Isobar Mechanism for Pion-Baryon Higher Resonances*

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Peierls isobar mechanism is extended to pion-hyperon interactions on the assumption that Y_1^* is the $\pi\Lambda$ isobar whose mass lies below the $\bar{K}N$ threshold and of spin parity $P_{\frac{3}{2}}$. It is shown that this mechanism, when combined with experimental data currently available, does suggest that a pion-hyperon resonance in the $I=1$ channel at c.m. energy 1645 Mev with spin-parity assignment $D_{\frac{3}{2}}$ is possible. This resonance state is expected to reflect in the total $K^- - p$ and $K^- - n$ (pure $I=1$) cross sections as resonance peaks centered at around 685-Mev/c K-meson lab momentum. Experimental implications of this isobar model for pion-baryon interactions as well as certain mathematical difficulties associated with the over-all validity of such a mechanism are also discussed.

IT has become evident in recent time that phenomenological analyses subscribe to the viewpoint that final states described by the (3,3) isobar model[1] play an important role at least in low- and intermediate-energy pion-nucleon interactions. Recently Peierls[2] proposed a mechanism which consists in taking seriously the concept that the (3,3) isobar is an unstable particle (N^*) of spin and isospin $\frac{3}{2}$ and has a complex mass; this simple model then appears to give good quantitative agreement with a description of the second pion-nucleon resonance. We extend here the analysis to pion-hyperon interactions on the assumption that Y_1^* is the $\pi\Lambda$ isobar whose mass lies below the $\bar{K}N$ threshold[3] and of spin-parity assignment $P_{\frac{3}{2}}$. It is shown that this mechanism when *combined* with experimental data currently available[4–6] does suggest that a pion-hyperon resonance in the $I=1$ channel at c.m. energy 1645 Mev with spin-parity assignment $D_{\frac{3}{2}}$ is possible.[7] This resonance state is expected to reflect in the total $K^- - p$ and $K^- - n$ (pure $I=1$) cross sections as resonance peaks centered at around 685-Mev/c K-meson lab momentum. We discuss also some experimental implications of this isobar model for pion-baryon interactions as well as certain mathematical difficulties associated with the over-all validity of such a mechanism.

For the pion-nucleon situation, the physical interpretation of Peierls' mechanism can be most readily understood in terms of the picture for isobar production [see Fig. 1(a)] $\pi+N\rightarrow\pi_2+N_2^*$; here the final observed products are (π_2,π_3,N_2) for the final-state three-body decay. In general a Dalitz plot of the kinetic energies T_{π_2} vs T_{π_3} for the final-state products from $\pi+N\rightarrow\pi_2+\pi_3+N_2$ shows bands $T_{\pi_2}=$constant and $T_{\pi_3}=$constant, corresponding to isobar formation. The Peierls enhancement corresponding to the intermediate nucleon pole N_1 occurs when the isobar produced (N_2^*) could have been formed by an intermediate pion π_1 and N_1 (both unobserved) such that N_1 and π_2 (observed) have the intermediate isobar mass N_1^*.

This occurs if and only if the above-mentioned bands *cross* in the physical region of the Dalitz plot. (To see this, note that the point of crossing is a point where both π_2 and π_3 have the resonant mass with respect to N_2; and further, that if π_3 and N_2 are *decay* products of the isobar, the isobar could have been *formed* by a π and a nucleon, of these same momenta; this π and nucleon are the unobserved π_1 and N_1 mentioned above. Their momenta are such that π_2 and N_1 also can form the isobar and thus invoke Peierls' pole. This situation prevails only if such a band crossing exists in the physical region. Since the final isobar produced decays at various angles with respect to its direction of motion in the over-all center-of-mass system, it is the entire region of the Dalitz plot which is enhanced, not just the point of crossing.

