Production and Decay of Intermediate Vector Mesons*

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The decay of the intermediate vector meson of the weak interactions W is discussed with emphasis on the modes $W \to 2\pi$, $W \to K+\pi$. It is noted that if the W mass is in the neighborhood of the observed π-π resonance at $\sim 30m_\pi^2$, then the mode $W \to 2\pi$ will dominate all others. The production of W's by π's and K's is also discussed in terms of a "peripheral collision" model.

INTRODUCTION

THE idea that the weak interactions are transmitted by a boson has been entertained almost from the time of the discovery of the weak interactions themselves. Since the establishment of the V-A theory and the nonconservation of parity it has become clear that these bosons must be vector particles, which, in the notation of Lee and Yang,[1] we shall call W mesons. These W mesons interact weakly with all particles taking part in the weak interactions. The charged mesons will also have electromagnetic interactions, but it is very unlikely that they have strong couplings since then effects like lepton production in p-$\bar{p}$ annihilation would probably have been observed.

The W mesons may be produced in a variety of ways. The production of W's by neutrons has been studied in detail theoretically[2] and experiments are now in process at CERN and Brookhaven which may exhibit charged W production if the W mass is of the order of the nucleon mass or less. In order to design an experiment to detect the W meson, in view of the very short lifetime ($\sim 10^{-17}$ sec), it is necessary to know what the branching ratio for different decay modes of the W are. It is possible to compute the two-body decay modes of the W directly in terms of parameters given by experiment, which we do in Sec. II. Both charged and neutral W mesons can also be produced by pions and K mesons incident on nucleons.[1,3] In Sec. III of this note we examine such production processes on the basis of a simple model.

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).
[2] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960).
[3] N. Dombey, Phys. Rev. Letters **6**, 66 (1961). In this paper, use of the conserved vector current hypothesis is made to relate the process $\pi^-+p \to n+\gamma$ to the vector part of the process $\pi^-+p \to p+W$. Some of the details of this calculation appeared to us to be wrong (in particular the peripheral diagram was not included and the isotopic *vector* part of the pion absorption process was not picked out), and hence we present our results here. Also no consideration of the W decays was made.

W DECAYS

The Lagrangian, $L_{Wl\nu}$, for charged W decays into leptons and neutrinos is given by

$$L_{Wl\nu} = ig\bar{l}\gamma_\lambda(1+\gamma_5)\nu W_\lambda + \text{H.c.}, \tag{1}$$

where g is defined so that

$$g/m_W = G_V^{\frac{1}{2}} 2^{-\frac{1}{4}}, \tag{2}$$

G_V is the vector β-decay coupling constant with $G_V M_N^2 \cong 10^{-5}$.

In terms of these definitions the rate for W decay into leptons is given[1] (in the limit of zero mass for the leptons) by

$$(g^2/6\pi)m_W \gtrsim 10^{17}\ \text{sec}^{-1}, \tag{3}$$

as $m_W > m_K$. The decay of W into 2π proceeds only through the vector part of the strangeness-preserving current V_λ.

Assuming that the current is conserved, and is proportional to the total isospin current for which there is now some experimental evidence,[4] the decay of the W^- into 2π's may be computed exactly in terms of $F_\pi(q^2)$, the electromagnetic form factor of the pion, which has been studied frequently in the literature.[5] The matrix element for this decay, $m_W \to 2\pi$, is given by

$$M_{W^-\to 2\pi} = \frac{\sqrt{2}(P_\lambda^- - P_\lambda^0)\epsilon_\lambda g F_\pi(m_W^2)}{(8m_W E_\pi^2)^{\frac{1}{2}}}. \tag{4}$$

The $\sqrt{2}$ factor is from isotropic spin considerations and the momentum *difference* $P_\lambda^- - P_\lambda^0$ arises because the current operator V_λ in the matrix element $\langle \pi^-\pi^0 | V_\lambda | 0 \rangle$ is supposed conserved. In the neighborhood of the π-π resonance the function $F_\pi(S)$ can be represented by

[4] A. R. Erwin, R. March, W. D. Walker, and E. West, *ibid.* **6**, 628 (1961); D. Stonehill, C. Baltay, H. Courant, W. Fickinger, E. C. Fowler, N. Kraybill, J. Sandweiss, J. Sanford, and H. Taft, *ibid.* **6**, 624 (1961).
[5] A recent experiment by T. Mayer-Kuckkuk and F. C. Michel (to be published), in which the β spectrum of B^{12} and N^{12} are compared, gives strong support to the current conservation hypothesis. The original suggestion of a conserved current, is, of course, due to R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958). See also S. S. Gershteim and J. B. Zeldovich, Soviet Phys.—JETP **29**(2), 576 (1957).

