

where  $C$  is the nuclear molar specific heat of the cobalt ions, and  $R$  is the universal gas constant. Using the expression for  $CT^2/R$  derived by Marshall<sup>23</sup> for ferromagnetic media, namely,

$$CT^2/R = \frac{1}{3}I(I+1)(g_N\mu_N H_N/k)^2,$$

where  $I$  is the nuclear spin (for  $\text{Co}^{59}$ ,  $I = \frac{7}{2}$ ),  $g_N$  is the nuclear  $g$  factor,  $\mu_N$  is the nuclear Bohr magneton, and  $H_N$  is the magnetic field at the nucleus, we computed  $H_N$  and obtained the value,

$$|H_N| = 410 \times 10^3 \text{ oe} \pm 10 \times 10^3 \text{ oe}. \quad (14)$$

This field is in excellent agreement with the theoretical calculations of  $H_N$  for a doubly ionized cobalt atom by Freeman and Watson.<sup>24</sup> They obtain the value

$$|H_N| \cong 435 \times 10^3 \text{ oe}. \quad (15)$$

The specific heat of manganese ferrite should also contain a nuclear contribution since it too has a large nuclear magnetic moment. However, nuclear specific heat data on manganese ferrite will not be as easy to interpret, since the manganese ions enter the crystal on both  $A$  and  $B$  sites with various valencies whereas cobalt enters the crystal on the  $B$  sites as a  $2+$  ion.

<sup>24</sup>A. J. Freeman and R. E. Watson, *Phys. Rev.* **123**, 2027 (1961).

Since ferrites have such small specific heats at low temperatures due to their large Debye temperatures and the absence of an electronic specific heat, this nuclear contribution should easily be distinguishable from the spin-wave and lattice specific heats without the necessity of going below  $1^\circ\text{K}$ .

We can conclude from these data that the  $T^{\frac{3}{2}}$  law for the magnetic contribution to the specific heat, as predicted by spin-wave theory, is valid. However, there is a serious discrepancy in the values of the exchange parameters when measured at low and high temperature.

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## Secondary Electron Emissions from Metal Surface by High-Energy Ion and Neutral Atom Bombardments

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To obtain  $\gamma_e$  for  $\text{H}_2^+$  ion bombardment on metal targets, an expression for the secondary electron emission coefficient by fast positive ion bombardment is obtained after adopting a method of calculation similar to that employed by Sternglass. It is shown that the experimental values of  $\gamma_e$  for  $\text{H}_2^+$  ion bombardment agree with the calculated values provided it is assumed that inside the target a hydrogen molecular ion is dissociated into a proton and a hydrogen atom, each having half the energy of the molecular ion. The dissociation cross section of a hydrogen molecular ion inside the target may be given by  $\sigma_d = K\sqrt{T}$ , where  $K = 1.2$ ,  $\sigma_d$  is expressed in units of  $10^{-17} \text{ cm}^2$ , and  $T$  is in Mev.

### I. INTRODUCTION

IT is well known that secondary electrons are emitted when metal surfaces are bombarded by positive ions. In the low-energy range (up to about 1 kev), the secondary electron emission coefficient  $\gamma_e$ , which is defined as the number of electrons emitted by the bombardment of one incident ion, varies directly with the ionization potential of the ion (potential ejection). Above 1 kev, the ejection depends primarily on the kinetic energy of the ion (kinetic ejection).

The secondary electron emission coefficient  $\gamma_e$  for high-energy hydrogen atom bombardment on a metal surface is also calculated and compared with experimental data. In the calculation, a neutral beam of hydrogen atoms is treated inside the target as composed of protons and electrons in addition to hydrogen atoms. Each of these three kinds of particles are capable of producing internal secondaries.

A fair agreement between the calculated and observed values of  $\gamma_e$  for  $\text{H}^+$ ,  $\text{D}^+$ ,  $\text{H}_2^+$ , and  $\text{H}^0$  bombardments has been obtained.

Secondary electron emission for potential ejection has been discussed theoretically by Oliphant and Moon,<sup>1</sup> Massey,<sup>2</sup> Shekhter,<sup>3</sup> Cobas and Lamb,<sup>4</sup> Varnerin,<sup>5</sup> and

<sup>1</sup>M. L. E. Oliphant and P. S. Moon, *Proc. Roy. Soc. (London)* **A127**, 388 (1930).

