For a more realistic uniform distribution, writing $R=(Ze^2/y)G(x)$, where $y=ka$, $x=\Delta a$, with a the rms radius, and defining $\beta=1/[(5/3)^{\frac{1}{2}}x]$, then

$$G(x)=\frac{x}{F(x)}[\tfrac{1}{2}\pi-\mathrm{Si}(1/\beta)+\beta^2(-150\beta^4+48\beta^2+5)$$
$$\times\sin(1/\beta)-\beta(-150\beta^4-2\beta^2+1)\cos(1/\beta)], \quad (17)$$

where $\mathrm{Si}(z)$ is the sine integral. $G(\Delta)$ has a zero for these finite charge distributions, and in the case of a distribution with an edge it also oscillates, like $F(\Delta)$, and generally out of phase. $G(\Delta)$ passes through its first zero before $F(\Delta)$.

Pine and Yount[14] have recently determined R in the scattering of 300-Mev positrons and electrons from Co through angles $\theta=10°$, 20°, 30°, 40°. Predictions for R from various nuclear models are compared with experiment in Fig. 5. Within the first zero of the form factor the uniform model gives a good prediction. To distinguish details between different realistic models, however, a better estimate of the importance of higher Born contributions must be made.

[14] J. Pine and D. Yount (private communication).

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Vector Mesons and Nucleon-Nucleon Potentials*

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The role of the π-π resonances in the nucleon-nucleon force is considered on a model which treats these resonances as particle exchanges. The nucleon form factors are used to obtain the resonance parameters; thus the coupling constant of the "particles" to the nucleon are the only free parameters. It is found that the qualitative features of the central, spin-orbit, and tensor potentials in all states can be reproduced, except that the central repulsive core has too long a range. These results suggest that π-π effects dominate rather than supplement the usual uncorrelated 2π, 3π contribution to the nucleon-nucleon force.

THERE is considerable current interest in the role of π-π resonances in other strong interactions. In this article we discuss the effect of the $J=1$, $T=1$ and $J=1$, $T=0$ resonances, considered as ρ and ω mesons, respectively, on nucleon-nucleon scattering. In particular, it is conjectured that π, ρ, and ω exchange are the dominant contributions to the long-range part of the nucleon-nucleon force.

Instead of using dispersion-theoretic techniques to compute the contribution of what would there appear as poles, we propose obtaining a useful orientation in terms of standard one-particle exchange potential theory. This procedure allows one to compare the results directly with phenomenological potentials.[1,2] Thus, as has been pointed out by Breit[3] and Sakurai,[4] the repulsive core and strong L-S forces needed to understand the scattering data can be qualitatively understood by a neutral vector meson. The difference in our approach lies in the use of the nucleon form factors. Including both "charge" and "magnetic" contributions to the meson-nucleon currents, we can identify the mesons in the nucleon structure as shown below.

The nucleon's electromagnetic structure has been shown to require a $J=1$ strong π-π interaction for its understanding. Bergia *et al.*[5] have given a simple discussion of the electromagnetic (e.m.) form factors, F, in the context of dispersion theory and found that $T=0$, $J=1$ resonances lead to ($\hbar=c=1$)

$$F_{1,2}{}^{S,V}=1-a_{1,2}{}^{S,V}+\frac{a_{1,2}{}^{S,V}}{1-q^2/m^2}, \quad (1)$$

where S-V refer to the isoscalar-vector, 1-2 to the charge-anomalous moment parts of the photon-nucleon current [e.g., see (2) below]; and m is the resonance energy, 1-a being the fraction of charge remaining in the core or short-range part of the cloud.

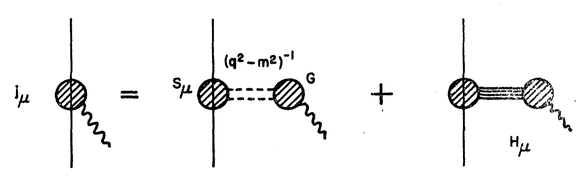

FIG. 1. The composition of the photon-nucleon vertex.