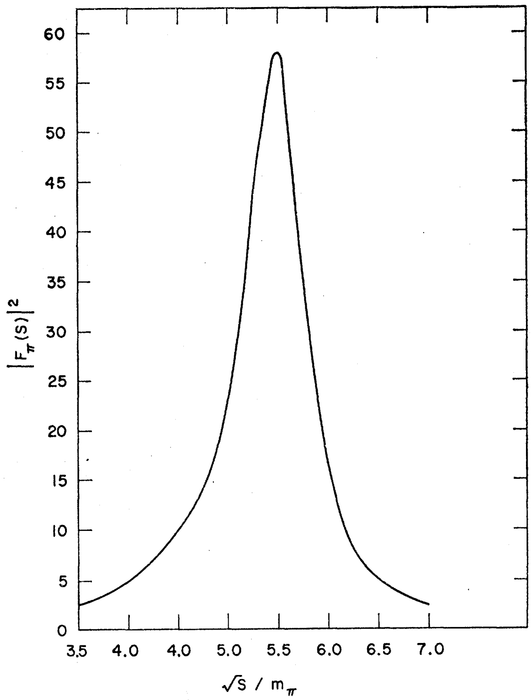

FIG. 1. A plot of $|F_\pi(S)|^2$, where S is the energy given in units of m_π^2. We have taken $S_r=30m_\pi^2$ and $\Gamma=0.72m_\pi$.

a resonance formula,

$$F_\pi(S)=S_r/[(S_r-S)-iS^{\frac{1}{2}}\Gamma]. \tag{5}$$

Here S is to be taken as m_W^2.

Experiment[6] seems to indicate that $S_r\sim30m_\pi^2$ and that Γ the full width at half maximum is about 100 Mev. The function $|F_\pi|^2$ rises to a maximum of about 58 at $S=S_r$ and falls off fairly sharply to about 2.5 at S, corresponding to a K mass on one side of the resonance and S corresponding to a nucleon mass on the other. In terms of $|F_\pi|^2$, the rate $R_W\to 2\pi$ is given by

$$R_{W\to2\pi}=(g^2m_W/24\pi)|F_\pi(m_W^2)|^2(1-4m_\pi^2/m_W^2)^{\frac{3}{2}}. \tag{6}$$

Thus the ratio of the leptonic to pionic rates is given by

$$R_{W\to\pi+\pi}/R_{W\to l+\nu}=\tfrac{1}{4}|F_\pi(m_W^2)|^2(1-4m_\pi^2/m_W^2)^{\frac{3}{2}}. \tag{7}$$

A plot of $|F_\pi(S)|^2$, Fig. 1, shows that the 2π mode will dominate the leptonic mode as long as $\sim4m_\pi\leq m_W\leq6.5m_\pi$, which gives a fairly wide range of values between the K mass and the nucleon mass.

Of course, if neutral W mesons exist as well, then they are unlikely to decay into leptons at all, because of the apparent absence of neutral lepton couplings in weak interactions. The rate of W^0 decay into 2π is related to the charged decay rate in the schizon theory of reference 1, the relation being $R_{W^0\to\pi^+\pi^-}=R_{W^-\to\pi^-\pi^0}=R_{W^+\to\pi^+\pi^0}$.

We conclude that if the W meson lies within a range of about 100 Mev around 750 Mev, an experiment designed to detect it could concentrate on the 2π decay mode. For instance, in the production of W by neutrinos, the event would resemble

$$\nu+p\to\pi^++\pi^0+e^-+p,$$

with the $\pi^+\pi^0$ having a very well defined Q value.

We may also consider the possible decay $W\to K+\pi$. The matrix element for this decay is given by

$$M_{W\to K+\pi}=\frac{g_K}{(8m_WE_KE_\pi)^{\frac{1}{2}}}\epsilon_\lambda\langle0|S_\lambda|K\pi\rangle. \tag{8}$$