<sup>2</sup>H. S. W. Massey, *Proc. Cambridge Phil. Soc.* **26**, 386 (1930).

<sup>3</sup>S. S. Shekhter, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **7**, 750 (1937).

<sup>4</sup>A. Cobas and W. E. Lamb, Jr., *Phys. Rev.* **65**, 327 (1944).

<sup>5</sup>L. J. Varnerin, Jr., *Phys. Rev.* **91**, 859 (1953).

Hagstrum,<sup>6</sup> and for kinetic ejection by Kapitza,<sup>7</sup> Sternglass,<sup>8</sup> Ross<sup>9</sup> and Izmailov.<sup>10</sup> In this paper, expressions for  $\gamma_e$  by high-energy positive ion and hydrogen atom bombardments have been deduced. The results are compared with the available experimental data for  $H^+$ ,  $H_2^+$ ,  $H^0$ , and  $D^+$  bombardments. It is found that the values of  $\gamma_e$  for  $H_2^+$  ion bombardments agree with the experimental values, provided it is assumed that hydrogen molecular ions are dissociated inside the target into protons and hydrogen atoms, each having half the energy of the molecular ion. A beam of high-energy  $H^0$  atoms incident on a metal surface has been treated as a mixture of protons, hydrogen atoms, and electrons inside the metal.

## II. EXPRESSION FOR $\gamma_e$ FOR FAST POSITIVE ION BOMBARDMENT

When a metal surface is bombarded by an ion, it can interact with atoms and weakly-bound electrons of the metal leading to the production of excited electrons inside the metal (internal secondaries). A theoretical calculation of Dalgarno and Griffing<sup>11</sup> for the ions passing through hydrogen atoms shows that in the high-energy region ionization is the most important process. Based upon this calculation it is assumed that the internal secondaries are produced by ionization of metal atoms.<sup>12</sup> The number of external secondaries is then calculated after considering the escape of the internal secondaries from the metal surface.

Considering ionization cross section instead of stopping power used by Sternglass<sup>8</sup> and following a method similar to that employed by him, it can be shown that in the high-energy range  $\gamma_e$ , the secondary electron emission coefficient, is given by

$$\gamma_e = 0.5N \sum_{nl} Q_{nl} [1/\alpha - e^{-al}/\alpha],$$

where  $N$  = number of metal atoms per cc,  $Q_{nl}$  = ionization cross section of the metal atom for the  $nl$  shell,  $\alpha$  = absorption coefficient, and  $l$  = range of ion inside the metal; since  $al$  is quite large,  $e^{-al}$  can be neglected in comparison with unity. Hence we obtain

$$\gamma_e = (0.5N/\alpha) \sum_{nl} Q_{nl}. \quad (1)$$

<sup>6</sup> H. D. Hagstrum, Phys. Rev. **96**, 336 (1954).

<sup>7</sup> P. Kapitza, Phil. Mag. **45**, 989 (1923).

<sup>8</sup> E. J. Sternglass, Phys. Rev. **108**, 1 (1957).

<sup>9</sup> O. V. Ross, Z. Physik, **147**, 210 (1957).

<sup>10</sup> S. V. Izmailov, Fiz. Tverdogo Tela, **1**, 1546 (1959).

<sup>11</sup> A. Dalgarno and W. G. Griffing, Proc. Roy. Soc. (London) **A232**, 423 (1955).

<sup>12</sup> Several observations can be cited in support of this assumption; firstly H. S. W. Massey and E. H. S. Burhop [*Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, 1952), Chap. IX.] showed that there are many features common to the ionization of an atom and the production of secondaries by ion bombardments. Secondly, it is found from the application of the adiabatic hypothesis of Massey that the energy at which the ionization cross section is maximum is of the same order as that for which the yield of secondaries is maximum.

TABLE I. Variation of  $\gamma_e$  with energy and mass of the incident ion.

Energy (Mev)	H <sup>+</sup>		D <sup>+</sup>		H <sub>2</sub> <sup>+</sup>	
	$\gamma_e$ for Al (cal)	$\gamma_e$ for Al (obs)	$\gamma_e$ for Al (cal)	$\gamma_e$ for Cu (obs)	$\gamma_e$ for Al (cal)	$\gamma_e$ for Al (obs)
0.7	1.43	1.31	2.64	...	3.71	3.71
1.0	1.08	1.08	1.92	1.41	3.31	3.24
2.0	0.59	0.68	1.18	1.06	2.19	2.24
3.0	0.41	...	0.824	0.85	1.6	...
4.0	0.32	...	0.604	0.71	1.46	...