* Supported in part by the U. S. Atomic Energy Commission.
[1] J. Gammel and R. Thaler, Phys. Rev. **107**, 291 (1957).
[2] P. Signell and R. Marshak, Phys. Rev. **109**, 1230 (1958).
[3] G. Breit, Phys. Rev. **120**, 287 (1960).
[4] J. J. Sakurai, Ann. Phys. **11**, 1 (1960).
[5] S. Bergia, A. Stranghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961).

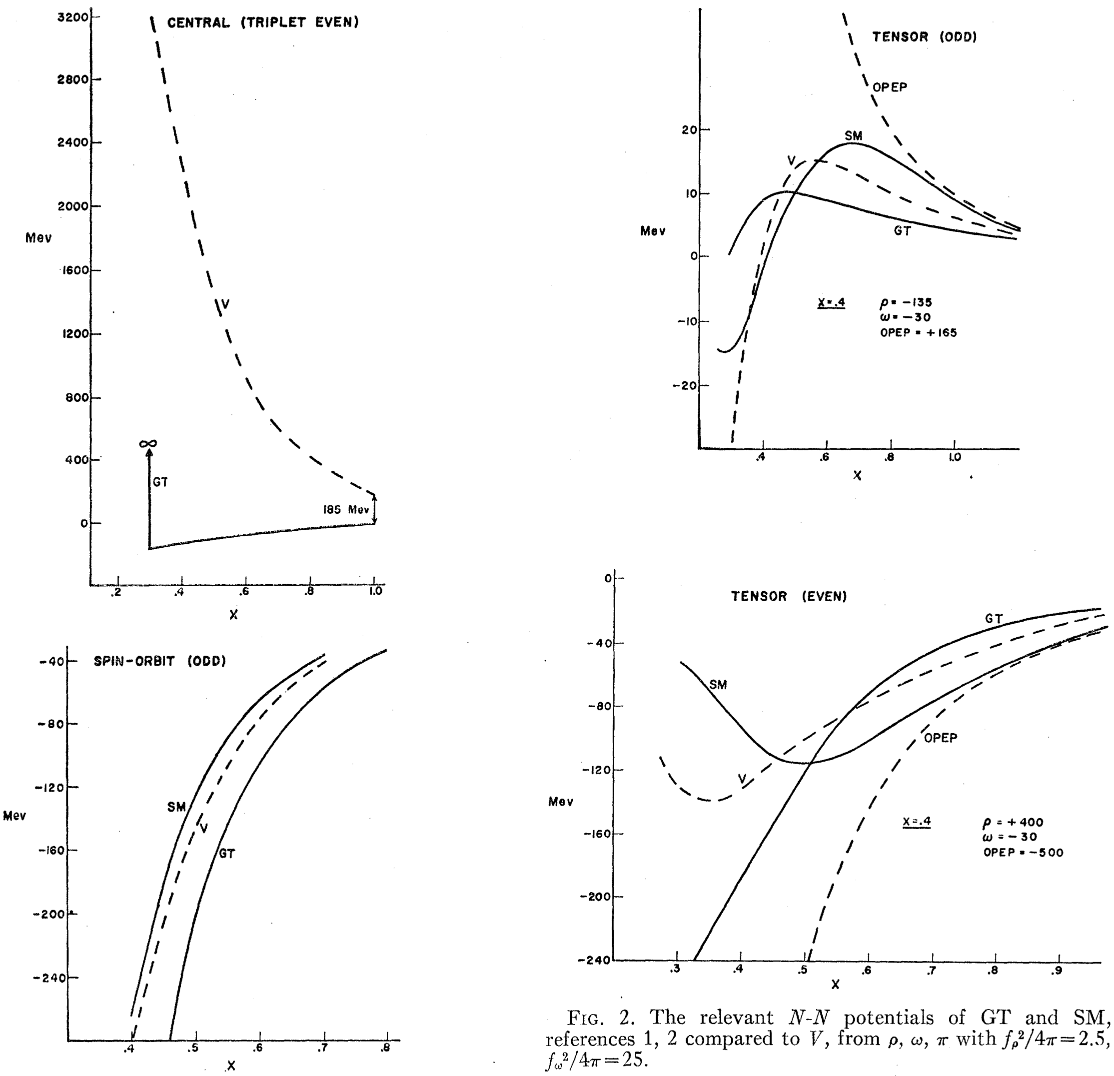

FIG. 2. The relevant N-N potentials of GT and SM, references 1, 2 compared to V, from ρ, ω, π with $f_\rho^2/4\pi=2.5$, $f_\omega^2/4\pi=25$.

Recent measurements,[6] appear to be consistent with this form, which can be obtained also from a vector meson theory. The photon-nuclear vertex j_μ is then composed of the two diagrams in Fig. 1, where S_μ, H_μ are conserved currents alike in form to j_μ, and H_μ refers to the high-mass (short-range) contributions from the cloud and core. The term of interest is S_μ

$$S_\mu = f\,\bar{u}(p')\left\{C_1(q^2)\gamma_\mu + \frac{K}{2m}C_2(q^2)\left(\frac{\boldsymbol{q}\gamma_\mu - \gamma_\mu\boldsymbol{q}}{2}\right)\right\}u(p), \quad (2)$$

where $\boldsymbol{q}\equiv\sum_\lambda \gamma_\lambda q_\lambda$. The form (1) indicates that the pole in F is due to the meson propagator $(q^2-m^2)^{-1}$ and $G(q^2)\approx G(0)$, $H(q^2)\approx H(0)$, and $C_1(q^2)/C_2(q^2)=a_1/a_2$. In this way the observed nucleon structure yields parameters $C_2=a_2/a_1$ and m, which determine all characteristics of the meson-nucleon system, except for a multiplicative constant—the coupling constant f. (Taking $C_1=1$ leads to a useful nomenclature, and indeed the difference between the resonance approximation in dispersion theory and Born approximation in vector meson theory is merely one of language.[7])