Here, the notation is similar to that of Eq. (3), except that in this decay we deal with S_λ, the strangeness changing weak current. The quantity $\langle0|S_\lambda|K\pi\rangle$ is the analytical continuation of the function $\langle K|S_\lambda|\pi\rangle$ which appears in K_{l_3} decays. As is well known

$$\langle K|S_\lambda|\pi\rangle=(K+\pi)_\lambda F^+(q^2)+(K-\pi)_\lambda F^-(q^2), \tag{9}$$

where F^+ and F^- are essentially real functions of the momentum transfer variable $(K-\pi)^2=q^2$, and $F^+(0)=1$ as a normalization condition. In the K_{l_3} decays $(K-\pi)_\lambda$ is the 4-momentum transferred to the leptons and hence, by a familiar argument, can be replaced in the expression for the K_{l_3} decay by the lepton mass which in the case of the electron decay may be neglected. If F^+ is approximated by a constant, as recent experiments suggest,[7] then the constant g_K is fixed by the K_{l_3} decay rate. In the analytical continuation appropriate to the W decay the momenta $(K+\pi)_\lambda$ and $(K-\pi)_\lambda$ change roles and in addition momentum conservation implies that $(K+\pi)_\lambda=P_{W_\lambda}$; the W four-momentum vector. But by the transversality condition, $\epsilon_\lambda P_{W_\lambda}=0$. Thus in the process $W\to K+\pi$ the function F^- does not contribute, and this decay is essentially related to the F^+ of the K_{l_3} decay evaluated at $q^2=-m_W^2$. In terms of $F^+(m_W^2)$ and $F_\pi(m_W^2)$ introduced in Eq. (3), the ratio of the 2π and the $K\pi$ decay rates may be written

$$\frac{R_{K\pi}}{R_{2\pi}}=\frac{1}{2}\frac{g_K^2}{g^2}\frac{|F^+(m_W^2)|^2}{|F_\pi(m_W^2)|^2}\times\left[\frac{[1-(m_K+m_\pi)^2/m_W^2][1-(m_K-m_\pi)^2/m_W^2]}{1-4m_\pi^2/m_W^2}\right]^{\frac{3}{2}}. \tag{10}$$

The phase-space and isotopic factors serve to reduce this ratio by something like a factor of ten and it is difficult to see any compensating factor arising from the form factor ratio. In fact, extrapolating from the K_{l_3} decay rate,[8] one has very likely $g_K^2/g^2\ll1$. Thus the

[6] See especially, J. Bowcock, N. Cottingham and D. Lurié, Nuovo cimento 16, 918 (1960), and other references given in this paper.

[7] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 255 (1961).

[8] An estimate of g_K^2/g^2 may be made from the K_{e3} decay rate, if we assume that the form factor F^+ defined above is approxi-

mode $W \to K+\pi$ is probably negligible compared to $W \to \pi+\pi$, unless the W mass is nearly coincident with that of the observed $K-\pi$ resonance,[9] in which case the factor $|F^+/F_\pi|^2$ may be very large. In view of the narrowness of the $K\pi$ resonance, this appears unlikely.

In view of these results, it is clear that any experiment designed to detect W production must recognize that the K-meson decay mode is likely to be very small, and so does not provide a useful signature of W production.

W PRODUCTION

Since the W is expected to have a lifetime of about 10^{-17} or 10^{-18} sec it is expected to be produced with a mass spectrum characterized by a half-width of perhaps the order of a kev or less. In the process $\pi+p \to W+p$, the recoil momentum distribution of the proton in the center-of-mass system should then exhibit a sharp peak corresponding to the approximate two-body kinematics appropriate to W production. Of course, this peak is superimposed on the background coming from processes like $\pi+p \to 2\pi+p$ and so forth. In fact, if the W has a mass in the neighborhood of the observed 2π resonance,[4] $30m_\pi^2$, it will be seen, in principle, as a very narrow spike on top of the broad, 2π resonance curve, whose width is $\sim$100 Mev. One way to distinguish the W production from the background would be to observe "prompt" leptons coming from the W decay. According to our previous estimates of decay rates, this will not be the dominant process for W mass around 750 Mev. It, therefore, would be useful to examine the possibility of distinguishing the sequence

$$\begin{array}{c}\pi+p \to W+p \\ \quad\;\;\downarrow \\ \quad\;\; 2\pi\end{array}$$

from the direct process $\pi+p \to 2\pi+p$.

