The disadvantage of using thin targets to reduce multiple electron scattering in conversion-electron studies should be more than offset by the higher efficiencies possible for electron counters as compared with those for gamma-ray detectors, and by the reduction in spurious background counts. Conversion-electron angular correlation studies for stripping reactions appear especially promising for tandem accelerators with sufficiently high energies. The possibility of conversion electron polarization measurements, yielding additional nuclear information, might also be mentioned in conclusion.

ACKNOWLEDGMENTS

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Alpha Particles from Be^9 and C^{12} by 25-Mev Alpha-Particle Bombardment

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Alpha-particle spectra from thin beryllium and carbon targets have been taken at 47.5° laboratory angle and an alpha energy of 25.41 Mev. The center-of-mass spectra of both nuclei indicate that the four-body reactions $Be^9(\alpha;3\alpha,n)$ and $C^{12}(\alpha;4\alpha)$ are greatly preferred over the three-body reactions $Be^9(\alpha;\alpha,n)Be^8$ and $C^{12}(\alpha;2\alpha)Be^8$. Several inelastic levels of both nuclei appear, and the recoil-decay alpha particles from the 9.61-Mev level of C^{12*} are seen. The Fermi statistical model is invoked in an attempt to establish that the multibody reactions are reasonably representative of the proportions of the two- and three-cluster configurations in the ground states of the two nuclei, and that the three-cluster configurations are preferred.

INTRODUCTION

WITHIN the past two years there has been a renewed interest in the alpha-particle model of the nucleus, which was first proposed over twenty years ago.[1–3] The more recent models do not precisely resemble the earlier models, but tend toward the form of the "cluster model," which takes into account the possible nucleon subgroups within the nuclear structure. In many of the light nuclei the natural subgroups are alpha particles, which have been used, for example, to calculate ground-state magnetic moments,[4,5] and thus the older and newer theories have a superficial similarity. However, the cluster model is designed to be compatible with the shell model, but with this difference; instead of having random azimuthal phase relations, the particles are no longer independent, but have a sufficiently strong pairing interaction to cause some phase grouping. The j-j coupling of the shell model is thus modified to a partially L-S coupled model. A particularly clear exposition of this idea has been given by Phillips and Tombrello,[6] in the case of the highest known level of the mass-5 nuclei. In this paper a more extreme assumption is made in the case of several levels in the light nuclei; specifically that a two-body (two-cluster) model will describe many of the ground- and low-excited levels, although these levels may be mixed states of two or more possible sets of cluster states. The Be^9 nucleus is believed to consist primarily of Be^8 [ground state (g.s.)] and a neutron, while the more highly bound internal structure of the Be^8 is attributed to two alpha particles in several rotational-vibrational states. The Be^8 structure has been examined by an ingenious calculation by Kallenopoulus and Wildermuth,[7,8] which tends to confirm the assumption.

Another nucleus which has received considerable theoretical attention is C^{12}. As in the case of all nuclei which can be said to consist of a whole number of alpha particles, there has been speculation that it might be fitted by a model of three alpha particles. In contrast to this, one might extend the two-cluster model of Phillips and Tombrello to attribute the ground state of C^{12} to an alpha and a Be^8 (ground) nucleus, although

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[1] D. R. Inglis, Phys. Rev. **50**, 783 (1936).
[2] H. Bethe, Phys. Rev. **53**, 842 (1938).
[3] L. R. Hafstad and E. Teller, Phys. Rev. **54**, 681 (1938).
[4] D. R. Inglis, Phys. Rev. **56**, 1175 (1939).
[5] K. Kendall, Bull. Am. Phys. Soc. **2**, 149, (1961).
[6] G. C. Phillips and T. A. Tombrello, Nuclear Phys. **19**, 555 (1960).
[7] T. Kallenopoulus and K. Wildermuth, Nuclear Phys. **7**, 150 (1958).
[8] K. Wildermuth and T. Kallenopoulus, Nuclear Phys. **9**, 449 (1958/59).

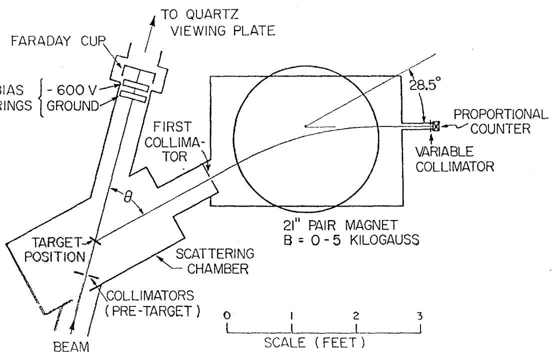

FIG. 1. Experimental apparatus.

this has not been explicitly done by these authors. Following their approach (as in the mass-5 case) to show shell-model equivalence, one might note that the shell model yields, for the C^{12} ground state, $(1s_{\frac{1}{2}})^2(1p_{\frac{3}{2}})^4$ for the configuration of both types of nucleon, and that the wave functions of the eight external particles have the same space dependence and are also influenced by particle pairing energies. It is plausible that two alphas can be formed from the pairing interactions, and the three alphas can all be in an S state. The Be^8 (g.s.) can, in principle, be formed from any pair of alphas, and it is possible that a Be^8 (g.s.) cluster exists within C^{12}.

The experimental investigation of the cluster model may proceed in a number of ways. One technique which may lead to a simple interpretation is the fragmentation of the ground-state nucleus and an examination of the reaction products. Gamma rays have been used on both beryllium and carbon, but dipole absorption of a photon necessarily changes the isotopic spin of the excited nucleus, and the ground-state configuration is obscured.[9] Particle bombardment, on the other hand, does not necessarily change the isotopic spin, although both $T=0$ and $T=1$ states can be produced if protons or neutrons are used as projectiles. Experiments done with protons and neutrons above 20 Mev have tended to yield alpha-particle spectra consistent with four-body phase-space distributions,[10–12] which suggests that the three-body model is preferred. However, the isotopic spin problem can be avoided if alpha particles are used as projectiles, since there are no bound $T=1$ states in the He^4 nucleus. They have certain other advantages as projectiles, being generally available at higher energy and having a shorter wavelength than a proton or neutron of the same energy.

In order to initiate a survey of the ground states of light nuclei, it was therefore decided to examine the spectrum of the alpha particles from Be^9 and C^{12}, using alpha particles from the Livermore 90-in. variable-energy cyclotron. Space limitations restricted the experiment so that only measurements at laboratory angles near 45° could be taken at this time. This geometry is not far from a 90° barycentric angle, which is a convenient angle for the investigation of such reactions.