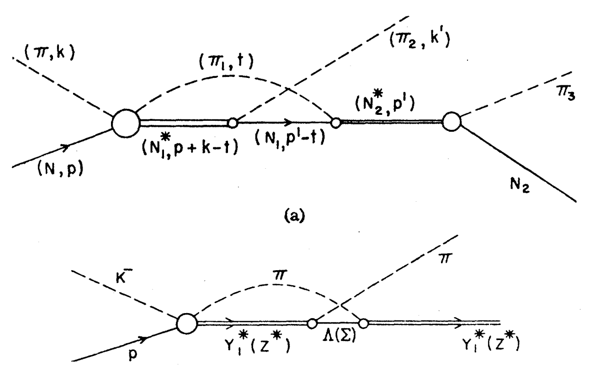

FIG. 1. (a) The contribution of the crossed one-nucleon diagram for pion-isobar scattering (πN^*) to the production process $\pi+N\rightarrow\pi+N_2^*$. (b) The contribution of the crossed one-hyperon (Λ or Σ) diagram for $\pi+Y_1^*$ (Z^*) scattering to the production process $K^-+p\rightarrow\pi+Y_1^*$ (Z^*).

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[1] R. M. Sternheimer and S. J. Lindenbaum, Phys. Rev. 123, 333 (1961).

[2] R. F. Peierls, Phys. Rev. Letters 6, 641 (1961).

[3] M. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 5, 520 (1960).

[4] J. P. Berge, P. Bastien, O. Dahl, M. Ferro-Luzzi, J. Kirz, D. H. Miller, J. J. Murray, A. H. Rosenfeld, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters 6, 557 (1961).

[5] R. H. Dalitz and D. H. Miller, Phys. Rev. Letters 6, 562 (1961).

[6] Leroy T. Kerth, Revs. Modern Phys. 33, 389 (1961).

[7] We expect the D state to be dominant for this resonance for K pseudoscalar, though perhaps far from pure; conceivably we might have the situation of D and P wave superimposed. For a scalar K meson, the resonance will contribute primarily to the $\bar{K}N$ $P_{\frac{3}{2}}$ state.

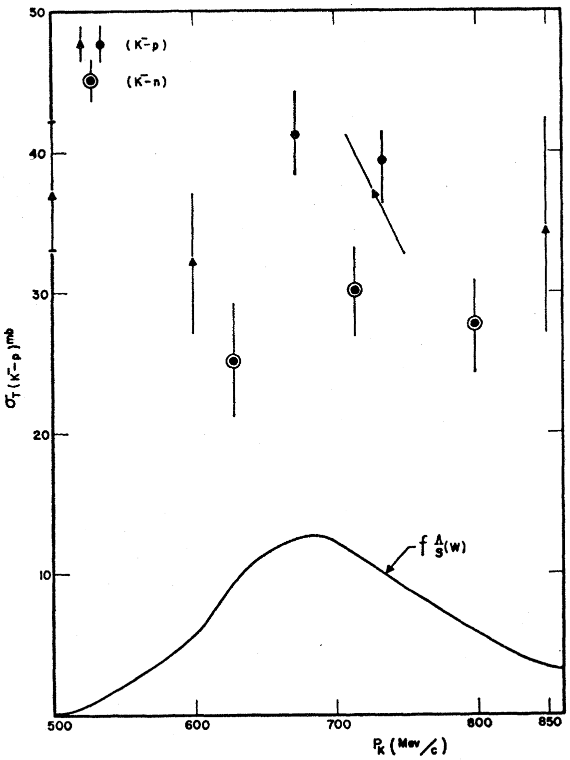

FIG. 2. The function $f_S{}^{\Lambda}(W)$ (with Λ pole) is plotted as a function of $K^- - p$ lab momentum from 500 to 850 Mev/c. The experimental points for $K^- - p$ and $K^- - n$ total cross sections are taken from reference 6.

