of units required to represent the interval) scales as $\lambda^{\frac{1}{2}}$. Also the metric tensor components transform as

$$\begin{aligned} g_{ij} &\rightarrow \lambda g_{ij}, \\ g^{ij} &\rightarrow \lambda^{-1} g^{ij}. \end{aligned} \tag{2}$$

Hence, the transformation of the spatial geometry is *conformal*.

The mass of a particle transforms as

$$m \rightarrow \lambda^{-\frac{1}{2}} m. \tag{3}$$

The electronic charge, the velocity of light, Planck's constant, and the electromagnetic four potentials are invariant under the transformation.

While λ may be taken to be an arbitrary function of coordinates, we may also limit ourselves to the case of λ a definite function of the scaler φ, of the Brans-Dicke formalism.[1] By so doing, rules of measure are established, for the scalar φ is locally observable. (It is measured with a φ meter, a black box having a pointer and a scale calibrated in units of MT^2L^{-3}. One type of black box contains a torsion balance deflected by the gravitational field produced by massive weights, held mechanically in fixed positions in the box.) Once φ, hence λ, is measured, the correction factor to be applied locally to a rod or clock is determined.

From a slightly different point of view, a unit of length can be established, in principle, in terms of the diameter of a planetary orbit in a solar system which can be transported (at least in time), alternatively in units of the diameter of the hydrogen atom, or with any combination of these two lengths. These various units are not in a fixed ratio in this theory but vary as functions of φ. It should be noted that it is necessary first to make a choice of the unit of length before a space-time geometry is established.

MACH'S PRINCIPLE, EXPRESSED IN TRANSFORMED UNITS

In reference 1 the gravitational coupling constant

$$Gm^2/\hbar c \sim 10^{-40} \tag{4}$$

(m the mass of some elementary particle) was interpreted not as a *fait accompli* presented to us by nature and unrelated to other parts of physics, but rather as a number determined, consistent with the requirements of Mach's principle, by the total mass distribution in the universe. Being a dimensionless number, it is invariant under a transformation of units.

It was emphasized in reference 1 that the formalism developed there was but a particular representation of the theory, a representation for which $\hbar$, c, and m were constant by definition and for which G was coordinate dependent, being determined by the scalar field φ. This representation had the great advantage that the equations of motion of "matter" in a known and given metric field were identical with those of General Relativity. For example, geodesic equations of motion are obtained for unchanged, zero-spin particles. These advantages are to be balanced against the disadvantage that the description of gravitational interactions is more complex, Einstein's field equation not being valid.

For the particular representation of the theory to be given here, the gravitational constant G, also $\hbar$, and c are constant by definition, and the rest masses of all elementary particles vary with position, being functions of φ, albeit in the same way, the mass ratios of different types of particles being constant.

As was mentioned above, this representation of the theory has the form of a "general relativity," Einstein's field equation being satisfied. The scalar φ of the theory plays the role of still another nongravitational field. If the field exists, its effects have not yet been observed with certainty.

Being a boson field generated by all matter, it is extraordinarily difficult to observe. The effect of nearby matter, in the laboratory, in generating this field is minor in comparison with the dominating influences of the enormously greater amounts of matter in the distant parts of the essentially static universe.

We turn now to the transformation of the formalism of reference 1 under the above described units transformation. This transformation, conformal in type, is similar to that discussed by Fierz[4] in his analysis of Jordan's theory.[2]

In reference 1, the equations of motion of matter and the gravitational field equations are derived from the variational principle

$$0=\delta\int\left[\varphi R-\omega\frac{\varphi_{,i}\varphi^{,i}}{\varphi}+\frac{16\pi}{c^4}L\right](-g)^{\frac{1}{2}}d^4x, \tag{5}$$

[4] M. Fierz, Helv. Phys. Acta. **29**, 128 (1956); see also the discussion by P. Jordan, reference 2, and Z. Physik **157**, 112 (1959), O. Heckmann, Z. Astrophys. **40**, 278 (1956), and the summary in D. R. Brill's article, Varenna summer school notes. 1961 (unpublished).

where R is the scalar curvature of the Riemannian space, is the Lagrangian density of matter (i.e., nongravitational fields) and φ is the above-mentioned scalar field, a field to be interpreted as part of the gravitational field which in this theory is described by both g_{ij} and φ. ω is a dimensionless constant of the order of unity. The Lagrangian density of matter is assumed to be completely standard, the usual scalar of special relativity generalized by replacing the Minkowskian metric tensor η_{ij} by the generally covariant metric tensor g_{ij}.