[9] V. L. Telegdi and W. Zünti, Helv. Phys. Acta **23**, 745 (1950).
[10] J. L. Need, Phys. Rev. **99**, 1356 (1955).
[11] H. B. Knowles, thesis, University of California Radiation Laboratory Report UCRL-3753, 1957 (unpublished).
[12] J. P. Jackson and D. I. Wanklyn, Phys. Rev. **90**, 381 (1953).

EXPERIMENTAL APPARATUS AND PROCEDURE

The alpha-particle beam from the cyclotron passed through two 20° bending magnets while being steered into the target chamber. The last straight leg, in which the target was positioned, was aligned optically, and the alignment was periodically checked during the experiment by examination of the position of the beam spot at the extreme end of the target leg. A quartz plate at this position fluoresced when struck by the beam and could be seen on a closed-circuit television. After passing through the target, the beam was collected on a carbon-backed, nylatron-insulated Faraday cup, located about 40 in. from the target. A bias ring two inches in front of the cup was maintained at −600 v to prevent loss of electrons, and the beam current was integrated by an "Eldorado" ac electrometer. The electrometer was calibrated at regular intervals, and the cabling from the cup tested for high resistance. (See Fig. 1.)

The beam was collimated to $\frac{1}{8}$ in. in width and about 1 in. in height before passing through the targets. Targets were $1\frac{1}{2}$ in. in diameter, and were mounted on a hub which could be rotated to one of four positions remotely, with a positioning error of less than $\frac{1}{32}$ in. The target normal was fixed at $22\frac{1}{2}°$ to the beam direction.

The beryllium target, prepared by vacuum evaporation of metallic beryllium, was approximately 0.9 mg/cm². Carbon targets, ranging from 0.5 to 1.0 mg/cm², were prepared by dipping glass plates into a solution of polystyrene in benzene, and removing the thin film by flotation (alpha particles scattered from hydrogen do not appear at angles greater than 15°). Both targets were unbacked.

Because it was necessary to analyze the spectrum of alpha particles to as low an energy as possible in order to examine the nature of the distribution, particular care was taken to keep the detecting and analyzing system "thin." Thus, a magnet was used to direct all particles of a given momentum into a thin proportional counter. The counter had a window of aluminized mylar, 0.00025-in. thick, was filled to 5 in. absolute pressure with 96% argon-4% carbon dioxide, and would accept alpha particles down to 2 Mev.

Momentum calibration of the magnet was done by the floating-wire technique. The particle orbit is defined by three points, namely, the target-beam intersection, the pre-magnet collimator slit, and the counter collimator. The latter is variable in width; at the counter the momentum acceptance of the slit is determined by the dispersion formula for a circular magnet:

$$\frac{\partial x}{\partial \epsilon}=\rho(1-\cos\theta)+L_2\left[\frac{\sin(\theta-\alpha)+\sin\alpha}{\cos\alpha}\right], \quad (1)$$

in which ρ is the mean bending radius, θ the mean bending angle, α the angle between the normal to the magnet edge and the beam entry or exit, L_2 the distance from the edge of the magnet to the counter, and $\delta\epsilon=\delta p/p$, the proportional momentum variation. A characteristic of magnetic analysis is that the proportional momentum interval taken is the same for all momenta. It is thus useful to be able to open the counter aperture when analyzing at low fields to avoid crowding together of the data in an energy region where fine structure does not occur. In this experiment the counter collimator could be opened sufficiently to accept a $\delta\epsilon$ of 2%. The intrinsic focal length of the magnet is about 10 meters, and it was impossible to place the target and counter at conjugate points. The resulting lack of focus is the primary cause of the resolution width, which is about 4% full width at half-maximum in momentum, and therefore reasonably competitive with scintillation techniques.

At any given magnet shunt-voltage value V_s (which is proportional to current), a particle entering the counter has a certain momentum per unit charge

$$Mv/Z \propto V_s. \quad (2)$$

The response of the counter is a pulse height H, roughly proportional to stopping power

$$H \propto Z^2/v^2 \propto M^2/V_s^2. \quad (3)$$

Thus, at any shunt-voltage value V_s, particles of masses 4, 3, and 2, can be, in principle, well resolved by pulse height. In order to keep the pulse sizes about the same at any value of V_s, the gain of the linear amplifier was empirically varied in a manner roughly proportional to V_s^2 (as indicated in Fig. 2). The output of the linear amplifier was fed into two discriminated scalers and a 256-channel pulse-height analyzer. At higher energies Landau effect caused the counter-pulse spectrum from alpha particles to spread from about 25 v to almost 70 v (for a 100-v maximum); at lower energies, the pulse was much narrower and more symmetrical. (See Fig. 2.) The discriminators on the scalers were set to accept the observed counter spectrum, and the algebraic difference between the counts recorded on the two scalers was taken to give the number of alpha particles. Tape records of the pulse-height-analyzer spectra were used to correct any runs in which there was a suspicion that the amplifier gain or the discriminators had been improperly set.

The magnet was varied so that contiguous momentum windows were counted in almost every case (e.g., when the counter collimator was set to 1% width, the

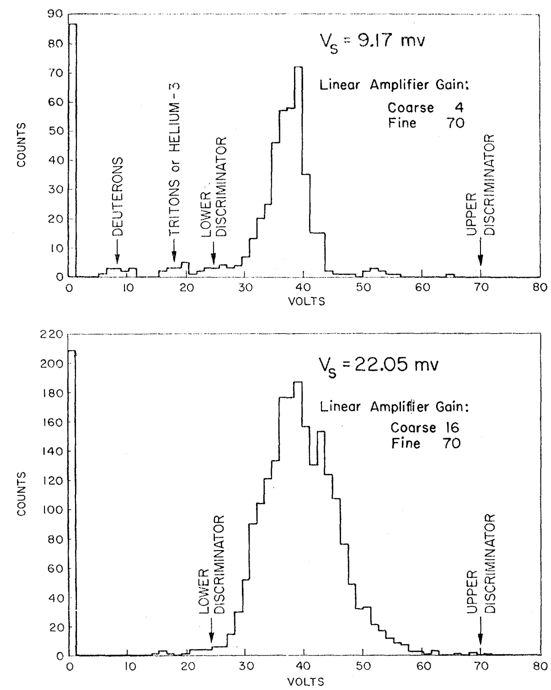

FIG. 2. Spectra showing Landau effect and discriminator setting.

magnet shunt voltage V_s was varied in 1% steps). Spectra were usually taken by steady reductions of V_s to avoid hysteresis effects.

