Energy and Orbital Angular Momentum Dependence of Coulomb-Distorted (d, p) Angular Distributions*

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Measurements are reported of the angular distributions of six proton groups from the $Pb^{206}(d,p)Pb^{207}$ reaction obtained at deuteron energies of 12.5 and 14 Mev. When these measurements were combined with previously published measurements, the energy dependence of the experimental angular distributions of eight proton groups from this reaction was obtained over the range of bombarding energies from 8.3 to 15 Mev. These proton groups correspond to neutron capture by Pb^{206} with orbital angular momentum transfers of 0, 1, 2, 3, 4, and 6. Preliminary distorted-wave calculations are reported that are intended to provide a qualitative comparison with the experimental results. When the best distorted-wave fit is obtained for the angular distribution of the ground-state reaction at 14 Mev, qualitative fits for the energy dependence of this group and the l dependence of other groups at 14 Mev are predicted by the theory without a change in parameters. With the particular parameters chosen, the predicted relative cross sections agree approximately with experiment with one exception; however the predicted absolute cross sections are too small. Different sets of deuteron parameters appear to give best fits to the elastic (d,d) and the (d,p) angular distributions, although a more extensive parameter search might yield a parameter set fitting both. The present comparison suggests that nuclear spectroscopic information may still be obtained in heavy elements by means of the deuteron stripping reaction, provided accurate experimental angular distributions are obtained with deuteron beams of high resolution and energies in the range of 15 to 25 Mev.

I. INTRODUCTION

THE deuteron stripping reaction has been successful as a spectroscopic tool for the study of the properties of nuclear energy levels in light nuclei. However, the original theoretical formulations of this process[1,2] included a number of approximations that invalidated its use for heavier nuclei. The subsequent development of the distorted-wave stripping theory[3] improved the details of the theoretical fits to the data in light nuclei, and suggested comparisons in heavier nuclei where Coulomb effects are serious. This paper examines certain representative (d,p) reactions in a heavy nucleus experimentally, and compares the results with distorted-wave stripping calculations. The qualitative success of this comparison suggests the possibility of extended use of the stripping reaction for nuclear spectroscopic purposes in medium and heavy nuclei.

In the simple theories of the (d,p) stripping reaction, the qualitative features of the experimental angular distributions of the emerging protons are predicted successfully for light nuclei by describing the incoming deuteron and outgoing proton by their plane-wave asymptotic limits. However, when detailed comparisons are made, certain difficulties are apparent. The minima in the oscillatory angular distributions are not zero as predicted, a failure originally attributed to a contribution from compound-nucleus formation. In a number of cases good fits can be obtained only by assuming an impure bound state configuration containing different orbital angular momenta. Backward peaks in the angular distributions are often observed, a discrepancy generally explained by assuming contributions from "heavy-particle stripping."[4] Although the simple plane-wave theories predict zero polarization for the emitted protons, very sizable polarizations are found. Finally, the reduced widths extracted from the observed cross sections are usually too small.

Although most of the differences in the results described above can be explained by various plausible modifications, the simple plane-wave theory cannot be applied to heavier nuclei. In the lower energy regions, complications involving the interference of compound nucleus formation and the stripping process are apparent in nuclei as light as carbon,[5] and these problems can be expected to become more serious as the nuclear level density starts to increase. Furthermore, the failure to include either Coulomb or nuclear interactions between the incident deuteron and the target nucleus or the exit proton and the residual nucleus limits the usefulness of the simple theory to the region of light nuclei.

A number of more detailed calculations that include the distortions of the incoming and outgoing waves by the Coulomb and nuclear potentials have been made by Tobocman and others.[3,6–9] These distorted-wave

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[1] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[2] A. B. Bhatia, K. Huang, R. Huby, and H. C. Newns, Phil. Mag. **43**, 485 (1952).

[3] W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955). Also W. Tobocman, *ibid.* **115**, 99 (1959).

[4] L. Madansky and G. E. Owen, Phys. Rev. **99**, 1608 (1955).

[5] M. T. McEllistrem, Phys. Rev. **111**, 596 (1958). Also M. T. McEllistrem, K. W. Jones, R. Chiba, R. A. Douglas, D. F. Herring, and E. A. Silverstein, *ibid.* **104**, 1008 (1956).

[6] J. Horowitz and A. M. L. Messiah, J. phys. radium **15**, 142 (1954).

[7] H. C. Newns and M. Y. Refai, Proc. Phys. Soc. (London) **71**, 627 (1958).

[8] W. R. Gibbs and W. Tobocman, Bull. Am. Phys. Soc. **6**, 295 (1961).

[9] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory, Physics Division Annual Progress Report, ORNL–3085, 1961 (unpublished).

calculations show that, in general, the repulsive Coulomb potential tends to shift the maxima of the characteristic stripping angular distributions to larger angles, while the attractive nuclear potential tends to shift the peaks to smaller angles. Therefore, a cancellation often results that causes the maxima of the angular distributions to look very much like the predictions of the simple plane-wave theory in the lighter nuclei, although the cross sections are off by fairly large factors.

These distorted-wave calculations are a distinct improvement over the simple theory for light nuclei.[10] Nonzero minima occur naturally in the calculations, and it is found that the assumption of an impure bound state containing different orbital angular momenta is usually not necessary. In some cases the backward peaks appear naturally, so that heavy-particle stripping is not necessarily required. Nonzero polarizations for $l\neq 0$ are predicted, even without the presence of spin-orbit contributions in the distorting potentials. Finally, the reduced widths are found to be considerably closer to the values obtained by other methods, e.g., neutron scattering.