According to Bethe,<sup>13</sup>

$$Q_{nl} = \frac{2\pi Z_i \epsilon^4 c_{nl} Z_{nl}}{m v^2 |E_{nl}|} \ln \left( \frac{2m v^2}{C_{nl}} \right), \quad (2)$$

where  $m$  = mass of the electron,  $Z_i \epsilon$  = charge of the ion,  $E_{nl}$  = ionization potential of the  $nl$  shell,  $Z_{nl}$  = number of electrons in the  $nl$  shell, and  $C_{nl}$  = certain mean of  $E_k - E_{nl}$  having a value of the same order as  $E_{nl}$ ,

$$c_{nl} = (Z_{\text{eff}}^2 / n^2 a_0^2) \int |x_{nl,k}|^2 dk.$$

The value of the  $c_{nl}$  ranges from 0.28 for the  $1s$  shell to 0.04 for the  $4f$  shell of the hydrogen atom. Substituting the value of  $Q_{nl}$  in Eq. (1) and replacing  $v$  by  $T$  and mass  $M$  of the incident ion, one obtains the secondary electron emission coefficient

$$\gamma_e = \frac{0.5N \pi Z_i^2 \epsilon^4 M}{\alpha m T} \sum_{nl} \frac{c_{nl}}{|E_{nl}|} \ln \left( \frac{4m T}{M C_{nl}} \right). \quad (3)$$

The value of  $\alpha$  is not known definitely. The values obtained from the experiments of Becker<sup>14</sup> and Partesch and Hallwachs<sup>15</sup> show that  $1/\alpha$  is about 100 A. To determine the value of  $\alpha$ , the calculated value of  $\gamma_e$  for 1-Mev ( $H^+$ -Al) emission is made to coincide with the observed value.

Assuming that the two outermost shells of Al are ionized by protons, we have

$$N \sum_{nl} Q_{nl} = 1.86 \times 10^6.$$

Since the experimental value of  $\gamma_e$  for 1-Mev ( $H^+$ -Al) emission is 1.08 (Aarset *et al.*<sup>16</sup>), the value of  $\alpha$  becomes  $8.50 \times 10^5$ , which shows that secondaries can come from a depth of about 120 A. In the above calculation, the value of  $c_{nl}$  is obtained from Bethe's table and  $C_{nl}$  is assumed to be  $\frac{1}{10}$  of  $E_{nl}$ .

In Table I, the variation of  $\gamma_e$  calculated from formula (3) with the energy of the ions is shown for an aluminum

<sup>13</sup> N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), 2nd ed., Chaps. IX and XI.

<sup>14</sup> A. Becker, Ann. Physik **2**, 249 (1929).

<sup>15</sup> A. Partesch and W. Hallwachs, Ann. Physik **41**, 247 (1913).

<sup>16</sup> B. Aarset, R. W. Cloud, and J. G. Trumpf, J. Appl. Phys. **25**, 1365 (1954).

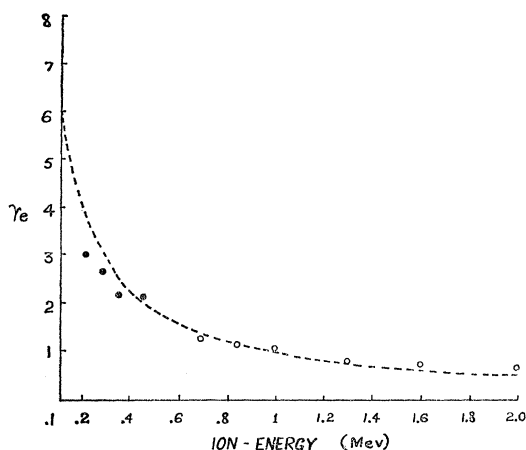


FIG. 1. Comparison of experimental values of  $\gamma_e$  for  $H^+$  ion with the calculated curve as obtained from Eq. (3). (●: obtained experimentally for Al target by Hill *et al.*<sup>17</sup> and ○: by Araset *et al.*<sup>16</sup> for the same target.)