Data reduction was done by an IBM-709 "Fortran" code. A preliminary calculation, using the magnet calibration and the parameters of the reaction, was used to determine the initial beam energy from the elastic peak. Using this derived information, the differential cross section per unit energy spectrum ($d^2\sigma/d\Omega dE$) was calculated in the laboratory system, and then both the energy and ($d^2\sigma/d\Omega dE$) spectrum were corrected for energy loss in the target. Especially at low energies, a spectrum is somewhat depressed from its true value by a target which is an appreciable fraction of the range at that energy, because the particles appear spread out over a larger energy interval. The target-corrected results were then transferred to the center-of-mass system by use of the relation: $p^{-1}(d^2\sigma/d\Omega dE)$ is an invariant.[13]

THE IDENTIFICATION OF SPECTRA

In the following section, it will generally be assumed that a transformation to the center-of-mass has been done, and that the energy and angles are the barycentric values, unless explicitly stated otherwise.

The multibody distributions have been calculated on the simple phase-space assumption, viz.,

$$P_3(E)dE=C_3[E(E_{\max}-E)]^{\frac{1}{2}}dE, \quad (4)$$

$$P_4(E)dE=C_4(E)^{\frac{1}{2}}(E_{\max}-E)^2dE, \quad (5)$$

[13] A. H. Rosenfeld, Phys. Rev. 96, 130 (1954).

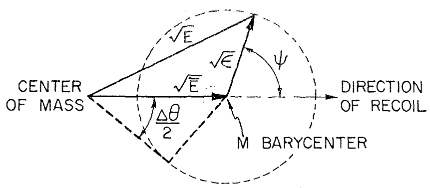

FIG. 3. Dynamics of stepwise decay.

primarily because the effects of the Coulomb force and other final-state interactions cannot be introduced in any simple way. Phillips[14] has suggested a technique for Coulomb-effect estimation; the result indicates that the difficulty of penetration of the Coulomb barrier suppresses the low-energy end of the spectrum, and, therefore, the high-energy end as well (if the process is in fact a multibody process). If the inhibition is sufficiently strong, something very much like stepwise decay is suggested and observations at lower bombarding energies tend to confirm this.[15]

When interpreting a spectrum of particles resulting from a multibody process, a broad energy distribution must be examined, rather than the more familiar set of peaks that result from two-body interactions. It is necessary in such a spectrum to measure particles at the lowest possible laboratory energy, and this introduces a secondary complication; the masking of the low-energy shape by superposed spectra, which are caused by the decay in flight of excited nuclei. If, as in this experiment, the projectile particle is the same as the selectivity detected particle, the presence of an inelastic peak permits one to infer the existence of a recoiling nucleus. Should the latter be energetically able to emit one of the detected particles, it is possible to establish, at the least, the *limits* of energy which the particle can have. In certain simple cases, the shape of the spectrum between these limits can also be calculated.

This energy calculation is explicitly noted: If a particle of mass M and kinetic energy E_0 breaks into masses m and m', with energy release Q, and ψ designates the angle of decay (in the barycenter of M) of mass m with respect to the original direction of motion of mass M, it can be shown that the kinetic energy of m is

$$E=\bar{E}+\epsilon+2(\bar{E}\epsilon)^{\frac{1}{2}}\cos\psi, \tag{6}$$

in which $\bar{E}=(m/M)E_0$ and $\epsilon=(m'/M)Q$. (See Fig. 3.) The minimum value E_1 and maximum value E_2 of E are given by

$$E_1=(\bar{E}^{\frac{1}{2}}-\epsilon^{\frac{1}{2}})^2, \tag{7a}$$

$$E_2=(\bar{E}^{\frac{1}{2}}+\epsilon^{\frac{1}{2}})^2. \tag{7b}$$

If, in addition, it is possible to assume isotropy of emission of M from the projectile-target barycenter and isotropy of m decay from the M barycenter, it is clear that the weighting of E depends only on the solid angle $2\pi\sin\psi d\psi$ into which m may decay. But since, by differentiation of Eq. (6),

$$dE\propto\sin\psi d\psi, \tag{8}$$

the E spectrum is a square (flat-topped) distribution, extending from E_1 to E_2. On the other hand, if either the reaction from which M originates or the decay it subsequently undergoes is anisotropic, any number of complicated spectra in E are possible. In the case of many of the excited states of both of the nuclei under consideration here, anisotropic (oscillatory) inelastic distributions are expected to occur, so one of the necessary conditions for a flat distribution is immediately voided; spins which are known to differ from zero for these levels may void the other. However, an estimate of the size of the distribution may be made, in principle. The recoil nuclei which may decay to contribute to the spectrum have a mean angle $\bar{\theta}$ (the original direction of M) and a total spread of the angle $\Delta\theta$ centered on $\bar{\theta}$ which can be seen by Fig. 3 to be

$$\Delta\theta=2\sin^{-1}(\epsilon/\bar{E})^{\frac{1}{2}}. \tag{9}$$

If $\epsilon\ll\bar{E}$, the $\Delta\theta$ will be narrow and there will be participation from nuclei recoiling over a small range of $\Delta\theta$ near $\bar{\theta}$, and the recoil-decay spectrum can be estimated in magnitude if the general variation in size of the inelastic single-particle peak over the range of the angles supplementary to $\bar{\theta}$ is known. Contrariwise, if $\epsilon\simeq\bar{E}$ recoil nuclei from almost all angles can participate, including those at very large backward angles which may come from large forward peaks, and it would not be surprising to find distributions several times larger than the differential cross section as measured from the single-particle peak. In effect, such a recoil-decay spectrum is an average, modified by the polarization of the recoil nucleus and its breakup anisotropy. (Polarization will tend to favor decay particles from nuclei recoiling into the plane defined by the beam and the counter position.) Calculation of such a spectrum would require additional information about anisotropy of decay from the barycenter of the excited state and a suitable average of the polarization produced by the beam. But the energy limits of such a spectrum as given in Eqs. (7) can always be calculated, and if isotropy is taken as the first-order assumption, certain additional general features of a distribution may appear. The calculated spectra shown in the next sections are based upon this premise. Dynamically, two- and three-body decay from an inelastically excited recoiling nucleus, and two-body decay from a recoil nucleus which has a three-body breakup energy distribution have been assumed to be isotropic in both stages. There is some evidence that near the 90° barycentric angle the multibody breakup of C^{12} is fairly flat.[11]

[14] G. C. Phillips (private communication, 1961).

[15] R. R. Spencer, G. C. Phillips, and T. E. Young, Nuclear Phys. 21, 310 (1960).