Mathematically, the interpretation of this pole effect is somewhat ambiguous. For the matrix element T of

$$(\pi,k)+(N,p)\rightarrow(\pi_2,k')+(N_2{}^*,p'), \tag{1}$$

where p, k are the energy-momentum four-vectors of the initial pion-nucleon system and p', k' those of the finally produced isobar $N_2{}^*$ and associated pion π_2, we have [see Fig. 1(a)]

$$T\sim\int\frac{\theta(t_0)\delta(t^2)\theta(p_0+k_0-t_0)\delta((p+k-t)^2-M^2)d^4t}{(p'-t)^2-m^2}$$
$$=\frac{1}{2|\mathbf{p}'|\sqrt{s}}\ln\left\{\frac{(M^2-m^2)\sqrt{s}-(p_0'-|\mathbf{p}'|)(s-M^2)}{(M^2-m^2)\sqrt{s}-(p_0'+|\mathbf{p}'|)(s-M^2)}\right\}. \tag{2}$$

Here M, m are the isobar mass and the nucleon mass, respectively, $s=(p+k)^2$, and the calculation is carried out on the assumption that the intermediate $(\pi_1,N_1{}^*)$ with energy-momentum $(t,\,p+k-t)$, respectively, are on their mass shells (ignoring the small contribution from the intermediate mass m_π for π_1) thus giving the mechanism the full benefit of contribution from the pole term N_1 with energy-momentum $p'-t$. The analytic expression for $|T|^2$ does exhibit a "pole-like" behavior at the position of the second resonance [incident pion kinetic energy $\sim$600 Mev (lab)] due to the logarithmic singularity arising from

$$(M^2-m^2)\sqrt{s}-(p_0'+|\mathbf{p}'|)(s-M^2)=0, \tag{3}$$

in the limit of an isobar stable against strong decay (M, real).[8] However, this effect largely disappears when we replace M^2 by $M^2+i\Delta$ in $|T|^2$, consistent with an unstable N^* in strong interactions; Δ here is determined from the position and width of the (3,3) resonance.[2] This certainly emphasizes a need for caution in dealing with pole effects involving logarithmic singularities and is admittedly a difficulty with the model in its present form.

The extension of Peierls' mechanism to pion-hyperon system is relatively straightforward. In Fig. 1(b) we illustrate the situation for $K^-+p\rightarrow\pi+Y_1{}^*$ and $K^-+p\rightarrow\pi+Z^*$; $Y_1{}^*$ and Z^* ($I=2, J=\frac{3}{2}$) being the conjectured first set of pion-hyperon isobars to correspond to the (3,3) pion-nucleon isobar,[9,10] where for $Y_1{}^*$ we have restricted ourselves to the Λ pole, since not much experimental evidence for $Y_1{}^*\rightarrow\pi+\Sigma$ has been found.[11] The explicit contribution of the pole term to the scattering amplitude $A(s,\bar{s},t)$ for

$$\begin{aligned}&\pi+Y_1{}^*\rightarrow\pi+Y_1{}^*,\\&\pi+Z^*\rightarrow\pi+Z^*,\end{aligned} \tag{4}$$

is then such as to give the following total transition probabilities for S-wave and P-wave pion-isobar scattering:

$$f_S(W)=\frac{g^4}{2q^2\Delta_1}\tan^{-1}\left(\frac{4q^2\Delta_1}{\Delta_1{}^2+(W^2-W_0{}^2)(W^2-W_0{}^2-4q^2)}\right),$$
$$f_P(W)=\frac{g^4}{8q^4}\left[\ln\frac{(W^2-W_0{}^2)^2+\Delta_1{}^2}{(W^2-W_0{}^2-4q^2)^2+\Delta_1{}^2}-\frac{2(W^2-W_0{}^2-2q^2)}{\Delta_1}\times\tan^{-1}\frac{4q^2\Delta_1}{\Delta_1{}^2+(W^2-W_0{}^2)(W^2-W_0{}^2-4q^2)}\right],$$
$$W_0{}^2=2M^2+2m_\pi{}^2-m_Y{}^2,\quad(Y=\Lambda,\Sigma)$$
$$\Delta_1=|\Delta|(1-\omega/E). \tag{5}$$

Here M^2 and Δ are connected with the mass of the unstable isobar M^{*2} by $M^{*2}=M^2+i\Delta$, W is the total

[8] However, it must be remembered that the matrix element T is multiplicatively proportional to g^2, the "coupling constant" for the $NN^*\pi$ vertex [determined by the $\pi-N$ cross section at the (3,3) resonance]. Since g^2 is proportional to the width of the (3,3) resonance in the approximation of a one-level formula, the limit that N^* is a stable isobar in strong interactions implies that g^2 and hence T both vanish.