target bombarded by  $H^+$ ,  $H_2^+$ , and  $D^+$ . It will also indicate the variation of  $\gamma_e$  with the mass of the ion. Values observed experimentally for an Al target bombarded by  $H^+$  and  $H_2^+$  ions by Hill *et al.*<sup>17</sup> and Aarset *et al.*<sup>16</sup> and for a Cu target bombarded by  $D^+$  ions by Akishin<sup>18</sup> are also given for comparison. In Fig. 1, experimental values for  $H^+$  ions are compared with the calculated values. For  $H_2^+$  ion bombardment, the  $\gamma_e$  are calculated after considering the splitting of  $H_2^+$  ions into protons and H atoms (see Sec. III). From Table I it follows that there is a fair agreement between the calculated and experimental values. It may be pointed out that the agreement is closer than what is apparent from the table. This is because according to Akishin the determination of  $\gamma_e$  for  $D^+$  ion bombardments is correct up to  $\pm 10\%$  and the error in the measurement of the incident ion energy is  $\pm 200$  kev.

It may be pointed out here that Sternglass<sup>8</sup> also obtained an expression for  $\gamma_e$ . Instead of using ionization cross sections, he utilized stopping power. Although for protons our value for  $\gamma_e$  agrees with that obtained by Sternglass, it may, however, be pointed out that the number of internal secondaries and the absorption coefficient obtained by us differ from those calculated by Sternglass. He assumed that the mean energy loss for the formation of a secondary is 25 ev. This value is small if we consider that the energy of an internal secondary is about this value. It may, however, be pointed out that in addition to ionization, the ion loses energy in producing excitation. According to Dalgarno and Griffing<sup>11</sup> the loss of energy due to excitation compared to ionization loss is about one-third at 1 Mev. Since close collisions are ineffective in producing external secondaries,<sup>8</sup> this ratio should be larger. Assuming the mean energy loss is 25 ev, the number of

internal secondaries is about seven times greater than the value obtained by us. Furthermore, Sternglass has assumed  $1/\alpha$  to be 15.5 Å which is only  $\frac{1}{8}$  of our value. It has been found experimentally<sup>14,15</sup> that for electron bombardments, its value is about 100 Å. Since escape problems of secondaries produced by the bombardments of electrons and ions are the same, this value of  $\alpha$  can be assumed also for ion bombardments. It agrees with our value but is about seven times the value used by Sternglass.

### III. SECONDARY ELECTRON EMISSION BY THE BOMBARDMENT OF HIGH-ENERGY $H_2^+$ IONS ON METAL

While traversing a target, high-energy  $H_2^+$  ions may dissociate into protons and hydrogen atoms, each having half the energy of a hydrogen molecular ion. Furthermore, the hydrogen atom may be ionized, yielding another proton and an electron. In this section,  $\gamma_e$  has been calculated after considering the splitting of  $H_2^+$  ions into protons and hydrogen atoms. It will be shown in the next section that for secondary electron emission, a hydrogen atom of energy  $T$  is equivalent to one proton of the same energy and an electron of energy  $T/1836$ .

At a depth  $x$  inside the metal, the change in the fraction of  $H_2^+$  ions after traversing a distance  $dx$  is given by

$$df_2 = -f_2\sigma_d N dx + f_1\sigma_T N dx,$$

where  $\sigma_d$  and  $\sigma_T$  are dissociation and recombination cross sections, respectively, and

$$f_1 + f_2 = 1.$$

Since  $f_1$  is quite small and  $\sigma_T$  is expected to be small in comparison with  $\sigma_d$ , we have

$$f_2 = e^{-N\sigma_d x}. \quad (4)$$

Assuming that the internal secondaries are produced by the ionization of metal atoms by the bombarding particles, the number of internal secondaries produced between depths  $x$  and  $x+dx$  is given by

$$d\gamma_i = NQ_{H_2^+} f_2 dx + NQ_{H^+} f_1 dx + NQ_{H^0} f_1 dx, \quad (5)$$

where  $Q_{H_2^+}$ ,  $Q_{H^+}$ , and  $Q_{H^0}$  are the ionization cross sections of metal atoms by bombardments of  $H_2^+$ ,  $H^+$ , and  $H^0$ , respectively.