These improvements indicated that a detailed experimental comparison should be made with the distorted-wave theory in heavier nuclei where Coulomb effects are important. This comparison should *not* be made where strongly energy-dependent nuclear distortions are expected, such as compound-nucleus direct-interaction interference effects in nuclei with intermediate level densities.[11] A logical choice seemed to be the lead region, where, if compound nucleus formation occurs at all, the high level density of the compound system should give a cancellation of interference terms and cause the nuclear distortions to vary slowly with energy.

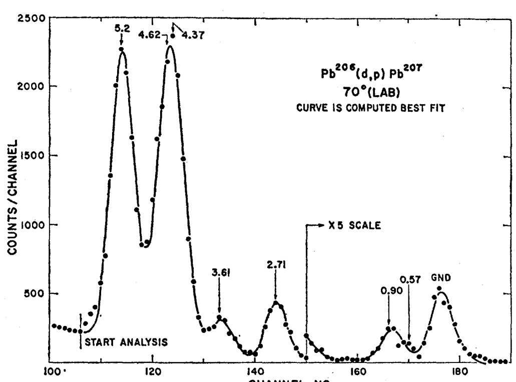

FIG. 1. Gated pulse-height spectrum of protons from the $Pb^{206}(d,p)Pb^{207}$ reaction obtained from the E scintillation counter. The single-hole states of Pb^{207} are quite weak, as indicated by the scale change of five. The solid curve is the best fit obtained by a least-squares analysis. The two unresolved pairs of groups were analyzed as if they were single groups.

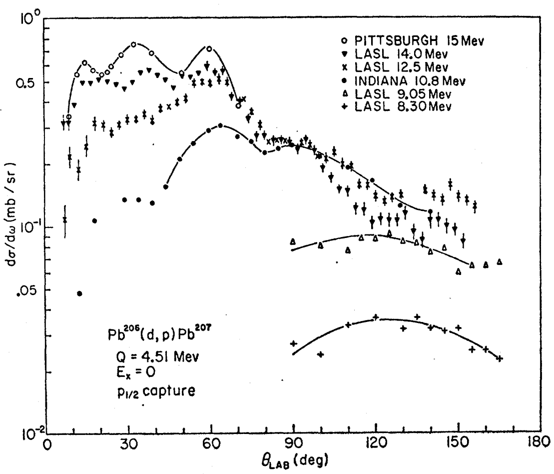

FIG. 2. Experimental angular distributions obtained at various laboratories and deuteron energies for the $Pb^{206}(d,p)Pb^{207}$ ground-state reaction, representing $p_{\frac{1}{2}}$ neutron capture. The results of the present experiment are shown by triangles and X's, and the previously published data are drawn in with solid curves.

Furthermore, the low-lying levels in the immediate vicinity of Pb^{208} are single-particle and single-hole states and are readily accessible to study by the stripping reaction with cyclotron techniques. The $Pb^{206}(d,p)Pb^{207}$ reaction is particularly useful because a variety of these states is available, corresponding to neutron orbital angular momentum transfers of 0, 1, 2, 3, 4, and 6.[12,13]

Aside from early exploratory investigations,[14,15] the first extensive attempt to establish the usefulness of stripping reaction in the lead region was carried out at Indiana University at a bombarding deuteron energy of 10.8 Mev.[12,13] Although serious distortions were obvious, assignments consistent with previous results and the shell model were possible based upon the maximum cross sections observed and their angular positions. Subsequent investigations at the University of Pittsburgh[16] at 15 Mev and at Los Alamos[17] at 9.1, 8.3, and 7.4 Mev also resulted in distorted angular distributions, but of a different character determined by the bombarding energy employed.

Deuteron energies of 14 Mev (the maximum available on the Los Alamos variable-energy cyclotron) and 12.5 Mev were chosen for the investigation reported in this paper to extend the energy above the original Indiana University experiment. The Pittsburgh University and low-energy Los Alamos results became available during the course of the work, so that the combination of all

[10] G. R. Satchler, Bull. Am. Phys. Soc. **3**, 298 (1961).

[11] See, for example, J. A. Kuehner, E. Almqvist, and D. A. Bromley, *Proceedings of the International Conference on Nuclear Structure, Kingston* (University of Toronto Press, Toronto, 1960), p. 378.

[12] M. T. McEllistrem, H. J. Martin, D. W. Miller, and M. B. Sampson, Phys. Rev. **111**, 1636 (1958).

[13] G. B. Holm, J. R. Burwell, and D. W. Miller, Phys. Rev. **118**, 1247 (1960).

[14] H. E. Gove, Phys. Rev. **81**, 364 (1951).

[15] N. S. Wall, Phys. Rev. **96**, 670 (1954).

[16] B. L. Cohen, R. E. Price, and S. Mayo, Nuclear Phys. **20**, 370 (1960).

[17] R. H. Stokes, Phys. Rev. **121**, 613 (1961).

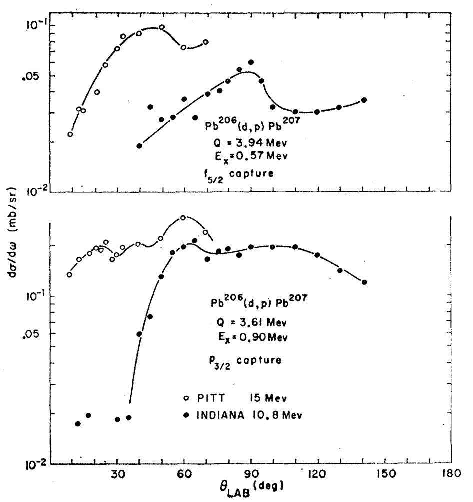

FIG. 3. Magnetic spectrometer results for the $Pb^{206}(d,p)$ reactions exciting the 0.57- and 0.90-Mev states of Pb^{207}. (Data obtained at Indiana and Pittsburgh.) These reactions correspond to $f_{\frac{5}{2}}$ and $p_{\frac{3}{2}}$ neutron capture, respectively.

four sets of experiments gave a consistent picture of the energy dependence of the angular distributions over the range of bombarding energies from 8.3 to 15 Mev. Distorted-wave stripping calculations were also carried out, and, for the same conditions, angular distributions similar to the experimental results were obtained. This comparison suggests that the stripping reaction can be used as a spectroscopic tool in the heavy-element region within certain limitations.