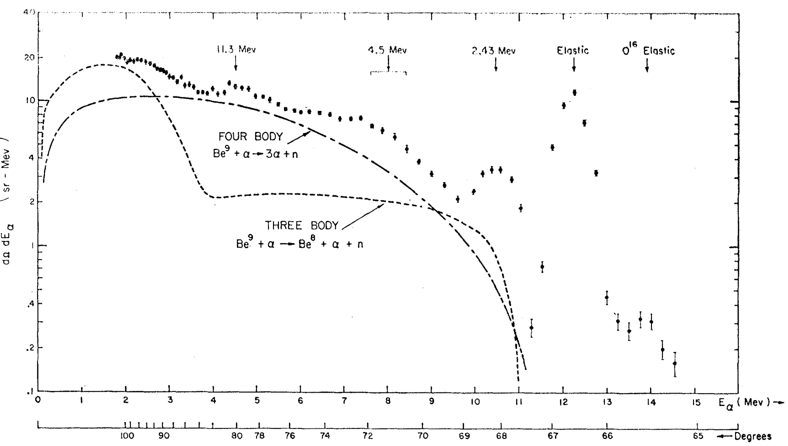

FIG. 4. Be^9 barycentric spectrum compared with three- and four-body spectra.

RESULTS

A. Beryllium

The barycentric alpha-particle spectrum from beryllium is shown in Fig. 4, together with the barycentric angles corresponding to each energy. The elastic peak lies above a low background (of the order of the target-out counting rate), and shows a very small satellite peak, which is explained as elastic scattering from the oxygen contamination of the target. The 2.43-Mev level lies over the high-energy limit of a continuously rising alpha spectrum, coming from the breakup of the beryllium nucleus. Other immediately discernible features are a peak corresponding to inelastic alpha scattering from the 11.3-Mev level of Be^9, well above the Coulomb barrier, and a ramp-shaped distribution at the extreme lower end of the spectrum. This feature is about as large as the continuous spectrum beneath it, a relation suppressed by the logarithmic vertical scale.

1. Direct Multibody Breakup

The continuous distribution under the peaks is the most significant feature of the spectrum. It was assumed to have one of two possible origins, the three-body reaction

$$Be^9+\alpha \rightarrow Be^8\ (\text{g.s.})+\alpha+n, \qquad \text{(I)}$$

or the four-body reaction

$$Be^9+\alpha \rightarrow 3\alpha+n. \qquad \text{(II)}$$

The expected spectra from these two reactions at the measured beam energy (25.41 Mev) were calculated. The assumption of isotropy is probably quite good in both cases, because of the reason previously stated, and because the zero-spin Be^8 nucleus decays isotropically. On Fig. 4 are plotted the relative shapes of the three- and four-body reactions; in the former the single-alpha distribution has been appropriately normalized to the superposed distribution of Be^8 decay (in flight) into two alphas. It is immediately evident that the four-body breakup dominates the multibody distribution. However, the peak of the three-body spectrum corresponds to the low-energy feature (which cannot be attributed to inelastic scattering because of the Coulomb barrier), and it is therefore possible that some amount of three-body breakup competes with the four-body breakup. Figure 5 shows a calculated energy spectrum in which a fraction of the three-body direct breakup reaction (I) combined with (II), the four-body direct breakup reaction, and is superposed on the experimental spectrum. It is seen that the over-all fit is generally somewhat better than for four-body breakup only, with particular note being taken of the improved fit immediately below the 2.43-Mev single-particle peak. The expected broad 4.5 level of Be^9 can now be seen more clearly. The normalized multibody distributions for this fit integrate to give

$$(d\sigma/d\Omega)_3 \simeq 10 \text{ mb/sr} \quad \text{for three-body breakup,}$$

and

$$(d\sigma/d\Omega)_4 \simeq 25 \text{ mb/sr} \quad \text{for four-body breakup,}$$

in the barycentric system. These differential cross sections represent the *maximum* amount of the three-body reaction (I) and the *minimum* of the four-body reaction (II). It should be remarked that, over most of the spectrum, (I) yields one alpha particle for every three from (II) and is consequently more difficult to identify in the absence of low-energy data. While the above combination of (I) and (II) represents one limiting explanation of the spectrum, it is not the only one possible. If the spectrum is assumed to arise only from the direct four-body reaction (II) (with the addition of the Be^{9*} decay discussed below), its differential cross section is

$$(d\sigma/d\Omega)_4 = 29.5 \text{ mb/sr},$$

which represents the maximum for this reaction.

2. Be^{9*} *Decay*

As an alternate explanation for the low-energy ramp, the decay in flight of the recoiling Be^{9*} nucleus should

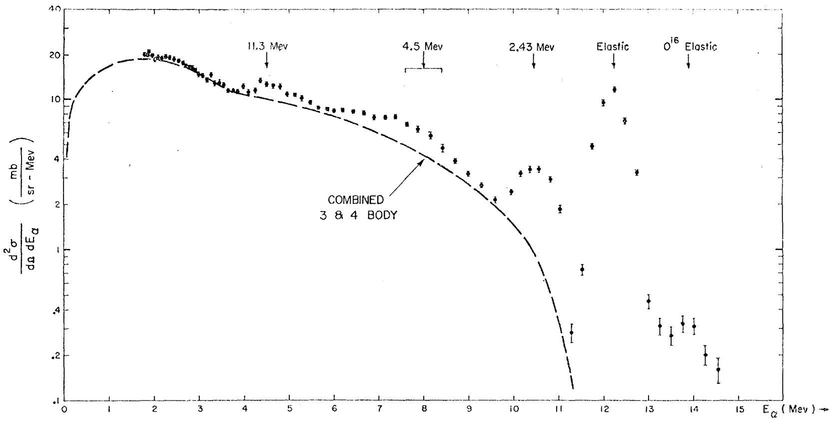

FIG. 5. Be^9 barycentric spectrum fitted to a mixture of three- and four-body decay.

be considered. Figure 6 shows a linear presentation of the barycentric spectra for the three dynamically possible decays (all normalized to 1 event/steradian):

$$Be^{9*}(2.43\ \text{Mev}) \to Be^8\ (\text{g.s.}) + n, \qquad \text{(a)}$$

$$Be^{9*}(2.43\ \text{Mev}) \to 2\alpha + n, \qquad \text{(b)}$$

$$Be^{9*}(2.43\ \text{Mev}) \to He^5\ (\text{g.s.}) + \alpha. \qquad \text{(c)}$$

These were calculated on the previously stated assumptions of double isotropy, which is clearly not correct, because the 2.43-Mev level is believed to be in a $\frac{5}{2}^-$ state. However, because reliance can be placed upon the energy limits, it is clear that if any of the recoil-decay spectra are to account for all of the ramp feature, the three-body decay (b) is a very unlikely explanation, because its spectrum extends to an energy well above 4 Mev. (A spectrum resulting from anisotropic decay of a nucleus in a high-spin state will probably show an increase at low and high values of ψ, that is, at maximum and minimum energies, over a spectrum calculated from the assumption of isotropy.) Of the two remaining possibilities (a) and (c), it has been shown that the decay through Be^8, (a), can account for, at most, 10% of all the decay of this level.[16] The decay (c) through He^5 cannot be excluded on the basis of its shape because, if it were to occur, the narrow peak would be spread out by the target thickness, and the distribution would somewhat resemble that shown for (a).