[9] T. D. Lee and C. N. Yang, Phys. Rev. **122**, 1954 (1961). D. Amati, A. Stanghellini, and B. Vitale, Phys. Rev. Letters **5**, 524 (1960).

[10] Leroy T. Kerth and Abraham Pais, University of California Radiation Laboratory Report UCRL-9706, 1961 (unpublished).

[11] M. Ferro-Luzzi and M. H. Alston, Revs. Modern Phys. **33**, 416 (1961).

pion-isobar energy in the c.m. system, ω and E are the pion and (real part) of isobar c.m. energy, respectively. For the $(Y_1^*\pi\Lambda)$ vertex, we have determined the coupling g from a width $\Gamma=50$ Mev for Y_1^* consistent with the best fit to over-all data[5] for this resonance.

For $\pi+Y_1^*$ scattering, the contribution from S wave due to the Λ pole $f_S^{\Lambda}(W)$ is plotted against K^- lab momentum from 500 Mev/c to 850 Mev/c. in Fig. 2. This energy-dependent function does show a substantial enhancement peaked around $P_K=685$ Mev/c (c.m. energy 1645 Mev) and could therefore show up in K^--p and K^--n total cross sections as a "resonance" from mechanism such as that shown in Fig. 1(b). The experimental $\bar{K}-N$ cross sections currently available[6] in this region are not sufficiently well-known to make a clear-cut decision but do not appear to be inconsistent with such a possibility (see Fig. 2). The P-wave contribution $f_P(W)$ in this momentum range is relatively small ($<20\%$), and thus we would expect by analogy with the pion-nucleon situation[2] that this pole effect will contribute dominantly to the D state in $\bar{K}-N$ interactions[7]; it is thus interesting to remark (i) the Dalitz and Miller analysis[5] of the mass distribution and the angular distribution from Y_1^* decay is consistent with a spin-parity assignment $P_{\frac{3}{2}}$ for this excited state if the production process is $K^-+p\ (D_{\frac{3}{2}})\rightarrow\pi+Y_1^*$ $(S_{\frac{3}{2}})$, and (ii) the excitation data for $Y_1^{*\pm}$ production[12] are consistent with a fairly sharp enhancement centered around $P_K=700$ Mev/c as is required for the model discussed here to be meaningful.

The isospin dependence of our approach can be inferred again from the Dalitz-Miller argument that Y_1^* production from K^--p,

$$\left|\frac{T_{I=0}(KY_1^*)}{T_{I=1}(KY_1^*)}\right|^2=\frac{3\sigma(Y_1^{*0})}{\sigma(Y_1^{*+}+Y_1^{*-}-2Y_1^{*0})},\qquad(6)$$

is principally in the $I=1$ state[13] at least for K^- lab momenta 760 Mev/c and 850 Mev/c.[5] In the spirit of our isobar mechanism for generating resonance we would therefore expect the 1645-Mev predicted resonance to be prominent in the $I=1$ channel (if Y_1^* production at around 700 Mev/c remain principally in $I=1$ state); this is, however, not a specific prediction of Peierls' model.