In order to show explicitly the transformation of the matter Lagrangian density, an example is constructed, for a system of charged particles interacting electromagnetically

$$L=-\frac{1}{(-g)^{\frac{1}{2}}}\sum\int[mc^2(-u^iu_i)^{\frac{1}{2}}+eA_iu^i]\delta^4(x-z)d\tau-\frac{1}{16\pi}F^{ij}F_{ij}. \quad (6)$$

Here the sum is over all particles, the particle position z is a function of the proper time τ, and the four-velocity is

$$u^i=dx^i/d\tau, \quad (7)$$

also

$$ds^2=g_{ij}dx^idx^j=-d\tau^2,$$
$$u^iu_i=-1, \quad (8)$$
$$g_{00}<0.$$

As usual

$$F_{ij}=A_{i,j}-A_{j,i}. \quad (9)$$

Measured in the new units defined above the mass of the particle becomes

$$\bar{m}=\lambda^{-\frac{1}{2}}m, \quad (9a)$$

or

$$m=\lambda^{\frac{1}{2}}\bar{m}.$$

In similar fashion, the other quantities in Eq. (6) are

$$\begin{aligned}
m&=\lambda^{\frac{1}{2}}\bar{m},\\
g_{ij}&=\lambda^{-1}\bar{g}_{ij},\\
g^{ij}&=\lambda\bar{g}^{ij},\\
d\tau&=\lambda^{-\frac{1}{2}}d\bar{\tau},\\
ds&=\lambda^{-\frac{1}{2}}d\bar{s},\\
(-g)^{\frac{1}{2}}&=\lambda^{-2}(-\bar{g})^{\frac{1}{2}},\\
u^i&=\lambda^{\frac{1}{2}}\bar{u}^i,\\
A_i&=\bar{A}_i,\\
F_{ij}&=\bar{F}_{ij},\\
F^{ij}&=\lambda^2\bar{F}^{ij},\\
e&=\bar{e},\\
c&=\bar{c},\\
\hbar&=\bar{\hbar},\\
\delta^4&=\bar{\delta}^4,\\
\varphi&=\lambda\bar{\varphi}.
\end{aligned} \quad (10)$$

For completeness φ, which has the dimensions of G^{-1} or $L^{-3}T^{+2}M^{+1}\sim T^{-2}$, is included in the above list. Making the above transformations we have

$$L=\lambda^2\bar{L}, \quad (11)$$

where $\bar{L}$ is L measured in the new units, obtained from L by replacing all quantities by barred quantities.

The transformation of the scalar curvature R under a conformal transformation is a well-known problem in Riemannian geometry. The result is[5] that

$$R=\lambda(\bar{R}+3\bar{\Box}\ln\lambda-\tfrac{3}{2}\lambda^{-2}\lambda_{,i}\lambda_{,j}\bar{g}^{ij}), \quad (12)$$

where

$$\bar{\Box}\ln\lambda=\frac{1}{(-\bar{g})^{\frac{1}{2}}}((-\bar{g})^{\frac{1}{2}}\bar{g}^{ij}\lambda^{-1}\lambda_{,i})_{,j}. \quad (13)$$

In similar fashion the last term in Eq. (5) is easily transformed to give

$$\frac{\varphi_{,i}\varphi^{,i}}{\varphi}=\lambda^2\frac{\bar{\varphi}_{,i}\bar{\varphi}^{,i}}{\bar{\varphi}}+2\lambda\lambda_{,i}\bar{\varphi}^{,i}+\bar{\varphi}\lambda_{,i}\lambda^{,i}. \quad (14)$$

