

Scattering of Photons by Protons*

LLOYD G. HYMAN†

Harvard University, Cambridge, Massachusetts

(Received August 8, 1961)

A previously proposed model to account for photon scattering from protons in the 50 to 300 Mev range is presented in detail. The scattering amplitude is composed of (1) Klein-Nishina amplitude, (2) resonance magnetic dipole, (3) resonance electric dipole, (4) Low term. Using the measured value of the π^0 lifetime = $(2 \pm 1) \times 10^{-16}$, which enters as a parameter in amplitude (4), the model is compared to experiment. The agreement is poor particularly at the scattering angle = $3\pi/4$.

I. INTRODUCTION

WE proposed previously¹ a relatively simple model to account for photon-proton scattering in the gamma-ray region 50 to 300 Mev. At the time we applied it only to the data for the scattering angle $\theta_s = \pi/2$, with what we believed to be considerable success. In subsequent calculations this model failed to fit the $\theta_s = 3\pi/4$ data with reasonable parameters.^{2,3} The purpose of this note is to record the details of the model.

II. SCATTERING AMPLITUDES

The model consists of four elementary scattering amplitudes, all computed in the barycentric system:

(1) Klein-Nishina amplitude⁴⁻⁶:

$$T_{KN} = \frac{e^2}{M} \frac{1}{(1+\beta)} \left\{ -\hat{e} \cdot \hat{e}' + \beta (\hat{e} \cdot \hat{k}') (\hat{e}' \cdot \hat{k}) \right. \\ \left. + \frac{1}{2} i \beta \hat{\sigma} \cdot (\hat{e}' \times \hat{e}) + \frac{1}{2} i \beta \hat{\sigma} \cdot (\hat{e} \times \hat{k}') \times (\hat{e}' \times \hat{k}) \right. \\ \left. + \frac{1}{2} i \beta [\hat{\sigma} \cdot (\hat{k}' \times \hat{e}') (\hat{e} \cdot \hat{k}') - \hat{\sigma} \cdot (\hat{k} \times \hat{e}) (\hat{e}' \cdot \hat{k})] \right\},$$

where \hat{e}, \hat{e}' are unit polarization vectors for incoming and outgoing γ rays, and \hat{k}, \hat{k}' are unit vectors along direction of incoming and outgoing γ rays. This amplitude is accurate to order β , where β is defined as $E_{lab}/(M + E_{lab})$, with E_{lab} the laboratory photon energy and M the nucleon rest mass.

(2) Resonance ($T = \frac{3}{2}, J = \frac{3}{2}$) magnetic dipole scattering amplitude⁷:

$$T_{\frac{3}{2}, \frac{3}{2}} = a_{33} \{ 2(\hat{k}' \times \hat{e}') \cdot (\hat{k} \times \hat{e}) - i \hat{\sigma} \cdot (\hat{k}' \times \hat{e}') \times (\hat{k} \times \hat{e}), \\ a_{33} = \frac{1}{6} \frac{e^2}{M} \left(\frac{\mu_p - \mu_n}{2f} \right)^2 \frac{m}{M} \frac{k^2}{\eta^3} \sin \delta_{33} e^{i\delta_{33}},$$

* Research supported by U. S. Atomic Energy Commission.

† Now at Argonne National Laboratory, Argonne, Illinois.

¹ L. G. Hyman, R. P. Ely, D. H. Frisch, and M. A. Wahlig, Phys. Rev. Letters **3**, 93 (1959).

² G. E. Pugh, R. Gomez, D. H. Frisch, and G. S. Janes, Phys. Rev. **105**, 982 (1957).

³ G. Bernardini, A. O. Hanson, A. C. Odian, T. Yamagata, L. B. Auerbach, and I. Filosofo, Nuovo cimento **18**, 1203 (1960).

⁴ F. E. Low, Phys. Rev. **96**, 1428 (1954).

⁵ M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

⁶ J. D. Walecka (private communication).