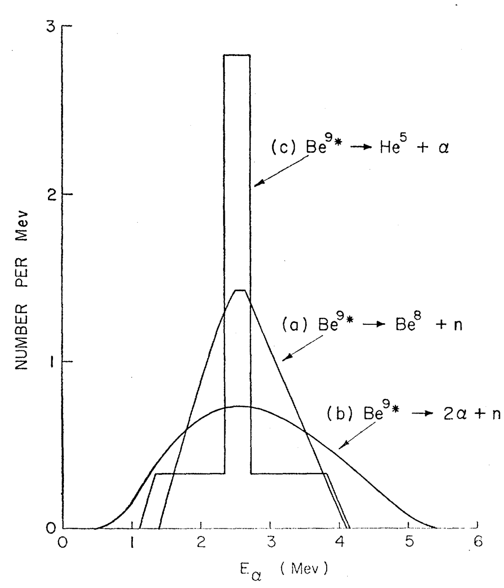

FIG. 6. Calculated decay in flight spectra of three possible modes of Be^{9*} (2.43 Mev).

The question of size remains, if the ramp structure is to be assigned to Be^{9*} decay, taking account of the two alpha particles per single event, the differential cross section derived from the ramp is about 10 mb/sr. In this case, the total angular spread $\Delta\theta$ from which alphas may come is relatively narrow (26°) and is centered about a mean value of 94° Be^{9*} barycentric recoil angle. This implies an average cross section of 10 mb/sr for inelastic alphas at barycentric angles $86°\pm13°$, which is to be compared to the directly measured differential cross section of 2.8 mb/sr at 63° barycentric angle. There are no reported measurements of this differential cross section at 25 Mev to which these values can be directly compared. If all of the ramp structure consists of Be^{9*} decay alpha particles, there is a three-fold increase in differential cross section between 63° and 86°; nor can this explanation be excluded. However, such applicable measurements as have been taken at 19 Mev suggest that it decreases or stays constant,[17] and the 48-Mev data of Summers-Gill[18] tends to confirm this, especially if the positions of the inelastic scattering maxima are reinterpreted for the lower-energy alpha particle with the same nuclear radius. It appears quite possible that, even if the differential cross sections are the same at the two angles, there will be some suppression of the size of the decay spectrum, because of the polarization of the recoil

[16] D. Bodonsky, S. F. Eccles, and I. Halpern, Phys. Rev. **108**, 1019 (1957).

[17] W. O. McMinn, M. B. Simpson, and V. K. Rasmussen, Phys. Rev. **84**, 963 (1951).

[18] R. G. Summers-Gill, Phys. Rev. **109**, 1591 (1958).

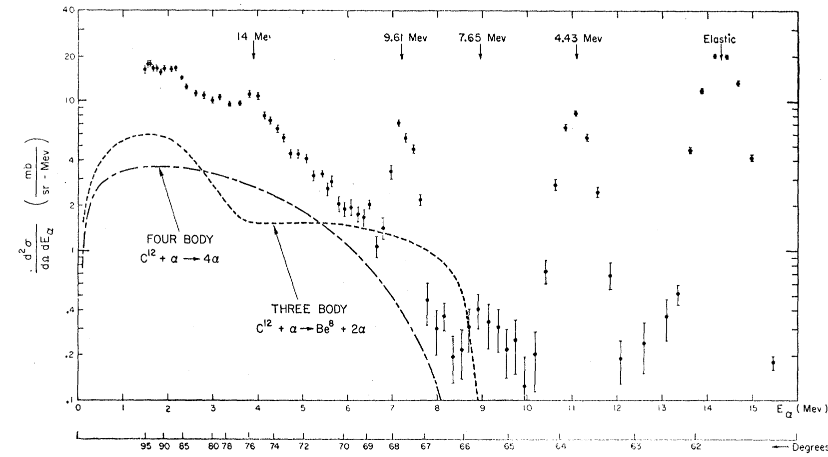

FIG. 7. C^{12} barycentric spectrum compared with three- and four-body spectra.

nucleus. It is therefore deemed more probable that the low-energy feature is caused by a mixture of (I) direct three-body breakup, and of Be^{9*} decay through either or both the (b) and (c) modes.

B. Carbon

The carbon data, which were taken at 25.41 Mev, are shown in Fig. 7, corrected to the barycentric system. Peaks corresponding to elastic scattering and the 4.43, 7.65, and 9.61-Mev levels are observed (the 7.65-Mev level was unusually low at this particular energy, in contrast to observations at other energies). Beneath the 7.65-Mev peak a continuum begins, which rises rapidly with decreasing energy, and upon which there appear no statistically significant peaks, with the obvious exception of the 9.61-Mev peak and, possibly, a peak indicating a level at about 14-Mev excitation. It appears at approximately the edge of the Coulomb barrier (which corresponds to 3.6-Mev barycentric alpha-particle energy). Although a C^{12} level has been reported near this value, another possible mechanism leading to a peak will be discussed below.

At the lowest energy, a feature somewhat like that observed on the beryllium spectrum appears, and identification of this feature requires a more extensive investigation of the possible reactions than was necessary for beryllium.