Since the escape probability of the internal secondaries is given by  $0.5 e^{-\alpha x}$  and  $f_1 = 1 - f_2$ , we obtain, after integrating Eq. (5) through the range of penetration  $l$  of bombarding of particle inside the metal,

$$\gamma_e = \frac{0.5N}{\alpha + N\sigma_d} (Q_{H_2^+} - Q_{H^+} - Q_{H^0}) + \frac{0.5N}{\alpha} (Q_{H^+} + Q_{H^0}).$$

(For the same reason as given in Sec. II,  $e^{-\alpha l}$  has been neglected.) Again, since

$$\gamma_{H^+} = (0.5N/\alpha)Q_{H^+},$$

<sup>17</sup> A. G. Hill, W. W. Buechner, J. S. Clark, and J. B. Fisk, *Phys. Rev.* **55**, 463 (1939).

<sup>18</sup> A. I. Akishin, *Zhur. Tekh. Fiz.* **28**, 776 (1958).

TABLE II. Variation of  $\gamma_e$  for  $H_2^+$  ion bombardment with energy. (Al target.)

Energy in Mev	$\gamma_e$ (calculated)	$\gamma_e$ (experimental)	Percent deviation of $\gamma_e$ (exp) from $\gamma_e$ (cal)
0.284	5.92	6.29	+5
0.426	5.15	5.46	+5.6
0.7	3.71	3.71	0
0.85	3.62	3.4	-6.5
1.0	3.31	3.24	-2.2
1.3	2.61	2.85	+8.4
1.6	2.39	2.58	+6.9
2	2.19	2.24	+2.3

we have

$$\gamma_e = \frac{1}{1 + (N/\alpha)\sigma_d} (\gamma_{H_2^+(T)} - \gamma_{H^+(T/2)} - \gamma_{H^0(T/2)}) + \gamma_{H^+(T/2)} + \gamma_{H^0(T/2)},$$

where  $\gamma_{H_2^+(T)}$  is the secondary electron emission coefficient for  $H_2^+$  ions in the absence of dissociation. We finally obtain

$$\gamma_e = \frac{\gamma_{H_2^+(T)} + \{\gamma_{H^+(T/2)} + \gamma_{H^0(T/2)}\} (N/\alpha)\sigma_d}{1 + (N/\alpha)\sigma_d}. \quad (6)$$

Experiments of Barnett *et al.*<sup>19</sup> show that for brass,

$$\gamma_{H^0}/\gamma_{H^+} = 1 + R,$$

where the value of  $R$  increases logarithmically with energy. In particular, its values at 200 kev and 1 Mev are 0.32 and 0.61, respectively. Since, in the high-energy range,  $\gamma_e$  is independent of the target material,<sup>16,17</sup> the above relation can be assumed for all metals. One then obtains

$$\gamma_e = \frac{\gamma_{H_2^+(T)} + \gamma_{H^+(T/2)}(2 + R)(N/\alpha)\sigma_d}{1 + (N/\alpha)\sigma_d}. \quad (7)$$

It is natural that in the high-energy range  $\sigma_d$  increases with energy. In the absence of any data the following simple relation, namely that  $\sigma_d$  varies directly with the velocity of the ion, has been assumed:

$$\sigma_d = K\sqrt{T}. \quad (8)$$

Expressing  $\sigma_d$  in units of  $10^{-17}$  cm<sup>2</sup> and  $T$  in Mev, it is found that for best fit between calculated and experimental values,  $K$  should be equal to 1.2.

It can be shown that in the high-energy range, we have,

$$\gamma_{H_2^+(T)} = 2\gamma_{H^+(T)} \ln\left(\frac{2m}{M} \frac{T}{C_{nl}}\right) / \ln\left(\frac{4m}{M} \frac{T}{C_{nl}}\right). \quad (9)$$

<sup>19</sup>C. F. Barnett and H. K. Reynolds, Phys. Rev. **109**, 355 (1958).

To calculate  $\gamma_e$  for various energies, the values of  $\gamma_{H^+(T)}$  and  $\gamma_{H^+(T/2)}$  are taken from the experimental values of Aarset *et al.*<sup>16</sup> and Hill *et al.*<sup>17</sup> while the value of  $R$  is obtained from experiments of Barnett *et al.*<sup>19</sup> (or extrapolation from their value) and  $K$  is taken to be 1.2. The calculated values of  $\gamma_e$  are shown in Table II, where the experimental values are also given for comparison.

The maximum deviation between observed and calculated values of  $\gamma_e$  is about 8%. Considering the various assumptions involved and the simple relation which has been assumed for the dissociation of molecular hydrogen ions into protons and hydrogen atoms, this deviation is not great. The actual relation between  $\sigma_d$  and  $T$  may be quite complicated.