⁷ W. J. Karzas, W. K. R. Watson, and E. Zachariasen, Phys. Rev. **110**, 253 (1958).

where k is the center-of-mass photon momentum in units of pion rest mass, η is pion momentum (measured in units of pion rest mass) in the center-of-mass system at the same total center-of-mass energy = $\omega_\eta = \omega_k$, f^2 is pion-nucleon coupling constant = 0.08, μ_p and μ_n are proton and neutron moments in units of nucleon Bohr magnetons, and δ_{33} is phase shift for $\pi^+ - p$ scattering through resonance $\frac{3}{2}, \frac{3}{2}$ state. δ_{33} is evaluated using a Chew-Low fit wherein $\cot \delta_{33} = \frac{3}{4} [\omega(1 - \omega/\omega_r)/f^2 \eta^3]$ with $\omega_r = 2.11$.⁸ The same formulas are used below meson threshold, where η and δ_{33} become imaginary and R is entirely real.

(3) Amplitude of electric dipole scattering from the tail of the $T = \frac{1}{2}, J = \frac{3}{2}$ resonance:

$$T_{\frac{1}{2}, \frac{3}{2}} = (e^2/M) a_{13} \{ 2\hat{e} \cdot \hat{e}' - i \hat{\sigma} \cdot (\hat{e}' \times \hat{e}) \}.$$

Given a_{13} at one energy, we extrapolate to other energies as follows: Assuming a one-level resonance form, we have

$$\sigma_{\gamma\gamma}/\sigma_{\gamma\pi} = \sigma_{\pi\gamma}/\sigma_{\pi\pi}, \quad \sigma_{\gamma\pi} \sim \sigma_{\pi\gamma} (\eta^2/k^2),$$

by detailed balance; therefore

$$\sigma_{\gamma\gamma} \sim (\sigma_{2\pi\gamma}/\sigma_{\pi\pi}) (\eta^2/k^2).$$

Thus, using the notation of partial widths,

$$\frac{\sigma_{2\pi\gamma}}{\sigma_{\pi\pi}} \sim \frac{\Gamma_\pi^2 \Gamma_\gamma^2}{\Gamma_\pi^2 [(\omega - \omega_r)^2 + \frac{1}{4} \Gamma^2]} \frac{1}{\eta^2},$$

or

$$\sigma_{\gamma\gamma} \sim \frac{\Gamma_\gamma^2}{k^2 [(\omega - \omega_r)^2 + \frac{1}{4} \Gamma^2]}.$$

On the tail,

$$\frac{1}{4} \Gamma^2 \ll (\omega - \omega_r)^2, \quad \Gamma_\gamma \sim (ka)^3 / [1 + (ka)^2];$$

therefore

$$a_{13} \sim k^2 / (\omega - \omega_r) (1 + k^2 a^2),$$

with a = range of interaction. We take $a = 0.88$ meson Compton wavelengths and $\omega_r = 4.12$, corresponding to a laboratory energy = 750 Mev. Because the phase shift is small, a_{13} is real in the region being discussed.

(4) Amplitude due to π^0 interaction with radiation

⁸ H. Y. Chiu and E. L. Lomon, Ann. Phys. **6**, 50 (1959).

field^{9,10}:

$$T_{\pi^0} = (e^2/M)a_L(\cos\theta)[(\hat{k}-\hat{k}') \cdot (\hat{e} \times \hat{e}') i \hat{\sigma} \cdot (\hat{k}-\hat{k}')],$$

$$a_L(\cos\theta) = \frac{1}{\lambda^{\frac{1}{2}}} \frac{M f^2}{m \pi} \frac{k}{(1-\cos\theta+1/2k^2)};$$

λ is the π^0 lifetime in units of $\tau_0 = (\mu c^2 \alpha^2 f^2 / \hbar 4\pi^2)^{-1} \approx 5 \times 10^{-17}$ sec. The cross section is found by taking the absolute square of the total amplitude, summing over final states, and averaging over initial states. The result is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left(\frac{e^2}{M c^2} \right)^2 \left\{ \frac{1}{(1+\beta)^2} (1 + \cos^2\theta + 2\beta \sin^2\theta \cos\theta) \right.$$

$$+ |a_{33}|^2 (7 + 3 \cos^2\theta) + |a_{13}|^2 (7 + 3 \cos^2\theta)$$

$$+ 4a_L^2 (1 - \cos\theta)^3 + \text{Re} a_{33} \left[20a_{13} \cos\theta - \frac{8 \cos\theta}{1+\beta} \right.$$