1. Multibody Reactions

As in the case of beryllium, both direct three- and four-body reactions were considered,

$$C^{12}+\alpha \rightarrow Be^8\ (\text{g.s.})+2\alpha, \qquad \text{(I)}$$

$$C^{12}+\alpha \rightarrow 4\alpha, \qquad \text{(II)}$$

and the calculated spectra are shown superposed on Fig. 7. There is no immediately evident good fit for either, but upon comparison with the high-energy part of the spectrum (7.5 Mev), the spectrum of the direct four-body reaction (II) corresponds to the measurement rather well, while the three-body spectrum (I) cannot be fitted anywhere. Three-body breakup from carbon will be more evident than from beryllium, because there should be two energetic alpha particles; it is pertinent that any significant amount of (I) added to (II) spoils the high-energy fit without materially improving the fit at lower energies. In particular, the low-energy rise could not be explained by the presence of (I). To set an upper limit on the direct three-body reaction,

$$(d\sigma/d\Omega)_3 \leq 1.7 \text{ mb/sr}.$$

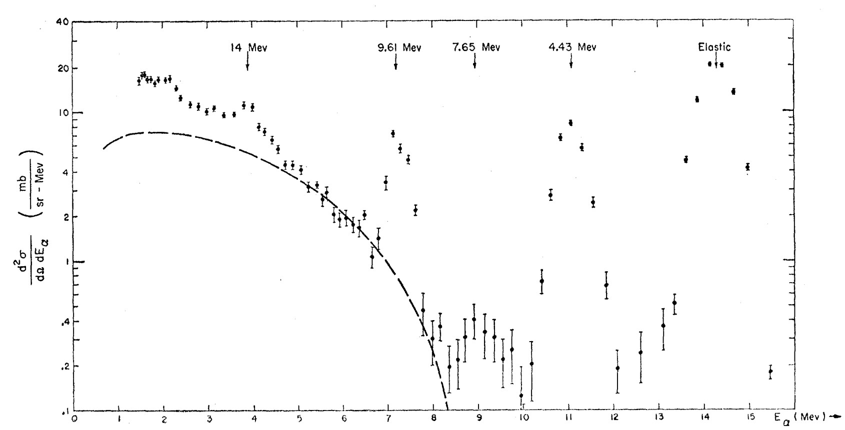

FIG. 8. C^{12} barycentric spectrum fitted to the four-body spectrum.

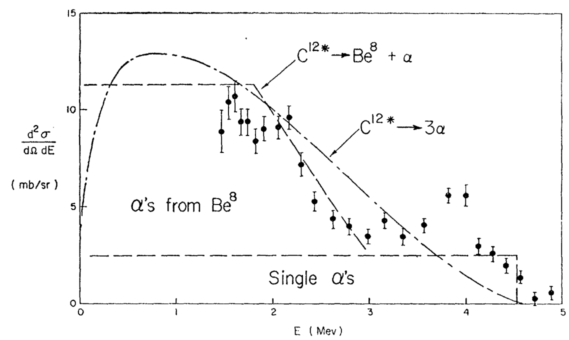

FIG. 9. Difference between fitted four-body spectrum and observed C^{12} spectrum against two possible modes of C^{12*} (9.61 Mev) decay.

The differential cross section for the four-body reaction to which (I) is to be added has a minimum value

$$(d\sigma/d\Omega)_4 \geq 8.2 \text{ mb/sr},$$

and a maximum value, if *no* three-body breakup is assumed, of

$$(d\sigma/d\Omega)_4 \leq 9.2 \text{ mb/sr}.$$

This value will be used subsequently as a background value, and is plotted against the data in Fig. 8. Below 4.5 Mev, the low-energy spectrum lies above the calculated spectrum for the four-body reaction, and it is this disparity which requires an explanation.

A third direct breakup reaction

$$C^{12}+\alpha \rightarrow Be^{8*}(2.90 \text{ Mev})+2\alpha, \qquad \text{(III)}$$

was also considered, but since its spectrum does not appear to fit any of the features, and did not extend above the barycentric energy 6.9 Mev, it will not be discussed further.

2. Possible Reactions Leading to Alpha Particles

The calculated spectra of the following alpha-producing reactions were also examined for their energy limits, and discarded because these limits were not compatible with any features:

$$C^{12}+\alpha \rightarrow 2Be^8 \text{ (g.s.)}, \qquad \text{(IV)}$$

$$C^{12}+\alpha \rightarrow 2Be^8(2.90 \text{ Mev}), \qquad \text{(V)}$$

$$C^{12}+\alpha \rightarrow Be^8 \text{ (g.s.)}+Be^8(2.90 \text{ Mev}), \qquad \text{(VI)}$$

$$C^{12}+\alpha \rightarrow p+N^{15*}; \quad N^{15*} \rightarrow \alpha+B^{11} \text{ (g.s.)}. \qquad \text{(VII)}$$

The reaction (VII) has been reported to be particularly troublesome at higher alpha energies,[19] but for 25-Mev bombarding energy the final alpha particle has less than 1.5-Mev barycentric energy.

3. C^{12*} Decay

The reaction

$$C^{12}+\alpha \rightarrow C^{12*}(9.61 \text{ Mev})+\alpha' \qquad \text{(VIII)}$$

is observed to occur, and to a much lesser degree, the reaction

$$C^{12}+\alpha \rightarrow C^{12*}(7.65 \text{ Mev})+\alpha', \qquad \text{(IX)}$$

which is reported to be about an order of magnitude lower in average differential cross section than (VIII) at 31.5 Mev.[20] Either of these can decay by

$$C^{12*} \rightarrow Be^{8*}(\text{g.s.})+\alpha, \qquad \text{(a)}$$

or

$$C^{12} \rightarrow 3\alpha, \qquad \text{(b)}$$

but Need[10] reports that the 9.61-Mev level goes only through (a), and this is confirmed by other data.[12] The barycentric energy maximum for both (IX a) and (IX b) alpha particles is about 1.80 Mev, and some part of the low-energy feature might arise from these events. However, as in any reaction in which the energy available for decay is much less than the recoil energy of the nucleus—that is, when the inelastic scattering level lies just above the breakup threshold—only particles in a narrow angular cone of recoil angle (small $\Delta\theta$) can be accepted (viz., Be^{9*}). This limitation, together with the small size of its inelastic cross section, suggests that the contribution of the 7.65-Mev level cannot be very large.

Of the two remaining possibilities, (VIII a) is expected to be the dominant mechanism. Figure 9 shows the difference between the observed spectrum and the extrapolated level of direct four-body breakup. The calculated spectra for processes (VIII a) and (VIII b) are superposed, both normalized to 10 mb/sr. The fit to (VIII a) is definitely preferred, because the calculated maximum energy of the single-alpha decay is at the energy at which the data depart abruptly from the four-body spectrum, and the calculated maximum alpha energy from the decay of Be^8 (g.s.) lies at the next observed rise. The departure of the shape between the limits of energy is attributable to the anisotropy of the recoiling C^{12*} nucleus and to its spin, which is believed to be 1. The nucleus will probably tend to favor forward and backward decay with respect to its direction of motion, and, depending upon the angular distribution of the recoil nuclei, is probably able to produce a maximum somewhere within the spectrum when the various distributions are averaged. This is believed to be a possible explanation of the apparent 14-Mev excitation.