Applying the present procedure to $\pi+Z^*$ [Fig. 1(b)], taking a width and position for Z^* compatible with global symmetry[10] ($\Gamma=140$ Mev, resonant energy $=1530$ Mev), we find that $f_S(W)$ is peaked around 1.15-Bev/c K^- lab momentum. While this shows remarkable agreement with the position of the $I=0$ resonance found in the K^--p total cross section,[6] it must be pointed out that (a) the mechanism represented by Fig. 1(b) can contribute only to the $I=1$ $\bar{K}N$ system, and (b) the magnitude of the enhancement in $f_S(W)$ is negligible (<2 mb) in comparison with the peak at a maximum of about 20 mb (after subtracting for the smooth nonresonant background from data) found for the experimental resonance. It can also be shown quite analogously that the recently found $I=0$ resonance at K^- momentum 400 Mev/c (c.m. energy 1525 Mev)[14] cannot contribute to the 1.1-Bev/c K^--p resonance. We have not considered the excited state[15] Y_0^* in the present framework; the evidence from the K^--deuterium reaction data[16] together with the fact that the Y_0^* production cross section at 850 Mev/c seems to be small compared with Y_1^* production are suggestive that this state may be related to the low-energy K^--p system as a virtual bound state $(J=\frac{1}{2})$,[17,18] and hence may not be directly connected with our considerations. In fact, calculations of $f(W)$ on the assumption that Y_0^* and Y_1^* are in the $J=\frac{1}{2}$ state, yield enhancements position-wise correct ($P_K\sim700$ Mev/c) but of negligible magnitude because of the substantially smaller coupling "g^2" involved.

It must be pointed out that Y_1^* production is quite substantial in the vicinity of the $I=0$ K^--p resonance at 1.1 Bev/c;[12] we can, however, not apply Peierls' mechanism at this energy since the resonant bands (unlike the situation at 700 Mev/c) for the $\pi^++\pi^-+\Lambda$ final state no longer intersect in the physical region of the Dalitz plot[11]—a prerequisite for the model to be operative. In fact, $f_S^{\Lambda}(W)$ and $f_P^{\Lambda}(W)$ at this energy show little energy dependence apart from being negligibly small. The proposal of Frazer and Ball[19] for explaining this higher $I=0$ K^--p resonance in terms of a rapidly rising inelastic cross section due to copious production of[20,21] K^* may well be pertinent to the physical picture. On the other hand, the very

[12] See especially Fig. 3 of reference 4 for the excitation data of Y^{*+} and Y^{*-} production.

[13] Equation (6) then implies that production of neutral Y_1^* is small at these energies, i.e., production cross section $\sigma(Y_1^{*0})$ $\ll\sigma(Y_1^{*+}+Y_1^{*-})$.

[14] M. Ferro-Luzzi, R. Tripp, and M. Watson, Lawrence Radiation Laboratory Internal Memo No. 310, 1961 (unpublished).

[15] M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **6**, 698 (1961); P. Bastien, M. Ferro-Luzzi, and A. H. Rosenfeld, *ibid.* **6**, 702 (1961).

[16] R. L. Schult and R. H. Capps, Phys. Rev. **122**, 1659 (1961).

[17] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters **5**, 425 (1959). J. Franklin, R. C. King, and S. F. Tuan, Phys. Rev. **124**, 1995 (1961).

[18] At the energy of the predicted resonance (K^- momentum 685 Mev/c), suppression of $\pi+Y_0^*$ production can be understood qualitatively if Y_0^* and Y_1^* spins are $\frac{1}{2}$ and $\frac{3}{2}$, respectively, since S-wave decay into $\pi+Y_0^*$ from the resonant state is forbidden and kinematic factors inhibit P-wave decay into $\pi+Y_0^*$ channel relative to S-wave decay into $\pi+Y_1^*$ channel.

[19] James S. Ball and William R. Frazer, Phys. Rev. Letters, **7**, 204 (1961).

[20] M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **6**, 300 (1961).

[21] An alternative explanation of this 1815-Mev K^--p $I=0$ resonance is, however, possible construed as a S-wave bound state of the K^* (interpreted as an $I=\frac{1}{2}$, $J=1$ vector meson) and nucleon. According to the vector theory of strong interaction, the K^*+N system (threshold 1822 Mev) is expected to be more strongly attractive in $I=0$ than $I=1$ channel and the bound system will thus contribute to a $\bar{K}N$ $I=0$ resonance in $D_{\frac{3}{2}}$. W. Krolikowski (private communication).