In order to get a more quantitative idea of the cross section for W production by pions, we have estimated it using a simple "peripheral collision" model; Fig. 2. The matrix element corresponding to this graph we write in the form

$$M=\sqrt{2}g_{\pi N}\frac{\bar{u}_{p'}\gamma_5 u_p}{(2E_k)^{\frac{1}{2}}}f_{\pi NN}(q^2)\frac{1}{q^2+m_\pi^2} \times g\frac{F_\pi((k+q)^2)}{(2E_W)^{\frac{1}{2}}}W_\lambda(k_\lambda-q_\lambda). \quad (11)$$

In this formula $g_{\pi N}$ is the pseudoscalar pion-nucleon coupling constant with $g_{\pi N}^2/4\pi \simeq 15$; $f_{\pi NN}(q^2)$ is the pion-nucleon vertex form factor defined so that $f_{\pi NN}(-m_\pi^2)=1$. The momentum transfers, $q=p-p'$, which are involved in our problem may be the order of a nucleon mass. Thus setting $f_{\pi NN}(q^2)\simeq 1$ is presumably a poor approximation. Not much is known about $f_{\pi NN}$ at such large momentum transfers but we shall assume that $f_{\pi NN}(q^2)=1$ for $q^2 \leq 16m_\pi^2$, and $f_{\pi NN}(q^2)=0$ for $q^2>16m_\pi^2$. This approximation is suggested from the theory of inelastic p-p collisions.[10] Our choice of metric is such that the transversality condition on W_λ, $P_{W_\lambda}W_\lambda=0$, implies the relation (summing over W polarizations)

$$\sum W_\alpha W_\beta=\left(\delta_{\alpha\beta}+\frac{P_{W_\alpha}P_{W_\beta}}{m_W^2}\right), \quad (12)$$

with P_{W_λ} the four-momentum of the W, and m_W its mass. The quantities g and F_π have been introduced in part II.

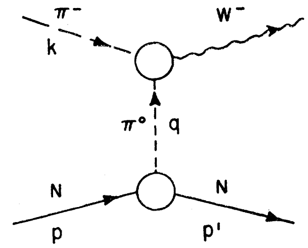

FIG. 2. The Feynman diagram representing "peripheral" production of the W in pion-nucleon collisions.

In terms of the invariant momentum transfer q^2, which in the rest system of the target nucleon is given in terms of the proton recoil energy E' by

$$q^2=2M_N(E'-M_N), \quad (13)$$

we find

$$\frac{d\sigma}{dE'}=\frac{1}{4\sqrt{2}}\frac{g_{\pi N}^2}{4\pi}G_F\frac{|f_{\pi NN}|^2|F_\pi|^2}{k^2}\frac{q^2}{M_N}\left(\frac{q^2+m_W^2}{q^2+m_\pi^2}\right)^2. \quad (14)$$

With the aid of the Mercury computer[11] at CERN we have plotted this function for a range of incident pion energies from 1 to 10 Bev and of possible W masses from $\frac{1}{2}$ to 1 nucleon masses. In Fig. 3 a typical spectrum is shown. In this plot, $F_{\pi NN}$ was set equal to one for all momentum transfers.

In computing the total cross section by integrating under this curve we have, as discussed above, cut off the function $f_{\pi NN}$ at $q^2=16m_\pi^2$. This probably underestimates the integral somewhat. For the case plotted in Fig. 1, $E_\pi=1.5M_N$, $m_W=0.75M_N$, the non-cutoff integral cross section is about 7×10^{-33} cm². This value does not include the effect of the π-π form factor enhancement, which would be expected to increase the cross section by a factor between 30 and 50 for this value of m_W. Cutting off the integral reduces the cross

mately constant over the spectrum of the K_{e3} decay. One finds from this $g_K^2/g^2=1/36$, which is consistent with the known result that strangeness-changing weak interactions are rather weaker than strangeness-conserving ones. See also L. B. Okun', Ann. Rev. Nuclear Sci. **9**, 61 (1959).

[9] M. Alston, L. W. Alvarez, P. Everhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters **6**, 300 (1961).

[10] We are grateful to Professor S. Drell for discussions on this point.

[11] The help of Mr. W. Klein of CERN was essential in carrying out the numerical work.

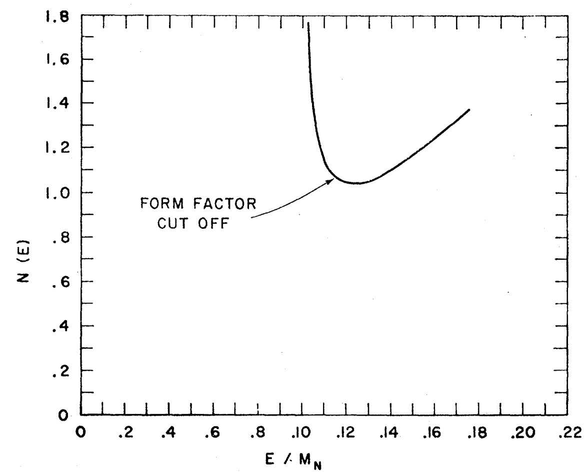

FIG. 3. A plot of the laboratory recoil momentum spectrum of the recoiling nucleon in the process $\pi+p \rightarrow W+p$ calculated in the approximation of Fig. 2. $M_W=0.75M_N$; $E_\pi=1.5M_N$.