The magnitude of the inelastic single-alpha peak is 3.71 mb/sr measured at 63°, which is smaller than the estimated 10 mb/sr of the recoil-decay spectrum at angles between 70° and 90°. But, in this instance, the ratio of $\epsilon/\bar{E}$ is about unity and $\Delta\theta \simeq 180°$, so that inelastically recoiling nuclei from almost any direction can project alpha particles into the observed distribution. The factor of three in the ratio of cross sections now becomes more acceptable. It is therefore reasonable that all of the low-energy features can be attributed to (VIII a).

[19] J. S. Blair (private communication, 1961).

[20] H. J. Watters, Phys. Rev. 103, 1763 (1956).

DISCUSSION

As noted, the spectra of the three- and four-body reactions were calculated without considering final-state interactions. Some of each measured spectrum was obscured so a complete comparison cannot be made. However, in the case of the high-energy part of the carbon spectrum, the fit of the four-body is good within statistics. This is a little surprising, since one high-energy alpha implies three low-energy alphas, which will certainly interact by Coulomb repulsion. Thus, this is one observable region in which a departure from the phase-space derivative spectrum is expected, and it is not observed. For beryllium, the discrepancy between the calculated four-body spectrum and the observed spectrum is, at least in part, probably attributable to the presence of some three-body breakup. This is, in effect, the inclusion of a very strong final-state interaction between two of the alpha particles. It is not immediately clear why it should appear in the nucleus with the lower atomic number and lower breakup threshold. But in both cases, the unmodified phase-space derivative spectra appear to fit the observations within the error.

It is not known whether the multibody breakup processes pass through a compound nucleus stage or result from direct interaction. Observations at a variety of angles are necessary to determine this. Of the two, direct interaction might be better reconciled with the lack of Coulomb effects on the spectra, but the compound nucleus as an intermediate state is also possible. Four-body breakup with 32-Mev protons appeared to have a differential cross section which was symmetrical around 90°.[11]

A strong case has been made for the stepwise breakup of nuclei through intermediate states:

$$A^* \to B^* + b + (E-Q),$$
$$B^* \to c+d+Q, \quad \text{stepwise}, \qquad (10a)$$

as opposed to

$$A^* \to b+c+d+E, \quad \text{three-body}, \qquad (10b)$$

by Phillips *et al.*[21] They suggest that the first decay is dependent on a generalized density of states in the B^* nucleus which decays as it is formed. This formulation of the process is quite closely related to the compound nucleus model, and has been successfully applied to breakup of the Be^9 nucleus by 5.5-Mev protons.[15] An attempt will be made below to develop an explanation for the apparent disparity between this result and that of the experiment reported here.

1. Compound Nucleus

The nucleus A^* formed by the interaction of the incident alpha particle with either Be^9 or C^{12} has a transition probability to the next state

$$w = (2\pi/\hbar)|H_{fi}|^2 S_n(E), \qquad (11)$$

in which $S_n(E)$ represents the density of states in the subsequent state, which may be either a two-, three-, or four-body state. The Coulomb effect is intrinsically included in Eq. (12), if the wave functions in H_{fi} are modified. It may be roughly estimated by a model which assumes that all nuclei have the same diameter d. Then the Coulomb barrier energy U_c for Be^9 or C^{12} in the two-, three-, and four-body states can be inferred by bringing the various fragments up to a common separation d. On this basis (for beryllium plus alpha),

$$(U_c)_2 = 8(e^2/d) \simeq 2 \text{ Mev},$$
$$(U_c)_3 = 8(e^2/d) \simeq 2 \text{ Mev},$$
$$(U_c)_4 = 12(e^2/d) \simeq 3 \text{ Mev},$$

and (for carbon plus alpha)

$$(U_c)_2 = 12(e^2/d) \simeq 3 \text{ Mev},$$
$$(U_c)_3 = 20(e^2/d) \simeq 5 \text{ Mev},$$
$$(U_c)_4 = 24(e^2/d) \simeq 6 \text{ Mev},$$

The Coulomb barrier thus immediately favors two-body over three-body and three-body over four-body production, especially at lower bombarding energies.

In deriving the energy density, the statistical model of Fermi (originally developed in order to estimate multiple pion production) appears to be applicable.[22] The assumption made is that the transition into a given number of particles n from a limited interaction volume V is proportional to $S_n(E)$, provided that the n particles exist. If $M_1, M_2, \cdots, M_n$ exist within V and can appear outside with a total kinetic energy E, the energy density is

$$S_n(E) = (V/8\pi^3\hbar^3)^{n-1} d\Phi/dE, \qquad (12)$$

in which Φ is the volume inside the $3(n-1)$ momentum-space surface satisfying conservation of energy (one of the n momenta goes to satisfy conservation of momentum). The densities of states for two-, three-, and four-body decay are, for different assumed kinetic energies E_2, E_3, E_4:

$$S_2(E_2) = \frac{V}{8\pi^3\hbar^3}\frac{4\pi p^2}{v} = \frac{V}{\sqrt{2}\pi^2\hbar^3}\left(\frac{M_1M_2}{M_1+M_2}\right)^{\frac{3}{2}} E_2^{\frac{1}{2}}, \qquad (13a)$$

$$S_3(E_3) = \frac{V^2}{16\pi^3\hbar^6}\left(\frac{M_1'M_2'M_3'}{M_1'+M_2'+M_3'}\right)^{\frac{3}{2}} E_3^2, \qquad (13b)$$

$$S_4(E_4) = \frac{V^3}{105\sqrt{2}\pi^5\hbar^9} \times \left(\frac{M_1''M_2''M_3''M_4''}{M_1''+M_2''+M_3''+M_4''}\right)^{\frac{3}{2}} E_4^{7/2}, \qquad (13c)$$

in which the two-body density-of-states function has been first stated in its more familiar form.

[21] G. C. Phillips, T. A. Griffy, and L. G. Biedenharn, Nuclear Phys. **21**, 327 (1960).

[22] E. Fermi, *Elementary Particles* (Yale University Press, New Haven, Connecticut, 1951), p. 79.

Generally, a multibody density-of-states function increases more rapidly with bombarding energy than does a two-body, but may still be small at any specific value of E_n because of the $V^{(n-1)}$ limitation. This is consistent with the uncertainty principle, because multiple particle formation within a limited volume permits a higher internal kinetic energy to be shared by a larger number of small momenta. In particular, the ratio of state densities of two- to three-body for the competing processes given in Eqs. (10a) and (10b) is calculable from Eqs. (13a) and (13b):

$$\frac{S_2}{S_3}\propto\frac{(E-Q)^{\frac{1}{2}}}{E^2V}\left(\frac{M_B{}^*}{M_cM_d}\right)^{\frac{3}{2}}, \tag{14}$$

(the mass *sums* are equal) indicating that the phase-space factor for low bombaring energy favors stepwise decay (especially if $Q\ll E$) from heavy nuclei ($V\propto A$) asymmetrically ($M_d\ll M_c\simeq M_B{}^*$). The matrix element must be included, as well as the angular momentum (the above is derived for zero-spin particles) in order to estimate the actual ratio of cross sections. The particles from multibody decay have less average momentum and will consequently have more difficulty decaying from a state of high spin.