Substituting Eqs. (11), (12), and (14) in Eq. (5) gives

$$0=\delta\int\left[\bar{\varphi}\bar{R}+3\bar{\varphi}\bar{\Box}\ln\lambda-\tfrac{1}{2}(3+2\omega)\bar{\varphi}\frac{\lambda_{,i}\lambda^{,i}}{\lambda^2}-2\omega\frac{\lambda_{,i}\bar{\varphi}^{,i}}{\lambda}-\omega\frac{\bar{\varphi}_{,i}\bar{\varphi}^{,i}}{\bar{\varphi}}+\frac{16\pi}{c^4}\bar{L}\right](-\bar{g})^{\frac{1}{2}}d^4x. \quad (15)$$

One should be reminded that λ in Eq. (15) may be regarded as a known and given function of coordinates, or alternatively as a function of φ. We are interested principally in a choice of λ, a function of φ, that results in $\bar{\varphi}$ being constant. This is

$$\lambda=\varphi/\bar{\varphi}, \quad (16)$$

where $\bar{\varphi}$ is constant, the value of φ at some arbitrarily chosen point. As $\bar{\varphi}$ is now constant, the variational equation becomes, after dropping the ordinary divergence $(-\bar{g})^{\frac{1}{2}}\bar{\Box}\ln\lambda$,

$$0=\delta\int\left[\bar{R}-\tfrac{1}{2}(2\omega+3)\frac{\lambda_{,i}\lambda^{,i}}{\lambda^2}+\frac{16\pi}{c^4\bar{\varphi}}\bar{L}\right](-\bar{g})^{\frac{1}{2}}d^4x. \quad (17)$$

λ must now be considered a dynamic variable, and it must be varied in Eq. (17). In addition to the above second term, λ occurs explicitly only in $\bar{m}=m\lambda^{-\frac{1}{2}}$, ($m$ constant).

To cast this variational principle in a form completely familiar, write

$$0=\delta\int\left[\bar{R}+\frac{16\pi G_0}{c^4}(\bar{L}+\bar{L}_\lambda)\right](-\bar{g})^{\frac{1}{2}}d^4x, \quad (18)$$

with $G_0=\bar{\varphi}^{-1}$ and

$$\bar{L}_\lambda=-\frac{(3+2\omega)c^4\lambda_{,i}\lambda^{,i}}{32\pi G_0\lambda^2}. \quad (19)$$

[5] J. L. Synge, *Relativity, The General Theory* (North-Holland Publishing Company, Amsterdam, 1960), p. 318.

Now λ is to be considered a "matter field" and the total Lagrangian density of "matter" is $\bar{L}+\bar{L}_\lambda$. Varying $\bar{g}_{ij}$ in Eq. (18) gives Einstein's field equations

$$\bar{R}^{ij}-\tfrac{1}{2}\bar{g}^{ij}\bar{R}=\frac{8\pi G_0}{c^4}\bar{\mathfrak{T}}^{ij}, \tag{20}$$

where

$$\bar{\mathfrak{T}}^{ij}=\bar{T}^{ij}+\bar{\Lambda}^{ij}=\frac{2}{(-\bar{g})^{\frac{1}{2}}}\frac{\partial}{\partial\bar{g}_{ij}}[(-\bar{g})^{\frac{1}{2}}(\bar{L}+\bar{L}_\lambda)] \tag{21}$$

is the energy-momentum tensor of matter. It satisfies the local conservation relation

$$(\bar{\mathfrak{T}}^{ij})_{;j}=0. \tag{22}$$

Varying Eq. (17) with respect to λ gives

$$\bar{\Box}(\ln\lambda)=-\frac{16\pi}{(2\omega+3)\bar{\varphi}c^4}\lambda\frac{\partial\bar{L}}{\partial\lambda}=\frac{8\pi}{\bar{\varphi}c^4(3+2\omega)}\bar{T}, \tag{23}$$

with

$$\bar{T}=\bar{T}_i{}^i=\bar{g}_{ij}\frac{2}{(-\bar{g})^{\frac{1}{2}}}\frac{\partial}{\partial\bar{g}_{ij}}((-\bar{g})^{\frac{1}{2}}\bar{L}). \tag{24}$$

The second equality of Eq. (23) follows explicitly from the form of Eq. (6).