section again by a factor of about 4. Hence the general order of magnitude of the cross section may be as large as 10^{-31} cm^2. In this illustration we have taken m_W quite close to the π-π resonance energy. For m_W the order of the K mass or the nucleon mass, $|F_\pi|^2\sim2.5$ so that a cross section of 10^{-31} cm^2 is very likely a quite optimistic upper limit.[12] As the cross section falls off with the square of the incident pion energy, there is nothing to be gained, from this point of view, by going to higher pion energies. We note that if we look specifically at the process

$$\begin{array}{c}\pi+p \rightarrow W+p \\ \qquad\qquad\downarrow \\ \qquad\qquad \mu\nu \text{ or } e\nu\end{array}$$

there is no enhancement from $|F_\pi|^2$ and the "partial cross section" for this will be about 10^{-33} cm^2.

We have also plotted the W angular distribution in the π-N center-of-mass system and Fig. 4 gives such a plot for the same choice of parameters as Fig. 3.

In order to have some idea of the background with which the W production must be compared, we consider the direct production of 2π, by the "peripheral model," which will have a similar angular distribution. This has been computed by Chew and Low.[13] Comparing their formula for 2π production with ours for W production, we find

$$\frac{\sigma_W}{\sigma_{\pi\pi}}=10^{-7}\frac{m_W^4}{m_P^2\Gamma_{\pi\pi}}|F_\pi(m_W^2)|^2, \tag{15}$$

where $\Gamma_{\pi\pi}$ is related to the width Γ of the $\pi\pi$ resonance by $\Gamma_{\pi\pi}=\Gamma E_r\sim4\mu^2$. For $m_W=750$ Mev, this becomes $\sigma_W/\sigma_{\pi\pi}\sim2\times10^{-5}$, indicating the difficulty of using the $\pi\pi$ decay mode to identify the W mesons.

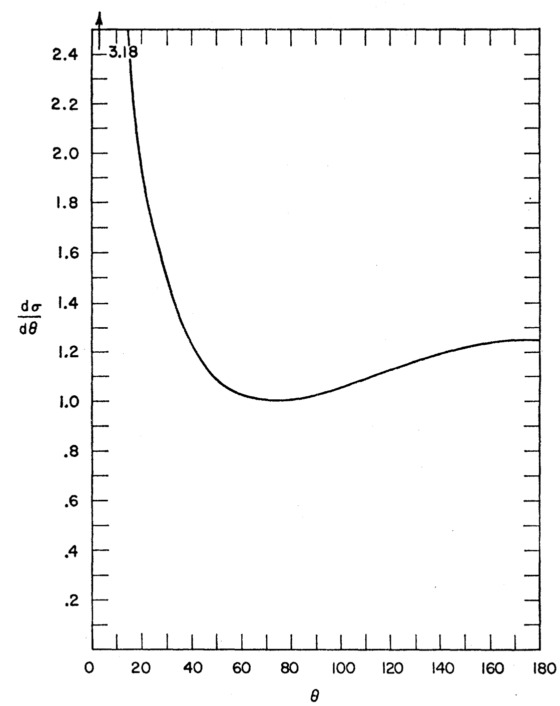

FIG. 4. A typical center-of-mass angular distribution for the process $\pi+p \rightarrow W+p$ calculated in the approximation of Fig. 2. $M_W=0.75M_N$; $E_\pi=1.5M_N$.

It may be remarked[14] that an essentially identical calculation may be made for the process $K+p \rightarrow W+p$. A similar formula is obtained except that the π-π form factor is replaced by the K-π form factor introduced in the previous section and $G_F/\sqrt{2}$ by g_K^2/m_W^2. Hence one cannot take advantage of the enhancement which came about from the π-π resonance. However, as discussed above, the W, if it has a mass in the region of the π-π resonance, will decay into two pions predominantly. Thus, the above reaction will look like $K+p \rightarrow \pi+\pi+p$ and may be much simpler to distinguish from the background than in reactions in which the W is produced by pions, despite the smaller cross section.