II. EXPERIMENTAL PROCEDURE

The details of the scattering chamber and detection apparatus have been described previously.[18] Deuterons of 12.5 or 14 Mev from the cyclotron were incident on an evaporated radiogenic lead target of thickness $\sim$3.8 mg cm^{-2}.[19]

The beam energy was determined by a range measurement in a nuclear emulsion, and was monitored with a beam energy monitor.[18] Protons emitted from the target were detected and separated from deuterons by a dE/dx and E counter system which could be rotated through the angular range from 0° to 169°. The detector acceptance angle was $\pm 1°$ at the forward angles and $\pm 2°$ at the backward angles. Bismuth absorbers were used at the forward angles to stop elastic deuterons; however, to insure no difficulties from multiple scattering, absolute cross sections were determined at larger angles without the use of absorbers. The spectrum of particles incident on the scintillation counter was displayed on a 100-channel analyzer.

Figure 1 shows a typical 100-channel gated pulse-height spectrum of protons from the $Pb^{206}(d,p)Pb^{207}$ reaction obtained in the present work. The solid curve represents the best fit to the spectrum obtained by a least-squares computer analysis.[20] This computer analysis determines the number of counts under Gaussian peaks superimposed on an exponential background. A similar analysis was made of the spectrum obtained at each laboratory angle. Angular distributions were determined in this way for groups corresponding to individual nuclear states in Pb^{207} in every case except for the 0.57- and 0.90-Mev states and the 4.37- and 4.62-Mev states which were not resolved.[21]

In order to provide a comparison between elastic scattering and stripping when the distorted-wave theory was used, deuteron elastic scattering angular distributions were also obtained experimentally with the identical arrangement by gating the 100-channel analyzer to accept only deuterons.

III. RESULTS

Figure 2 represents a compilation of the angular distributions measured for the $Pb^{206}(d,p)Pb^{207}$ ground-state reaction at various laboratories. The results labeled "Pittsburgh" refer to the 1960 paper by Cohen, Price, and Mayo[16]; "LASL (14 and 12.5 Mev)" to the present work; "Indiana" to the 1958 paper by McEllistrem, Martin, Miller, and Sampson[12]; and "LASL (9.05 and 8.30 Mev)" to the 1961 paper by Stokes.[17] It should be emphasized that absolute cross sections are plotted in all of the experimental figures, so that changes in absolute cross sections as well as changes in angular dis-

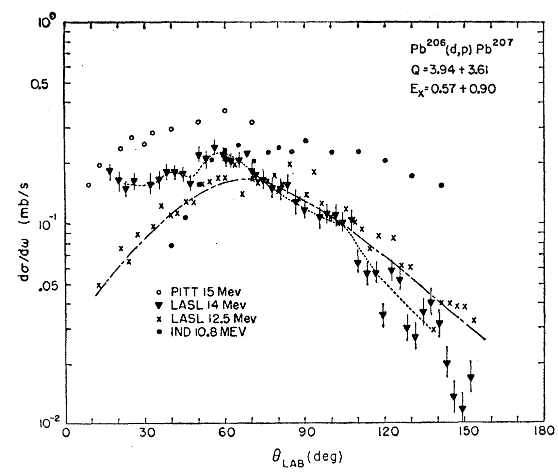

FIG. 4. Angular distributions obtained for the sum of the proton groups leading to the 0.57- and 0.90-Mev states of Pb^{207}, which were not resolved in the present scintillation counter experiment (see Fig. 1). The Indiana and Pittsburgh results of Fig. 3 have been summed and replotted here for comparison, and some curves sketched to aid in data identification.

[18] H. E. Wegner and W. S. Hall, Phys. Rev. **119**, 1654 (1960).

[19] The radiogenic lead sample was 88% Pb^{206}. The target thickness was determined both by weighing and observing the Rutherford scattering yield. Correction for the 88% composition is included in the cross sections plotted in the figures.

[20] P. McWilliams, W. S. Hall, and H. E. Wegner, Rev. Sci. Instr. (to be published).

[21] The 5.20-Mev state of Pb^{207} is also probably a doublet which was not resolved even in the Indiana and Pittsburgh magnetic spectrometer experiments described in references 12 and 16.

tribution shapes are of interest. The error in absolute cross section in the present experiment is due to target nonuniformities and background uncertainties and amounts to $\pm 15\%$. The error in relative cross section is due to statistics and analysis limitations caused by the background and limited resolution. These errors in relative cross section are indicated in the figures or are less than the size of the points.

Figure 2, which represents $p_{\frac{1}{2}}$ neutron capture to the ground state of Pb^{207}, shows the most detailed oscillatory structure of all the angular distributions. At 15-Mev deuteron energy, three prominent peaks appear at forward angles. The two forward peaks persist at 14 Mev, but start to disappear at 12.5 Mev. At 10.8 Mev the third peak remains, but the more forward structure has largely disappeared. At the backward angles, the structure does not change appreciably from 14 to 10.8 Mev, and a fourth peak at about 90° maintains essentially constant cross section. At energies of 9.05 and 8.30 Mev, the backward cross sections decrease, although a broad maximum moves toward backward angles at lower energies.