We now calculate the contribution of ρ, ω to the N-N potential in a manner similar to that used to obtain the familiar one-pion exchange potential (OPEP).[8] In order to carry through this procedure it is necessary to reduce the invariant Feynman amplitudes to nonrelativistic form,[9] keeping terms to lowest order in the momentum $(P/2M)^2\approx5\%$. Then the scattering amplitude (3) yields straightforwardly the

[6] R. Hofstadter and R. Herman, Phys. Rev. Letters **6**, 293. (1961). From this data we obtain: $m_\rho=4.5m_\pi$, $C_2^\rho=3.70$; $m_\omega=3.0m_\pi$, $C_2^\omega=0.64$.

[7] See especially the discussion in Revs. Modern Phys. **33**, 457 (1961).

[8] Suppl. Progr. Theoret. Phys. (Kyoto) **3** (1956).

[9] See, for instance, S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, Illinois, 1955), Vol. I, p. 32.

potential (4):

$$(ME)^{\frac{1}{2}}/2\pi S_\mu(p,p')(q^2-m^2)^{-1}S_\mu(-p,-p')|_{NR}$$

$$\cong -2M/4\pi\left\langle u_1^\dagger u_2^\dagger \left| \int d^3x\, e^{-p'x}V(x)e^{ipx} \right| u_1u_2 \right\rangle, \quad (3)$$

$$\text{(central)} \qquad V(x)=Y(x)\left[1+\tfrac{2}{3}(1-C_2)^2\left(\frac{m}{2M}\right)^2 \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right],$$

$$\text{(spin-orbit)} \quad V(x)=-\frac{1}{x}\frac{d}{dx}Y(x)(1-C_2)\left(\frac{m}{2M}\right)^2 8\mathbf{L}\cdot\mathbf{S}, \qquad (4)$$