$$\left. - 4a_L (1 - \cos\theta)^2 + \beta (2(1 - \cos\theta) - 3 \sin^2\theta) \right]$$

$$+ a_{13} \left[\frac{-4}{1+\beta} (1 + \cos^2\theta) + 4a_L (1 - \cos\theta)^2 \right.$$

$$\left. + \beta (1 - 2 \cos\theta - 3 \cos^2\theta + 4 \cos^3\theta) \right]$$

$$\left. - 2a_L \beta (1 - \cos\theta)^3 \right\}.$$

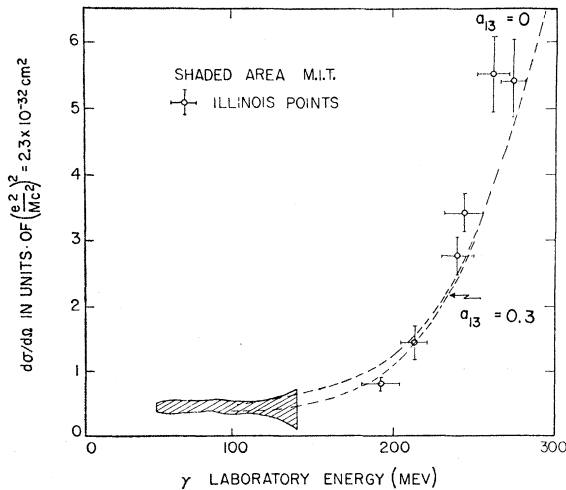


FIG. 1. Differential cross section at $\theta = \pi/2$ vs laboratory energy. The low term is in with perturbation theory phase. $\tau_{\pi^0} = 2 \times 10^{-16}$ sec.

⁹ F. E. Low (private communication).

¹⁰ M. Jacob and J. Mathews, Phys. Rev. **117**, 854 (1960).

III. DISCUSSION

The primary purpose of this model at the time was to see if we could place a lower limit on the π^0 lifetime, which enters as a parameter in amplitude (4). We took the relative sign of the amplitude for process (4) given by perturbation theory¹¹; this sign yielded a lower limit for the lifetime, $\tau_{\pi^0} \geq 10^{-18}$ sec. Subsequent calculations by Lapidus and Chou Kuang-chao indicate that the term has opposite phase from that taken by us¹²; this would have given $\tau_{\pi^0} \geq 10^{-17}$ sec. Since then, the value of $\tau_{\pi^0} = (2 \pm 1) \times 10^{-16}$ sec has been measured.^{13,14} Using the lifetime $\tau_{\pi^0} = 2 \times 10^{-16}$ sec and regarding a_{13} at a given energy as a free parameter to be extrapolated to other energies by a one-level resonance technique, we arrive at the set of curves shown in Figs. 1 and 2 for $\theta_s = \pi/2$ and $3\pi/4$, with the perturbation theory sign of π^0 interaction amplitude a_L . If instead we take the Lapidus-Chou sign for a_L , we obtain the curves shown in Figs. 3 and 4. The fit at $\theta_s = 3\pi/4$ is poor in all cases; an a_{13} value of approximately 0.2 with the Lapidus-Chou phase for a_L is roughly the best fit at $3\pi/4$ with $\tau_{\pi^0} = 2 \times 10^{-16}$ sec.

The model suffers from the following defects:

(a) According to Jacob and Mathews,¹⁰ when the dispersion integrals are evaluated to include the $T = \frac{1}{2}$, $J = \frac{3}{2}$ resonance, the resonance has little effect below 250 Mev. This implies that a value $a_{13} = 0.2$ at 250 Mev

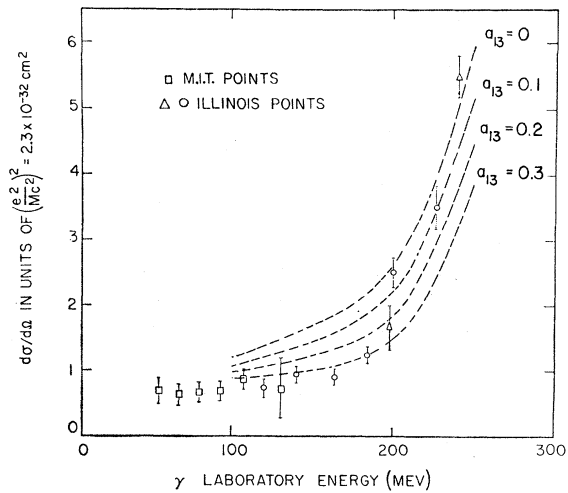


FIG. 2. Differential cross section at $\theta = 3\pi/4$ vs laboratory energy. The low term is in with perturbation theory phase. $\tau_{\pi^0} = 2 \times 10^{-16}$ sec.