The solid curves of Fig. 3 represent the Pittsburgh and Indiana magnetic spectrometer data for the first- and second-excited states of Pb^{207}, corresponding to $f_{\frac{5}{2}}$ and $p_{\frac{3}{2}}$ neutron capture, respectively. The $l=3$ transition in the upper half of the figure shows less structure than either of the $l=1$ transitions, but the $l=3$ peaks are more obvious than will be seen in subsequent figures. The $p_{\frac{3}{2}}$ capture plotted in the lower half of the figure shows the same triple-peaked forward structure at 15-Mev deuteron energy observed in the $p_{\frac{1}{2}}$ ground-state capture, except that the forward peaks are not as pronounced.[16]

Figure 4 shows the results of the present scintillation counter work that was not able to resolve the first- and

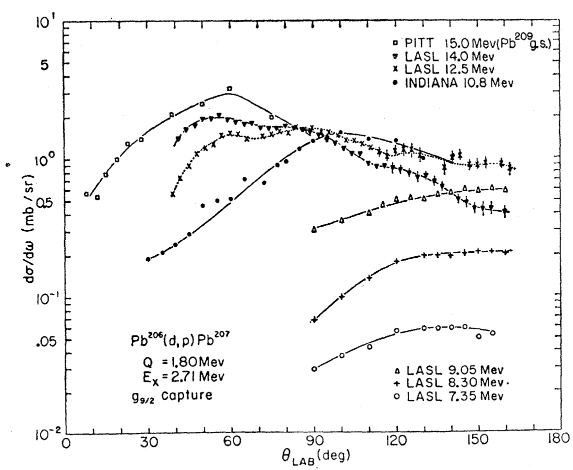

FIG. 5. Experimental results obtained for the $Pb^{206}(d,p)$ reaction leading to the first single-particle state of Pb^{207} at 2.71-Mev excitation, representing $g_{9/2}$ neutron capture. The Pittsburgh data shown in this and subsequent figures actually represent their results for (d,p) reactions exciting the corresponding single-particle states of Pb^{209}, as explained in the text.

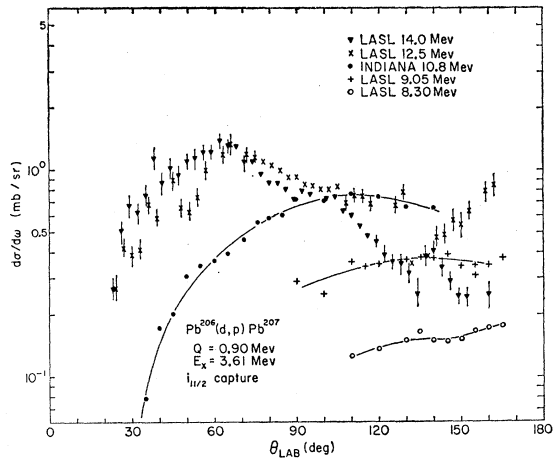

FIG. 6. Experimental angular distributions obtained for the $Pb^{206}(d,p)$ reaction corresponding to capture of an $i_{11/2}$ neutron into the 3.61-Mev state of Pb^{207}.

second-excited states. For comparison, the Indiana and Pittsburgh results for these two states were added and are also plotted on the figure. Although the errors on these weak states are large, it is apparent that the forward angle cross sections decrease and the backward angle cross sections increase as the deuteron energy is lowered from 15 to 10.8 Mev.

The three states discussed represent single-hole states in Pb^{207}. Figure 5 shows the results obtained for the first single-particle state in Pb^{207} at 2.71-Mev excitation, which corresponds to $g_{9/2}$ neutron capture. All Pittsburgh data shown in this and subsequent figures were obtained for the single-particle states of Pb^{209} that correspond to the single-particle states of Pb^{207} studied here.[22] The forward angle points of the present work are omitted for all angular distributions corresponding to levels with $Q<+2$ Mev. In this energy range it became possible for elastically scattered deuterons stopped in the absorber in front of the counter to produce (d,p) reactions in large enough quantities to obscure the protons from the target.

Less structure is observed for the $l=4$ transition shown in Fig. 5 than for the $l=1$ and $l=3$ transitions of the previous figures. However, the decrease of the forward angle cross sections with decreasing deuteron energy is apparent, followed by a decrease in the absolute cross section at back angles with a broad maximum moving toward 180°. A similar over-all behavior is exhibited in Fig. 6 for the group corresponding to $i_{11/2}$ neutron capture to the 3.61-Mev level of Pb^{207}.

Neutron assignments for the next three states studied are not certain. The Indiana group[12,13] has proposed that the 4.37-Mev state in Pb^{207} is $d_{\frac{5}{2}}$, the 4.62-Mev state is $s_{\frac{1}{2}}$, and the 5.20-Mev state is $d_{\frac{3}{2}}$ and $g_{7/2}$ superimposed. On the other hand, the Pittsburgh group[16] suggests somewhat different assignments for the corresponding

[22] See Fig. 6 of reference 13 for a summary of the corresponding states of Pb^{207}, Pb^{209}, and Bi^{210}.