#### IV. SECONDARY ELECTRON EMISSION BY BOMBARDMENT OF HIGH-ENERGY HYDROGEN ATOMS

Since in the high-energy region the loss cross section  $\sigma_{01}$  is many times larger than the capture cross section  $\sigma_{10}$ , a hydrogen atom of energy  $T$  while traversing a target is ionized, yielding one proton of the same energy and an electron of energy  $T/(M/m)$ . Thus

$$H^0(T) \rightarrow H^+(T) + \bar{e}(T/1836). \quad (M/m = 1836)$$

Hence, a neutral beam of hydrogen atoms can be treated inside the target as composed of protons and electrons in addition to hydrogen atoms. Each of these three kinds of particles are capable of producing internal secondaries.

Consider now a beam of hydrogen atoms penetrating through a metal. The fractional concentration of neutral atoms and protons at a depth  $x$  is given by<sup>20</sup>

$$f_0 = \frac{\sigma_{10}}{\sigma_{10} + \sigma_{01}} + \frac{\sigma_{01}}{\sigma_{10} + \sigma_{01}} e^{-N(\sigma_{10} + \sigma_{01})x}, \quad (10)$$

and

$$f_1 = \frac{\sigma_{01}}{\sigma_{10} + \sigma_{01}} - \frac{\sigma_{01}}{\sigma_{10} + \sigma_{01}} e^{-N(\sigma_{10} + \sigma_{01})x}. \quad (11)$$

Since each hydrogen atom produces one proton and one electron, the number of electrons is equal to  $f_1$ .

The number of internal secondaries produced between depths  $x$  and  $x+dx$  is given by

$$d\gamma_i = N f_0 Q_{H^0} dx + N f_1 Q_{H^+} dx + f_1 A dx, \quad (12)$$

where  $A$  represents the number of internal secondaries produced by electrons in traveling through unit distance. Substituting the values of  $f_0$  and  $f_1$  from Eqs. (10) and (11) in Eq. (12), multiplying by the escape

<sup>20</sup>S. K. Allison, Revs. Modern Phys. **30**, 1137 (1958).

TABLE III. Comparison of theoretical and experimental values of the ratio  $\gamma_{H^0}/\gamma_{H^+}$ .

Energy in kev	$\gamma_{H^0}$	$(\gamma_{H^0}/\gamma_{H^+})_{\text{cal}}$ for Al	$(\gamma_{H^0}/\gamma_{H^+})_{\text{obs}}$ for brass	Percentage deviation
200	3.70	1.20	1.32	+9.0
300	3.13	1.27	1.39	+8.7
400	2.75	1.36	1.45	+6.2
500	2.47	1.43	1.49	+4.0
600	2.24	1.50	1.52	+1.3
700	2.05	1.56	1.55	-0.7
800	1.91	1.59	1.57	-1.3
900	1.82	1.65	1.59	-3.8
1000	1.79	1.66	1.61	-3.1

probability, and then integrating, we get

$$\gamma_{H^0} = \frac{0.5N}{\alpha} Q_{H^0} \left\{ \frac{\sigma_{10}}{\sigma_{10} + \sigma_{01}} + \frac{\sigma_{01}}{(\sigma_{10} + \sigma_{01})[1 + (N/\alpha)(\sigma_{01} + \sigma_{10})]} \right\} + \frac{0.5}{\alpha} (NQ_{H^+} + A) \left\{ \frac{\sigma_{01}}{\sigma_{01} + \sigma_{10}} - \frac{\sigma_{01}}{(\sigma_{01} + \sigma_{10})[1 + (N/\alpha)(\sigma_{01} + \sigma_{10})]} \right\}. \quad (13)$$

Now

$$\gamma_{H^+} = (0.5N/\alpha)Q_{H^+},$$

and

$$\gamma_{\bar{e}} = 0.5A/\alpha.$$

Hence,

$$\gamma_{H^0} = \frac{0.5N}{\alpha} Q_{H^0(T)} \left[ f_{0\infty} + \frac{f_{1\infty}}{1 + (N/\alpha)\sigma_{01}(1 + f_{0\infty}/f_{1\infty})} \right] + [\gamma_{H^+(T)} + \gamma_{\bar{e}(T/1836)}] f_{1\infty} \times \left[ 1 - \frac{1}{1 + (N/\alpha)\sigma_{01}(1 + f_{0\infty}/f_{1\infty})} \right], \quad (14)$$

where

$$f_{0\infty} = \sigma_{10}/(\sigma_{10} + \sigma_{01}) \quad \text{and} \quad f_{1\infty} = \sigma_{01}/(\sigma_{01} + \sigma_{10})$$

are the equilibrium fractions of protons and hydrogen atoms, respectively.