If the transformation relation

$$\bar{T}=\lambda^{-2}T \tag{25}$$

is substituted in Eq. (24), one obtains

$$\Box\lambda=\frac{8\pi}{\bar{\varphi}c^4(3+2\omega)}T, \tag{26}$$

which is equivalent to Eq. (13) of reference (1).

While the representation of the theory used here has certain advantages over that of reference 1, it is clumsy in some ways. For example, freely falling matter does not move on geodesics of the geometry, although light rays still follow null geodesics. Also, the measures provided by rods and clocks are not invariant in this geometry. For example, in this formalism the gravitational red shift appears only partially as a metric phenomenon, the remainder of the effect being described as due to a "real" change in the energy levels of an atom with λ.

Consider the motion of an unchanged spinless particle in the gravitational field. The equations of motion obtained from the variational principle, Eq. (15), are

$$\frac{d}{d\bar{\tau}}(\bar{m}\bar{g}_{ij}\bar{u}^j)-\tfrac{1}{2}\bar{m}\bar{g}_{jk,i}\bar{u}^j\bar{u}^k+\bar{m}_{,i}=0. \tag{27}$$

The last term represents a nongravitational force and results in a nongeodesic motion of the particle. The inverse of the units transformation applied to Eq. (27) gives the expected geodesic equation of the old geometry

$$\frac{d}{d\tau}(mg_{ij}u^j)-\tfrac{1}{2}mg_{jk,i}u^ju^k=0. \tag{28}$$

Note that the gravitational constant measured by a Cavendish experiment would not be G_0 of Eq. (20) as it does not include the effect of the "nongravitational" interaction with the λ field.

While dimensional arguments were used above to obtain the λ dependence of inertial mass, the existence of such a dependence is a consequence of dynamical considerations and is not a separate assumption. Quite generally, the mass of a particle varies, being a function of the potential, if it interacts with a scalar field. This can be seen by starting with the variation principle

$$0=\delta\int[m(-\bar{u}^i\bar{u}_i)^{\frac{1}{2}}+m\psi]d\bar{\tau}, \tag{29}$$

where $m\psi$ is the scalar potential and the mass m is assumed to be constant. The variations in Eq. (29) may not be taken arbitrarily but must be subject to the constraint

$$\bar{u}^i\bar{u}_i=-1. \tag{30}$$

The resulting equation of motion is identical with Eq. (27) with

$$\bar{m}=m\psi. \tag{31}$$

If the variational equation is taken as

$$0=\delta\int\bar{m}(-\bar{u}^i\bar{u}_i)^{\frac{1}{2}}d\bar{\tau}. \tag{32}$$

The same equation of motion is obtained, as the Euler equation, without the necessity for introducing the constraint explicitly.

SUMMARY AND CONCLUSION

The field equations compatible with Mach's principle which were previously formulated,[1] are here transformed in such a way that the required modification appears as part of the nongravitational field, Einstein's field equations being valid. The rest masses of all particles are affected by an interaction with a scalar field. This interaction reduces the masses of the particles and the gravitational coupling constant, Eq. (4), may be interpreted as small because the particle mass $\bar{m}$ is reduced drastically by interaction with the field, generated by the enormous amounts of matter in the universe.

In similar fashion the relation[6]

$$GM/Rc^2\sim 1 \tag{33}$$

is understandable. (M is the mass of the universe out to visible limits, and R, the Hubble radius, is a measure of the radius of this visible portion.) Measured in the new units, the masses of elementary particles adjust themselves, through the scalar field generated by all the

[6] D. W. Sciama, Monthly Notices Roy. Astron. Soc. **113**, 34 (1953), R. H. Dicke, Am. Scientist **47**, 25 (1959), R. H. Dicke, Science **129**, 621 (1959).

other matter, in such a way that the ratio M/R stays constant and of the order of magnitude of $G^{-1}c^2$.[1]

APPENDIX 1

Transformation of the Metric of a Riemannian Space to that of a Flat Space

As a first step a coordinate system, time orthogonal, is chosen. This can be done, at least for a finite coordinate patch, by erecting a family of geodesic curves normal to any space-like surface and using it to define a second space-like surface everywhere equidistant from the first. Corresponding points on the two surfaces, labeled by the same space-like coordinates x^1, x^2, x^3, are points joined by the same normal geodesic curve. The time coordinate is assigned different values on each of the two surfaces. The procedure can be iterated to assign coordinates to all points in the coordinate patch.