CONCLUSION

The main result of the work described above is that interactions of the W with two pions may be considerably enhanced if the W has a mass in the neighborhood of the observed $J=1$, $T=1$, two-pion resonance. This could be a significant effect in experiments designed to produce and detect W's. In view of the importance of the W to the theory of weak couplings, an

[12] We have also estimated the magnitude of the cross section from nonperipheral graphs (see reference 3). The cross section coming from such terms seems to be of the same general order of magnitude as that of the peripheral graph, but without the enhancement coming from the π-π resonance. The estimates of the nonperipheral terms are, of course, more difficult and uncertain.

[13] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).

[14] We are grateful to Dr. M. A. B. Bég for discussions of this point.

intensive experimental search for it certainly seems warranted.

ACKNOWLEDGMENTS

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Reaction, $\pi^-+p\rightarrow e^++e^-+n$, as a Means of Measuring the Electromagnetic Form Factor of the Charged Pion*

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The success of the single pion exchange approximation in explaining the experimental results for the reaction $\pi+n\rightarrow\pi+\pi+n$ in the region of the two-pion resonance suggests that the single pion exchange contribution to the reaction $\pi+n\rightarrow e^++e^-+n$ should be dominant for center-of-mass energies of the electron-positron system in the region of this resonance. A measurement of the rate of this reaction in this region should, therefore, provide direct information about the charged-pion electromagnetic form factor in the neighborhood of the two-pion resonance. We have calculated the differential cross section for the reaction $\pi^-+p\rightarrow e^++e^-+n$, assuming a counter experiment. Our estimates indicate that counting rates of the order of ten per hour are attainable by accelerators capable of producing negative-pion beams with an average flux of 10^6 pions per second.

I. INTRODUCTION

A RESONANCE in the two-pion $J=1$, $T=1$ state as a means of explaining the relatively rapid change of the nucleon electromagnetic form factors with momentum transfer has been proposed by Frazer and Fulco.[1] Since then there has been a considerable amount of experimental effort devoted to the search for this resonance by examining processes with at least two pions in the final state. Recently, Walker and his co-workers, and Pickup *et al.*[2] have obtained the strongest evidence for this resonance by a careful examination of the processes

$$\pi^-+p\rightarrow\begin{cases}\pi^++\pi^-+n\\ \pi^0+\pi^-+p.\end{cases}\qquad(1)$$

In analyzing their data, these groups find that for cases where the momentum transferred to the recoil nucleon is small, it is consistent to assume that the dominant contribution to process (1) in the resonant region comes from the single pion exchange graph shown in Fig. 1(a). A comparison of the relative rates of process (1) with each other and with the alternate reaction, $\pi^-+p\rightarrow\pi^0+\pi^0+n$ supports the notion that the strong two-pion interaction occurs in the $T=1$ state. The observed branching ratio for reaction (1) in the region of resonance also lends further support to the result that the single pion exchange graph gives the dominant contribution to this process.[3]

It is the purpose of this paper to point out that, in light of these results, the measurement of the process

$$\pi^-+p\rightarrow e^++e^-+n\qquad(2)$$

is now very much worth doing. The observed dominance of the single pion exchange graph [Fig. 1(a)] for pion production strongly suggests that the corresponding graph for electron pair production, shown in Fig. 1(b), would also be dominant, again in the region of small momentum transfer and provided we restrict ourselves to center-of-mass energies of the electron-positron system close to that of the two-pion resonant energy. If this is true, we have a means of measuring the charged-pion electromagnetic form factor for the most interesting values of the invariant four-momentum transfer. This could be done without recourse to a difficult extrapolation procedure,[4] making it practical to use counters to observe the electron pair.

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[1] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959); Phys. Rev. **117**, 1609 (1960).

[2] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961); E. Pickup, D. K. Robinson, and E. O. Salant, *ibid.* **7**, 192 (1961).

[3] In the resonance region, for $\Delta^2<10$, Pickup *et al.*[2] find $\sigma(\pi^0,\pi^-)/\sigma(\pi^+,\pi^-)=0.42\pm0.06$.[2] The ratio predicted by the single pion exchange model is one half. Another simple explanation for this observed value of the branching ratio is that this reaction proceeds predominantly through the state of total isotopic spin 3/2. This would predict, however, much too large a cross section for the reaction $\pi^++p\rightarrow\pi^0+\pi^++p$, i.e., $\sigma(\pi^0,\pi^+)=(9/2)\sigma(\pi^+,\pi^-)$.

[4] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1640 (1959).