From Eqs. (13b) and (13c) the actual ratio of S_3 to S_4 can now be estimated for $Be^9+\alpha$ and $C^{12}+\alpha$ under the conditions of this experiment. In both cases, the mass sums are equal, the resulting kinetic energies are equal (within 96 kev), and

$$M_1'=M_1''=M_\alpha;\quad M_2'=M_2''=M_n \quad \text{for} \quad Be^9+\alpha,$$

and

$$M_1'=M_1''=M_\alpha;\quad M_2'=M_2''=M_\alpha \quad \text{for} \quad C^{12}+\alpha.$$

The ratio for both compound nuclei may be written

$$\frac{S_3(E)}{S_4(E)}=\frac{105\sqrt{2}}{16}\frac{\pi^2\hbar^3}{V}\left(\frac{M_{Be^8}}{M_\alpha M_\alpha}\right)^{\frac{3}{2}}E^{-\frac{3}{2}}, \tag{15}$$

in which V is estimated to be the volume of the compound nucleus of mass A

$$V=\tfrac{4}{3}\pi(A^{\frac{1}{3}}r_0)^3=\tfrac{4}{3}\pi A\,(\alpha\hbar/2m_ec)^3, \tag{16}$$

where α is the fine-structure constant and m_e the electron mass. The computed ratios are

$$(S_3/S_4)_{Be}=0.91 \quad (E=15.9\text{ Mev}),$$
$$(S_3/S_4)_{C}=1.31 \quad (E=11.7\text{ Mev}).$$

These are quite close to unity at the bombarding energy used. This indicates that, if the decay is through a compound nucleus, the products are fairly representative of the ground-state configuration, and, on the basis of the phase-space factor, four-body decay is not greatly favored over three-body decay. Different intensities of the two reactions then indicate different values of the matrix element H_{fi} of Eq. (12). The matrix element represents, in part, an overlap integral which is expected to be larger if the initial state resembles the final state. The experimentally determined preference for four-body over three-body would then indicate that the Be^8 (g.s.) cluster tends to disappear within both nuclei. Moreover, the stronger suggestion of three-body decay from Be^9 than from C^{12} may reflect the nonzero spin of the Be^9 ground state. In general, then, the compound nucleus model suggests that the configurations $(2\alpha+n)$ for Be^9 and (3α) for C^{12} are preferred for the ground states over a two-cluster model in which a Be^8 is formed.

2. Direct Interaction

Twentyfive-Mev alpha particles are expected to produce knock-on collisions on the nuclear surface and are sufficiently above the Coulomb barrier to penetrate the nucleus. The tentatively identified three-body reaction on Be^9 may result from a knock-on collision between the incident alpha and the neutron. The four-body reactions may also be explained as direct interaction phenomena, if the incident alpha collides with one of several pre-existing internal alpha clusters, after which one of the two subsequently recoils with sufficient momentum to knock out a third.

One indication of this process would be roughly equal total cross sections for beryllium and carbon. The Coulomb distortion of the wave function of the incident alpha is greater for carbon, so a ratio of 2:3 is not expected. The differential cross sections reported here do not indicate equal cross sections, but if the reactions result from direct interaction, the total cross section cannot be inferred. On this model, again, the alpha clusters must pre-exist to be ejected, and direct interaction indicates that the ground states of the two nuclei in question have a smaller probability for Be^8 clustering than for multiple alpha formation. On the assumption that the interaction volume is not the compound nucleus, but the volume given by

$$V=\tfrac{4}{3}\pi(\lambda\!\!\!^{-}+R_0)^3, \tag{17}$$

(in which $\lambda\!\!\!^{-}=\hbar/p_\alpha$, the wavelength of the incident alpha, and R_0 is the radius of the target nucleus), Eqs. (13) are still applicable. The volume in Eq. (17) is slightly smaller than that given by Eq. (16), but is approximately the same value at 25-Mev incident energy. The ratios of two-, three-, and four-body phase-space are about the same, and again, the strength of the reaction depends on the matrix element.

CONCLUSIONS

Using either the compound nucleus model or the direct interaction model, both Be^9 and C^{12} have less Be^8 (g.s.) clustering inside than is expected on the basis of the two-cluster models. While the nuclei are in an alpha-particle-like configuration, three subgroups are usually present; in Be^9, two alpha particles and a neutron, and in C^{12}, three alpha particles.

Be^9 shows more evidence of the three-body reaction through Be^8 than does C^{12}. This could result either from a more pronounced tendency to form into two clusters, from direct interaction of the alpha with the neutron, or from a higher centrifugal barrier for the four-body reaction in the $Be^9+\alpha$ compound nucleus. Coulomb barriers also favor three-body breakup, so the preponderance of four-body indicates that it is a more fundamental mode. Between 67° and 100° barycentric angles for Be^9, the following differential cross sections are found:

$$Be^9+\alpha \rightarrow 3\alpha+n,$$
$$29.5 \text{ mb/sr} \geq d\sigma/d\Omega \geq 25 \text{ mb/sr} \quad (\pm 3\text{mb/sr});$$
$$Be^9+\alpha \rightarrow Be^8(\text{g.s.})+\alpha+n,$$
$$10 \text{ mb/sr} \geq d\sigma/d\Omega \geq 3 \text{ mb/sr} \quad (\pm 5 \text{ mb/sr});$$

while the C^{12} breakup shows, between angles 65° and 95°:

$$C^{12}+\alpha \rightarrow 4\alpha, \quad 9 \text{ mb/sr} \geq d\sigma/d\Omega \geq 8 \text{ mb/sr} \quad (\pm 1 \text{ mb/sr});$$
$$C^{12}+\alpha \rightarrow Be^8(\text{g.s.})+2\alpha,$$
$$1.7 \text{ mb/sr} \geq d\sigma/d\Omega \geq 0 \quad (\pm 2 \text{ mb/sr}).$$

(The upper figure for four-body breakup corresponds to the lower figure for three-body breakup for each nucleus.)

However, the 9.61-Mev level of C^{12} appears to decay exclusively through

$$C^{12*}(9.61 \text{ Mev}) \rightarrow Be(\text{g.s.})+\alpha,$$

showing that the two-cluster model is applicable to at least one excited state of carbon.

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