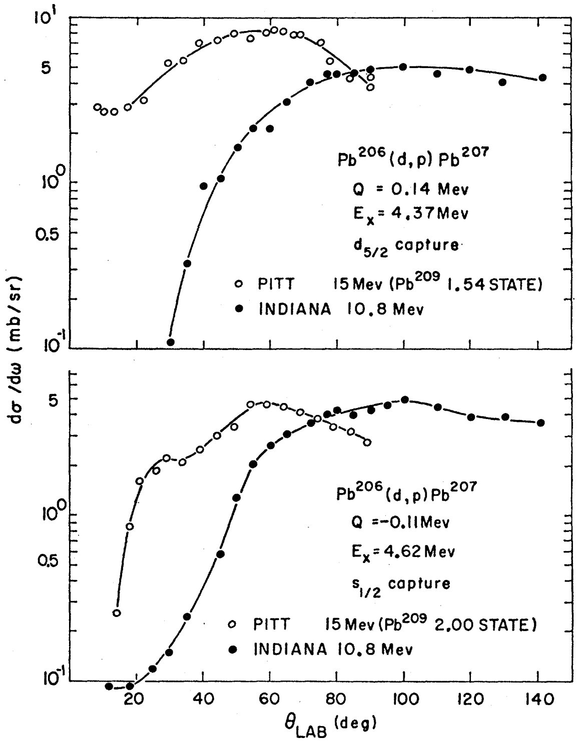

FIG. 7. Magnetic spectrometer results for the $Pb^{206}(d,p)$ reactions exciting the 4.37- and 4.62-Mev states of Pb^{207}, obtained at Indiana and Pittsburgh. The neutron capture assignments are assumed in this work to be $d_{\frac{5}{2}}$ and $s_{\frac{1}{2}}$ as discussed in the text.

states of Pb^{209}. Translating these assignments to the corresponding states of Pb^{207} under discussion here, assuming a one-to-one correspondence, the 4.62-Mev state is $g_{7/2}$, whereas the $s_{\frac{1}{2}}$ state is unresolved either from the 4.37-Mev or from the 5.20-Mev state. The principal argument of the Pittsburgh group against an $s_{\frac{1}{2}}$ assignment for the equivalent to the 4.62-Mev state is that the cross section drops rapidly at forward angles. However, as will be shown later, this behavior is expected for $s_{\frac{1}{2}}$ capture from distorted-wave stripping calculations in this region.[9,10] Therefore, in the following discussion, the Indiana assignments will be quoted.

The previously published magnetic spectrometer results for the 4.37- and 4.62-Mev states of Pb^{207}, corresponding to $d_{\frac{5}{2}}$ and $s_{\frac{1}{2}}$ neutron capture, respectively, are shown in Fig. 7. The rapid decrease in cross section at forward angles for the assumed $l=0$ transition is apparent, although a leveling off at very small angles is suggested. Because the Los Alamos scintillation counter results did not resolve these two states, the angular distributions for the doublet are shown in Fig. 8. Again the Indiana and Pittsburgh results were summed for these two states and plotted in Fig. 8 to provide a reasonable comparison. The systematic changes of the forward and backward angle cross sections follow the general behavior mentioned in the discussion of previous figures.

The results obtained for the group corresponding to the 5.20-Mev state of Pb^{207} are shown in Fig. 9. This group is apparently a doublet, not resolved in the magnetic spectrometer measurements, made up mostly of $d_{\frac{3}{2}}$ along with some $g_{7/2}$ neutron capture. Figures 7 and 8 show that the angular distributions are not appreciably different from the other $l=2$ case.

IV. DISTORTED-WAVE STRIPPING CALCULATIONS

To provide a comparison with the experimental results of the previous section, distorted-wave stripping calculations were carried out with the code developed at the Rice Institute by Gibbs and Tobocman and adapted for the Los Alamos computers by Swartz and Rodberg. The experimental data covering the complete angular region and exhibiting the most structure were those for the ground-state angular distribution measured at 14-Mev deuteron energy. This distribution was calculated initially with the usual optical-model parameters for the proton and similar parameters for the deuteron. With these initial conditions it was found that changes in the deuteron potential affected the (d,p) angular distributions more strongly than similar changes in the proton potential. The proton parameters were then arbitrarily held at the usually accepted values determined by elastic scattering and the deuteron parameters were adjusted in order to achieve a fit. One of the most pronounced effects found in this adjustment was the increase of oscillatory structure in the (d,p) angular distribution as the imaginary part of the deuteron potential (W_d) was decreased.

The parameters were interdependent to the extent that unexpected changes in the predicted angular distributions occurred when more than one parameter at a time was changed. Because the deuteron parameters

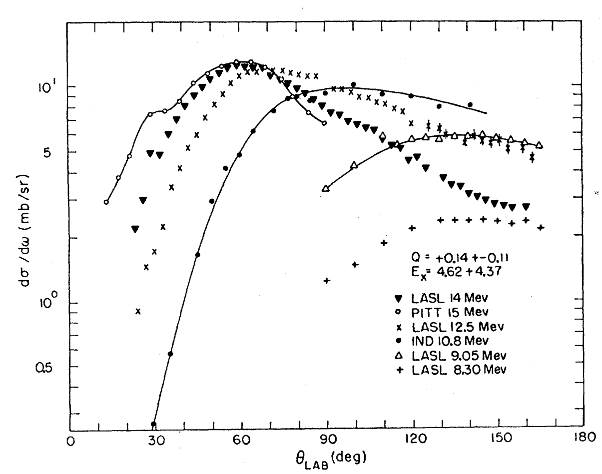

FIG. 8. Angular distributions obtained for the sum of the proton groups leading to the 4.37- and 4.62-Mev states of Pb^{207}, which were not resolved in the present or earlier Los Alamos scintillation counter work. For comparison, the Indiana and Pittsburgh data in Fig. 7 have been summed and replotted in this figure. The curves are not those of the original authors, but are sketched in this figure for easier data identification.