$$\text{(tensor)} \qquad V(x)=Y(x)\left(\frac{1}{3}+\frac{1}{x}+\frac{1}{x^2}\right)(1-C_2)^2\left(\frac{m}{2M}\right)^2 S_{12},$$

$$Y(x)=\frac{f^2}{4\pi}\frac{e^{-x}}{x}, \quad x=mr.$$

$V(x)$ in $(\mathbf{L}\cdot\mathbf{S}_1)(\mathbf{L}\cdot\mathbf{S}_2)$ and $(\boldsymbol{\sigma}_1\cdot\mathbf{P})(\boldsymbol{\sigma}_2\cdot\mathbf{P})$ appear in order $(P/2M)^4$, and are neglected here.

With the inclusion of the isopin dependence and the parameters m, C_2 appropriate to ρ, ω (see reference 6) we now attempted to fit the long-range part of the empirical potential[1,2] with *only* ρ, ω, π exchange, disregarding contributions from the 2π, 3π continuum.

The main features found are:

(i) $f_\omega^2 \gg f_\rho^2$ explains the repulsive core in the central potential and the large attractive $\mathbf{L}\cdot\mathbf{S}$ force. *Fortunately* $C_2^\omega<1$, *otherwise the sign of* $\mathbf{L}\cdot\mathbf{S}$ *would be reversed.*

(ii) The *tensor* coefficients given by e.m. structure have $(1-C_2^\rho)^2 \gg 1 \gg (1-C_2^\omega)^2$ so that ρ *is enhanced* and ω diminished as is necessary for ρ to prevail over OPEP in both isospin states.

(iii) The spin, isospin symmetries of ρ, ω are such as to always give the proper behavior in sign; this is what allows one to include large amounts of ρ, ω for any empirical potential of the general character of those shown, provided the relations in (i), (ii) hold.

(iv) The central potential is too large at large distances due to the low mass $m_\omega \approx 3m_\pi$. ($m_\omega \approx 5-6\, m_\pi$ rectifies this difficulty.)

Despite the crudeness of this approach, the results indicate that the dominant features of the N-N force at low energy may indeed be primarily due to ρ, ω, π exchanges.

This suggests an essential simplification in that the 2π, 3π continuum effects may not be important except for contributions from the exchange of a 2π $T=0$, $J=0$ state, which gives rise to a central attraction only. The mass spectrum of this state would have to be concentrated below $3m_\pi$ in order to cancel the long-range ω repulsion without affecting the desirable shorter range property. Perhaps the Booth, Abashian, and Crowe effect[10] at $2.2m_\pi$ is related to this required feature. It is interesting to note the rough agreement of $f_\rho^2/4\pi=2.5$ with that predicted by Sakurai[4] from S-wave π-N scattering and the width of the 2π resonance. $f_\omega^2/4\pi\approx 25$ must also be compatible with other nucleon interactions.

In the light of very recent experimental evidence[11] for a $T=0$, $J=1$ 3π resonance at $5.6m_\pi$ and the 2π peak at $5.4m_\pi$ we have attempted to obtain fits to the N-N potential with these masses for ω, ρ. See Fig. 2. We find that results roughly similar to those above can be obtained provided one takes C_2^ρ, C_2^ω as free parameters. Generally one can say only that $f_\omega^2/4\pi>10$, $f_\rho^2/4\pi<3$ but, more important, the corresponding values of C_2^ω, C_2^ρ yield form factors badly in disagreement with those of reference 6. However, Durand[12] has argued that the earlier analysis of the deuteron e.m. scattering can be questioned. It thus appears that an understanding of the "intermediate region" of the N-N potential is not an unreasonable hope for the near future.

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[10] N. Booth, A. Abashian, and N. Crowe, Phys. Rev. Letters **7**, 35 (1961).
[11] B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters **7**, 178 (1961).
[12] L. Durand, Phys. Rev. **123**, 1393 (1961).