It was shown by Hall<sup>21</sup> that for metallic foils  $f_{1\infty}$  is several tens times greater than  $f_{0\infty}$  for energies greater than 200 kev. Hence in the energy range under consideration  $f_{0\infty}$  can be neglected in comparison with  $f_{1\infty}$ ; we then have

$$\gamma_{H^0} = \frac{0.5N}{\alpha} Q_{H^0} \frac{1}{1 + (N/\alpha)\sigma_{01}} + (\gamma_{H^+(T)} + \gamma_{\bar{e}(T/1836)}) \left[ 1 - \frac{1}{1 + (N/\alpha)\sigma_{01}} \right],$$

<sup>21</sup> T. Hall, Phys. Rev. **79**, 504 (1950).

assuming  $f_{1\infty}$  to be unity, or

$$\gamma_{H^0} = \frac{[\gamma_{H^+(T)} + \gamma_{\bar{e}(T/1836)}](N/\alpha)\sigma_{01} + (0.5N/\alpha)Q_{H^0(T)}}{1 + (N/\alpha)\sigma_{01}}. \quad (15)$$

To calculate  $\gamma_{H^0}$  at different energies, Eq. (15) is utilized. Experimental values of  $\gamma_{H^+}$  are taken from Hill *et al.*<sup>17</sup> and Aarset *et al.*,<sup>16</sup> while values of  $\gamma_{\bar{e}}$  are taken from Brunning *et al.*<sup>22</sup> To calculate  $\sigma_{01}$  for a neutral hydrogen beam penetrating through aluminum, Bohr's relation<sup>23</sup> for  $\sigma_{01}$  is utilized. According to this relation

$$\sigma_{01} \propto Z^{\frac{2}{3}},$$

$Z$  being the atomic number of the target. Hence we get

$$\sigma_{01(A1)} = \left( \frac{Z_{A1}}{Z_{Ar}} \right)^{\frac{2}{3}} \sigma_{01(Ar)}.$$

Using experimental values of  $\sigma_{01(Ar)}$  given by Barnett *et al.*,<sup>19</sup> the values of  $\sigma_{01(A1)}$  for the 200-kev to 1-Mev energy range are calculated.

In the case of hydrogen atom bombardment, the high-energy beam is rapidly changed to protons and hence the contribution to secondary electrons by bombardment of  $H^0$  is small. Therefore, the second term in Eq. (15) can be neglected in comparison with the first term. Thus, we obtain

$$\gamma_{H^0} = \frac{[\gamma_{H^+(T)} + \gamma_{\bar{e}(T/1836)}](N/\alpha)\sigma_{01}}{1 + (N/\alpha)\sigma_{01}}.$$

The calculated values of  $\gamma_{H^0}$  are shown in Table III. The values of  $\gamma_{H^0}/\gamma_{H^+}$  are also shown and are compared with the experimental values obtained by Barnett *et al.*<sup>19</sup> for brass target.

The maximum deviation of the calculated and observed ratios of secondary electron emission coefficients by neutral hydrogen atoms and protons bombardments is 9%, which is not large because experimental error can be as great as 7%.<sup>19</sup> Furthermore, it should be noted that the experimental data for  $\gamma_{H^+}$  and  $\gamma_{\bar{e}}$  are obtained by different investigators, who might not have performed their experiments under identical conditions. Since the positive error decreases with energy and at the most is 9%, it can be concluded that the production of the internal secondaries by neutral atoms is very small as compared with those produced by protons and electrons. It is to be noted that  $\gamma_{H^0}/\gamma_{H^+}$  calculated for Al is compared with the experimental values for brass. As the agreement is quite good, it indicates that like  $\gamma_{H^+}$ , the coefficient  $\gamma_{H^0}$  is also independent of the target material in the high-energy range.

<sup>22</sup> H. Brunning and J. H. De Boer, Physica **5**, 17 (1938).

<sup>23</sup> N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **18**, No. 8 (1948).