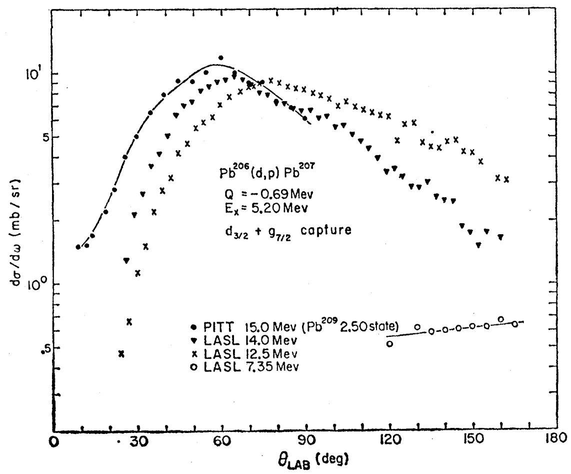

FIG. 9. Experimental results obtained for the $Pb^{206}(d,p)$ reaction leading to the 5.20-Mev state of Pb^{207}. This state is apparently a $d_{\frac{3}{2}}+g_{7/2}$ doublet that has not been resolved in any of the measurements to date.

giving the best fit to the elastic scattering data did not predict a reasonable fit for the ground-state (d,p) angular distribution, the parameters were arbitrarily adjusted to achieve the best (d,p) fit commensurate with reasonable computer time requirements. In particular, the average slope throughout the angular range and the correct angular location of maxima and minima were fitted with a visual estimate of quality of fit.

The purpose of this preliminary investigation was to determine if the distorted-wave theory could predict the general features of the angular distributions corresponding to the different angular momenta and the energy dependence through the region of the Coulomb barrier. A careful analysis would require the determination of the best compromise fits for the different l values and energies and is beyond the scope of this paper.

With this philosophy in mind the potential parameters yielding the best (d,p) fit[23] for the ground state at 14 Mev were then held fixed and two comparisons made: (a) The deuteron energy alone was varied for the ground-state case, and (b) the l, J, and Q values were varied to correspond to the other levels studied at 14 Mev. Because the optical-model parameters are energy dependent to some degree and spin-orbit effects in the distorting potentials (not included in this code) could affect the angular distributions in an unknown way, the comparisons in this paper must be considered as qualitative. However, for a first attempt the results are quite interesting, since the theory predicts most of the salient features of the various experimental angular distributions and indicates that these techniques can probably be used to good advantage in the study of nuclear spectroscopy in the heavy-element region. These results also indicate that this method of calculation seems to be an improvement over the simple Butler model and accounts properly for the Coulomb and nuclear effects in stripping at least to a first approximation.

Figure 10 shows a comparison of the experimental angular distributions for the ground-state reaction ($p_{\frac{1}{2}}$ neutron capture) at various deuteron energies and the angular distributions predicted from the distorted-wave calculations. Experimental points are included for comparison on the single theoretical curve (14 Mev) where the original fit was obtained. The 14-Mev theoretical curve was arbitrarily normalized for best fit, and the same normalization was used in plotting the other theoretical curves in the figure. The three distinct peaks in the forward angle region are not predicted to have as large an oscillation amplitude as observed; however, the general slope of the curve and the angular location of maxima and minima are satisfactory. As the deuteron energy is decreased from 15 to 10.8 Mev, the theoretical curves also show a decrease in the forward cross section, damping of the forward oscillatory structure, persistence of the third and fourth peaks, and a

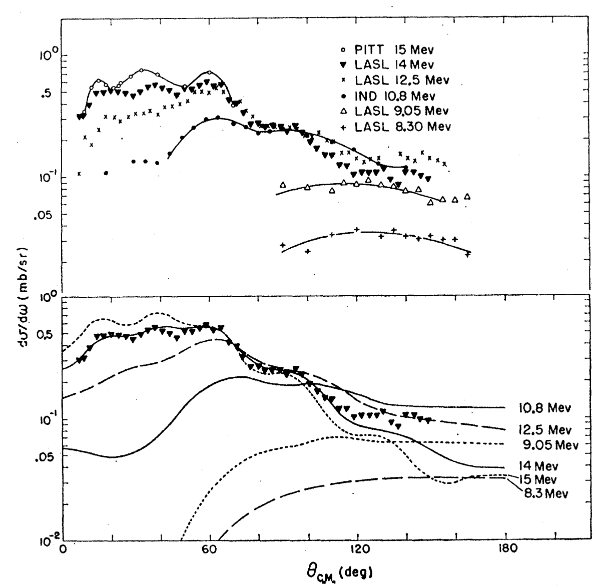

FIG. 10. A comparison of the results of the distorted-wave stripping calculation described in the text and the experimental results of Fig. 2 as the bombarding deuteron energy is varied. The parameters of the calculation and the normalization were adjusted for best fit to the 14-Mev data, as shown by the 14-Mev theoretical curve and data points in the lower half of the figure. The parameters (defined in footnote 23) yielding this fit were $R_n=6.5$ f, $V_d=-50$ Mev, $W_d=-25$ Mev, $A_d=0.7$ f, $R_d=7.5$ f, $V_p=-60$ Mev, $W_p=-8$ Mev, $A_p=0.55$ f, and $R_p=7.12$ f. The other theoretical curves in this figure were obtained by changing the deuteron energy in the calculation, with the normalization and other parameters the same.

[23] The parameters for this fit are given in the caption of Fig. 10. These parameters are expressed in the usual notation, where R_n is the matching radius for the internal and external wave functions, V_d and W_d are the real and imaginary parts of the optical potential representing the deuteron-target interaction, R_d and A_d the radius and diffuseness parameter of this potential, and V_p, W_p, R_p, and A_p the analogous optical parameters for the proton—residual-nucleus interaction.