fact that the $\pi-\pi$ $(I=1, J=1)$ resonance does not show up strongly in a Dalitz plot for $(\pi^+\pi^-n)$ or $(\pi^+\pi^-\Lambda)$ final states is indicative that the (3,3) isobar N^* and Y_1^* are very strong at low and intermediate energies and dominate (obscure) the effects of the $\pi-\pi$ resonance.[22] In fact, the threshold for production of the $(\pi-\pi)$ resonance is some 170 Mev above the position of the second pion-nucleon resonance N^{**}, and it is hard to reconcile this with the cusp effect from inelastic contributions according to the Frazer-Ball mechanism.[23] Thus a situation may arise whereby the conjectured 685-Mev/c $K^- - p$ resonance (and the second pion-nucleon resonance N^{**}) are generated essentially by the pion-isobar formalism rather than a mechanism based on the production of the $J=1$, $I=1$ pion-pion resonance,[19] while at high energies and for higher partial waves the strip approximation method of Frazer and Ball becomes operative.[24]

In Table I we have summarized the list of pion-hyperon resonances including the theoretically predicted resonant state here discussed, which can be identified from the $K^- - p$ or $K^- - n$ systems. The pattern of isobars does not agree well with the positions predicted from global symmetry on the basis of a phenomenological mass formula which has in any case no place for the 1525-Mev $I=0$ resonance,[10] but we do see the same alternating sequence of $I=1$ and $I=0$ resonances there obtained.

We conclude by emphasizing those experimental determinations of greatest interest to our present considerations: (1) A compilation of the excitation data for $\pi+N \to \pi+N^*$ production in the $I=\frac{1}{2}$ state (employing methods similar to the Bose statistics analysis of Dalitz and Miller[5] for $K^-+p \to \pi+Y_1^*$) in a neighborhood of the pion-nucleon second resonance N^{**}. This production data should show a sharp peak to correspond to the second resonance, if Peierls' mechanism is physically meaningful. (2) More extensive compilation of $\pi+Y_1^*$ excitation data between $p_K=500$ to 900 Mev/c to identify a similar situation for the pion-hyperon isobar system. (3) It is important to improve total cross-section data for $K^- - p$ and especially $K^- - n$ (pure $I=1$) interactions between 0.5 to 1.0 Bev/c to search for this $I=1$ excited state at $p_K=685$ Mev/c.

TABLE I. A list of pion-hyperon resonances identifiable from the $\bar{K}N$ system. The spin-parity assignments are not known experimentally and the above insertions are thus tentative but would appear reasonable if the (Λ,Σ) parity is even and K pseudoscalar; in particular those at 1645 and 1815 Mev are made on the basis of the predictions of the present model and that of Ball and Frazer,[19] respectively.

Mass (in Mev)	Isotopic spin	Spin and parity
1385(Y_1^*)	1	$P_{\frac{3}{2}}$
1525	0	$P_{\frac{3}{2}}(D_{\frac{3}{2}})$
1645	1	$D_{\frac{3}{2}}$
1815	0	$D_{\frac{5}{2}}$

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[22] Remarks of W. Selove, Revs. Modern Phys. **33**, 426 (1961).

[23] This argument may be somewhat weakened if a neutral vector meson (or a $\pi^+\pi^-\pi^0$ resonance) ω' of mass 550 Mev and narrow width exist, since the $\omega'+N$ threshold is $\sim$1490 Mev. Professor Sakurai (private communication) has pointed out, however, that the sharp peak anomaly of L. Hand and C. Schaerf [Phys. Rev. Letters, **6**, 229 (1961)] in $\gamma+p \to \pi^++n$ coincide with the threshold for $\gamma+p \to \omega'+p$ and suggest that this anomaly (rather than N^{**}) is associated with the Ball-Frazer cusp effect through ω' production.

[24] It is of interest to note that both models emphasize the importance of inelastic processes either through the formation of isobars $\pi+N^*$ or that of (π,π) resonance production $(\pi,\pi)+N$. There thus appears an underlying unity shared between the two approaches derived from a basic three-body interaction $\pi+\pi+N$.