Let g_{ij} be the metric tensor in a particular time orthogonal coordinate system. Then

$$g_{0\alpha}=0, \quad \alpha=1, 2, 3. \tag{34}$$

Introduce the tensor $T_i{}^j$ having the inverse $\tilde{T}_i{}^j$

$$T_i{}^j\tilde{T}_j{}^k=\delta_i{}^k, \tag{35}$$

such that for *this particular coordinate system* the tensor (interpreted as a matrix) is orthogonal,

$$T_i{}^j=\tilde{T}_j{}^i. \tag{36}$$

It is a well-known theorem of matrix algebra that any symmetric matrix can be diagonalized by an orthogonal transformation. Hence, it is always possible to so choose $T_i{}^j$ that

$$T_k{}^ig_{ij}T_l{}^j=\bar{g}_{kl}, \tag{37}$$

with $\bar{g}_{kl}$ purely diagonal in this coordinate system.

Corresponding to the coordinate intervals dx^i, one can define new intervals through the transformation

$$d\bar{x}^i=\tilde{T}_j{}^idx^j. \tag{38}$$

It must be emphasized that this transformation does not generally represent a coordinate transformation.

The infinitesimal separation of two neighboring points is ds with

$$ds^2=g_{ij}dx^idx^j. \tag{39}$$

This can now be given as

$$ds^2=\bar{g}_{ij}T_m{}^iT_n{}^j\tilde{T}_k{}^m\tilde{T}_l{}^ndx^kdx^l=\bar{g}_{ij}d\bar{x}^id\bar{x}^j. \tag{40}$$

As $\bar{g}_{ij}$ is diagonal, the intervals $d\bar{x}^i$ are all mutually orthogonal. Now by redefining the measure of time, and of length along the three mutually perpendicular space-like directions $d\bar{x}^\alpha$, $\bar{g}_{ij}$ can be transformed into any other diagonal tensor of the same signature. In particular, it can be transformed into the Minkowskian metric tensor η_{ij}. The resulting measure of interval between the two points is

$$d\bar{s}^2=\eta_{ij}d\bar{x}^id\bar{x}^j. \tag{41}$$

Because the original coordinate system was time orthogonal, the orthogonal transformation

$$d\bar{x}^i=\tilde{T}_j{}^idx^j \tag{42}$$

represents a space-like local rotation, hence the Minkowskian tensor η_{ij} is invariant under this transformation. Consequently,

$$d\bar{s}^2=\eta_{ij}d\bar{x}^id\bar{x}^j=\eta_{ij}dx^idx^j. \tag{43}$$

Note that Eq. (43) gives a new measure of interval and a *new metric tensor* for the *old coordinate system.*

The new measure of interval leads to a flat space with a Minkowskian metric tensor. It should be noted that the transformed coordinate intervals are mutually orthogonal both before and after units are redefined. Hence, the criteria for orthogonality of these vectors are independent of units and the condition of local orthogonality may be meaningfully imposed.

Null geodesics are generally *not* invariant under this transformation of units. The velocity of light varies, being a function of both coordinates and spatial directions. Physically, with the redefined units, space might be considered to have some of the electromagnetic properties of an anisotropic medium. However, these properties can be eliminated by a units transformation and they are without a physical significance, invariant under this group. It should be noted that the same objection, based on considerations of invariance under units transformations, can be leveled against the reading of physical significance into the geometrical invariants. These "invariants," such as the scalar curvature, are not invariant under a units transformation.

Because of the various nonequivalent ways of establishing standards of mass, length, and time within the framework of the Brons-Dicke theory, invariance under the units transformation group is particularly important. This is of lesser importance in standard general relativity. However, even here it is possible, in principle, to construct rods and clocks whose units are dependent upon some scalar field variable such as a curvature "invariant" or a Maxwell invariant.