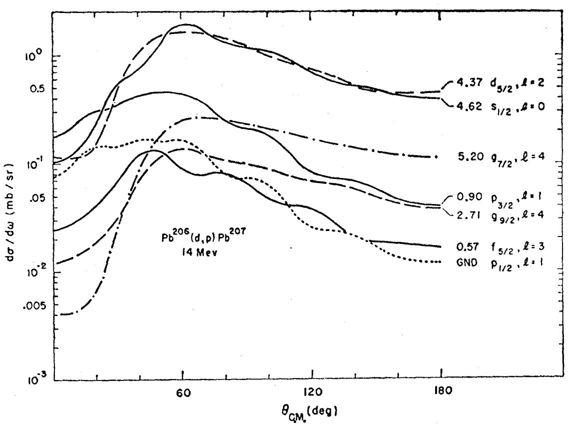

FIG. 11. Angular distributions predicted by distorted-waʌe stripping calculations. The parameters listed in the caption of Fig. 10 are those used except for appropriate Q value and the l and k of the captured neutron. This is the only figure in which the actual cross-section magnitudes predicted by the calculation are plotted; in both Figs. 10 and 12 a common arbitrary normalization of the theoretical curves is used. See footnote 26 for corrections to be applied to the cross sections of the single-hole states.

constant differential cross section at 90°. In addition, at still lower energies the backward cross sections decrease in a manner similar to experiment. The relative magnitudes of the curves are fairly well predicted throughout the deuteron energy range studied.

Figure 11 shows the predicted 14-Mev angular distributions for all[24] of the states of Pb^{207}.[25] Because two pairs of the states were not resolved in the present work, the curves of Fig. 12 were obtained from Fig. 11 by adding the theoretical predictions for the unresolved doublets. In addition, all of the theoretical angular distributions of Fig. 12 are plotted with the same normalization (that giving the best fit to the ground-state data). Qualitative agreement is found between the *shapes* of the theoretical angular distributions and the 14-Mev experimental data (replotted in Fig. 12 from Figs. 2 through 9). It is interesting to note in Figs. 11 and 12 that the oscillatory structure of the various states is distinctive, although small, and that in the case of the 4.37–4.62 Mev doublet the out-of-phase character results in a smooth summed curve in agreement with the experiment.

V. DISCUSSION

There are three aspects to a comparison between experiment and this type of theory: (a) angular distribution *shapes*, (b) *relative* cross sections, and (c) *absolute* cross sections. It would be surprising if an initial attempt of this type should give agreement on all three, since considerable effort has been required in the past to obtain reasonable agreement for all three even in the light nuclei. However, certain aspects of the comparison are quite encouraging.

It has been pointed out in the previous section that good agreement between the *shapes* of the angular distributions was obtained. This is apparent both in Fig. 10, where only the deuteron energy was changed in the calculation of the angular distribution of one particular state, and in Fig. 12, where only the Q's, l's, and j's corresponding to different states were changed. If the theoretical and experimental curves are superimposed, some deviations are observed. In some cases, the predicted average slope is somewhat greater or less than observed, and these detailed differences change in a complex way with various parameter changes. However, considering the preliminary nature of the analysis, the over-all agreement for the angular distribution shapes seems to be quite satisfactory.

The shape comparisons indicate the possibility of determining l values from observed angular distributions. The best-fit parameters may be determined with known levels and then applied to unknown levels and nearby nuclei. Such a procedure may be of use in nuclear spectroscopy. This procedure will become easier as the deuteron energy is raised, because the characteristic structure becomes more pronounced. (See Fig. 10.)

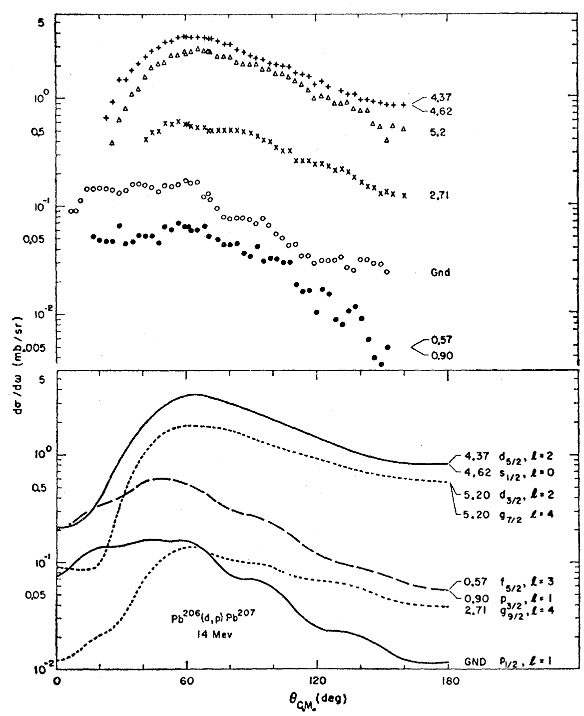

FIG. 12. A comparison of the angular distributions predicted by distorted-wave calculations and the results of the present experiment at 14 Mev. The theoretical curves were obtained from Fig. 11 as described in the text, and the 14-Mev experimental points are reproduced from Figs. 2 through 9 for comparison.

[24] The present code does not include l values as high as 6, so that the $i_{11/2}$ neutron capture to the 3.61-Mev state of Pb^{207} is not included.

[25] The code generates absolute cross sections assuming that the neutron is captured into a pure single-particle state, and therefore, in principle, predicts an upper limit for the absolute cross section. However, the experimental cross sections in this preliminary fitting study were always larger than predicted.

Hence, for nuclei in the lead region, deuteron energies of 20 to 25 Mev would be very useful.

In the 10- to 15-Mev region it is worth noting that the *details* of the oscillations of the predicted curves for different groups are quite different, although the over-all gross features are similar. Therefore, in order to do nuclear spectroscopy in these energy and mass regions, it will be necessary to obtain the experimental angular distributions very accurately out to large angles.

The second aspect of the comparison, relative cross sections, gives only moderate agreement. In Fig. 10, the 14-Mev curve was arbitrarily normalized to fit the data, and the same normalization was used for the rest of the curves. The two halves of this figure show good relative cross section agreement as the deuteron energy is decreased by a factor of 2, in spite of the fact that the parameters of the distorting potentials were kept fixed.