¹¹ F. Villars (private communication).

¹² L. I. Lapidus and Chou Kuang-chao, Joint Institute for Nuclear Research Report USSR D-681, 1961 (unpublished).

¹³ A. V. Tollestrup, S. Berman, R. Gomez, and H. Ruderman, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 27.

¹⁴ R. G. Glasser, N. Seeman, and B. Stiller, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, New York, 1960), p. 30.

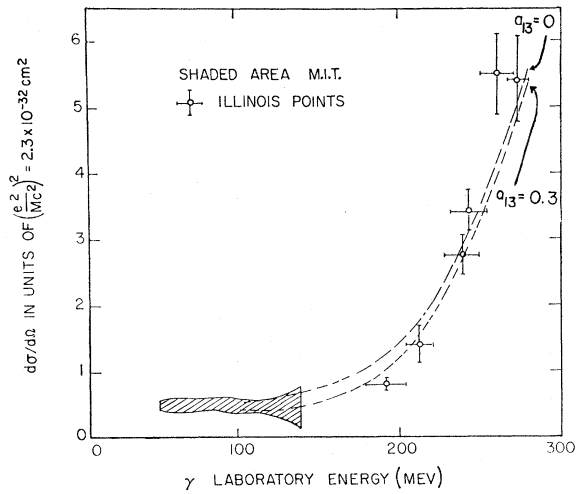


FIG. 3. Differential cross section at $\theta = \pi/2$ vs laboratory energy. The Low term is in with Lapidus-Chou phase. $\tau_{\pi^0} = 2 \times 10^{-16}$ sec.

is unrealistically large. Berkelman,¹⁵ using a one-level resonance technique and the photoproduction data from the second resonance, estimates a_{13} at 250 Mev of the order 0.1, which in the present model affects the $\theta_s = 3\pi/4$ cross sections noticeably. The discrepancy concerning the importance of a_{13} between the dispersion-theoretic approach of Jacob and Mathews and this model may arise because there may be nonresonant

¹⁵ K. Berkelman, Istituto Superiore di Sanita Report ISS-61/13 (unpublished).

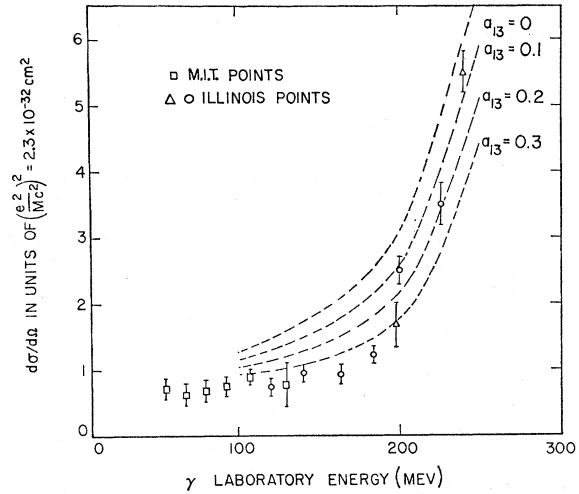


FIG. 4. Differential cross section at $\theta = 3\pi/4$ vs laboratory energy. The Low term is in with Lapidus-Chou phase. $\tau_{\pi^0} = 2 \times 10^{-16}$ sec.

contributions to a_{13} which are as important in this energy range as the resonance contributions.

(b) Terms which correspond to electric dipole scattering through the $J = \frac{1}{2}$ state have been left out. Perturbation-theory calculations by Karzas, Watson, and Zachariasen⁷ indicate the $J = \frac{1}{2}$ terms to be of the same order as the terms we have included.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge the many contributions to this research by Professor D. H. Frisch. I wish to thank Dr. Earle Lomon for helpful discussions.