On the other hand, Fig. 12 does not show good agreement of relative cross sections for all of the various angular momentum states measured at 14 Mev. Fair agreement is found for the relative cross sections of the ground state, the 5.20-Mev doublet[11] state, and the 4.37–4.62 Mev doublet, but with this normalization the 2.71-Mev state is a factor of four larger than predicted.[26,27] However, the agreement in Fig. 12 may be fortuitous, because it was found that small changes in various parameters could affect the relative (and absolute) cross sections in a complex way. The details of these changes were not studied except for the ground state. It is possible that a different set of parameters could predict the same general features and cross section of the ground state, and yet predict different relative cross sections and shapes for the other levels.

The most stringent comparison between experiment and theory concerns the absolute cross sections. The present preliminary fits fail seriously in this aspect of the comparison. Although the predicted absolute cross sections should represent an upper limit, the observed absolute cross sections were in all cases[28] considerably larger, as can be seen by careful comparison of the observed cross sections with Fig. 11. How serious these discrepancies really are cannot be assessed until more detailed attempts are made to fit all of the data.[29] Both

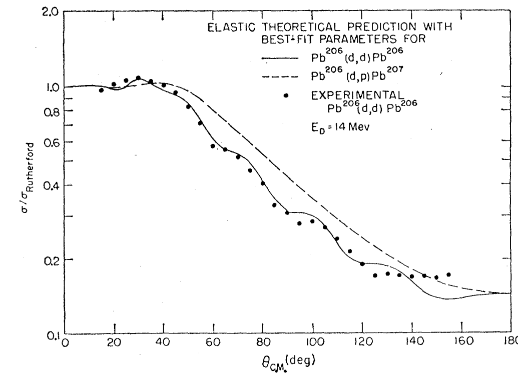

FIG. 13. A comparison of the 14-Mev deuteron elastic scattering angular distribution obtained in the present experiment (plotted as ratio to Rutherford) and the predictions of the distorted-wave calculations. Two theoretical curves are plotted, corresponding to the deuteron parameters providing the best fit to the experimental ground state (*d,p*) angular distribution (see caption of Fig. 10) and also those providing the best fit to the experimental elastic scattering angular distribution shown here ($V_d = -50$ Mev, $W_d = -7$ Mev, $A_d = 0.55$ f, and $R_d = 9.0$ f).

Tobocman[30] and Satchler[31] have pointed out that increasing R_n will raise the predicted cross section. This effect was observed; however, the quality of fit was decreased. An extensive effort to attain a best fit and hold absolute cross section was not attempted and is beyond the scope of this paper.

Figure 13 shows a comparison of the experimental elastic deuteron angular distribution at 14 Mev with the angular distributions predicted from the parameters which fit the elastic data best and from those which fit the (*d,p*) data best. In order to fit the elastic data, the real part of the deuteron potential (V_d) was arbitrarily held constant. The imaginary part of the potential (W_d) was decreased from -25 to -7 Mev; the radius of the potential well (R_d) was increased from 7.5 to 9.0 fermis; and the diffuseness parameter (A_d) was decreased from 0.7 to 0.55 fermi.

The best elastic deuteron fit was achieved with 0.5-fermi changes in R_d during the search and could be improved with smaller increments. R_d affects the "breakpoint" where the curve departs from Rutherford and also the angular location of maxima and minima. A_d affects the average slope of the curve from the "breakpoint" and W_d determines the amplitude of oscillation in the observed structure. These new parameters which fit the elastic data best resulted in a (*d,p*) angular distribution completely unlike the data (i.e., no peak position correlation, violent oscillations, etc.) It appears at present that no other set of *reasonable* proton parameters combined with the best-fit elastic deuteron parameters will predict the experimental (*d,p*) distributions as well as those used. Unless the reason for the discrepancy is the failure to include some other detail in the calculation, such as spin-orbit terms in

[26] Figures 11 and 12 do not take account of the fact that the cross section for the stripping reaction to the single-hole states is proportional to the square of the corresponding two-hole amplitudes in the ground state of Pb^{206}. When this factor is included, the relative cross section of the 0.57–0.90 Mev doublet is brought into rough agreement in Fig. 12. Theoretical and experimental estimates of these amplitudes have been made in references 12, 16 and 27, but a careful analysis of the present type should in principle be able to give the relative amplitudes of the various two-hole configurations in Pb^{206} fairly accurately.

[27] W. W. True and K. W. Ford, Phys. Rev. **109**, 1675 (1958).

[28] This assumes that the theoretical predictions for the single-hole states are appropriately reduced as already mentioned.[25]

[29] G. R. Satchler (private communication) has emphasized that this calculation assumes that the effective *n-p* interaction in the (*d,p*) reaction is the same as the free *n-p* interaction.

[30] W. Tobocman (private communication).

[31] G. R. Satchler (private communication).

the distorting potentials or more general shapes for the absorbing potential, it would appear that a different effective potential is required by the deuteron in elastic scattering from that required in stripping within the framework of this calculation.[32]

In conclusion, the present comparison between theory and experiment shows good agreement for stripping angular distribution shapes in a heavy nucleus under conditions of strong Coulomb distortions. Relative cross sections of one particular state as a function of deuteron energy are very well predicted, but only partial agreement is found for the relative cross sections for states of different angular momentum character. The measured absolute cross sections are considerably larger than those predicted by the calculation. If more detailed fits are made to all of the experimental data, better agreement may be obtained. The results suggest that nuclear spectroscopic information may be obtained in heavy elements by means of the deuteron stripping reaction, provided accurate experimental angular distributions are obtained with deuteron beams of high resolution and energies in the range of 15 to 25 Mev.

[32] It is quite possible that the best-fit elastic deuteron parameters could be used with proton parameters quite different than those determined by elastic scattering and achieve a (d,p) fit as good as presently found. This possibility was not explored and the proton parameters were arbitrarily held at values determined by elastic scattering.

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