Boson Photoproduction Experiments*

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A general discussion is presented of all possible experiments on photoproduction of spin-zero bosons from spin-$\frac{1}{2}$ fermions. Both scalar and pseudoscalar bosons are considered. The M matrix is constructed in terms of four particularly convenient invariants. There are altogether 64 different experiments possible. Of these, 16 are needed to measure all the bilinear combinations of the coefficients of the invariants in the M matrix. To determine the coefficients themselves (not necessarily unambiguously) seven experiments have to be carried out. The 64 observables are given in terms of the bilinear combinations of the coefficients. An extensive list of experiments is then given each of which determines the parity of the boson participating in the process. All of these experiments require polarized photon sources, and even the simplest one requires the measurement of recoil fermion polarization and the use of a polarized target, although not simultaneously. Sets of seven experiments are then constructed to determine the four coefficients in the M matrix. The simplest of these sets implies the same experimental requirements as the simplest parity experiment. The simplest set of 16 experiments giving all bilinear combinations of the coefficients involves in addition also simultaneously recoil nucleon polarization measurements and the use of polarized targets.

I. INTRODUCTION

PHOTOPRODUCTION of bosons from fermions has for some time been one of the most important tools in our understanding of elementary particle reactions. From the very beginning of pion physics production by photons (together with pion-nucleon scattering) has been the main source of information on the basic pion-nucleon force.[1] Recently, as the energy range available reached the Bev region, and the higher pion-nucleon resonances were found, photoproduction again played an important role.[2] A similar situation is expected to occur for K-meson photoproduction, especially when the energies of photon-producing machines extends beyond the present 1.5 Bev. But even today, K-meson photoproduction experiments contribute significantly to our guesses as to what the parity of the K meson is.[3]

So far, boson photoproduction experiments have been restricted to the simplest three kinds. Differential cross sections with unpolarized photons and unpolarized fermion targets have been measured extensively for some time.[1] The polarization of the recoil fermion under similar circumstances has also been measured.[4] Finally, some experiments have been carried out measuring the differential cross section for polarized photons with unpolarized targets.[5] It is clear, however, that more types of experiments could be carried out with improved experiments techniques. It is also apparent that experimental techniques are, in fact, developing fast. The creation of polarized photon beams has made good progress,[5] and several groups are working on the construction of polarized nucleon targets.[6] It seems therefore appropriate at this time to give a general discussion of all possible experiments involving the photoproduction of a spin-zero boson from a spin-$\frac{1}{2}$ fermion. Such a discussion is the subject of the present paper.

Some photoproduction experiments have been discussed previously.[2,7] The present paper extends these considerations in several respects. We will be concerned with *all* possible experiments, and will list subsets that can determine the production matrix. We will give the results for both scalar and pseudoscalar bosons (assuming the parity of the fermions to be even by definition) and will suggest experiments determining the boson parity. Finally, we present our results in terms of a particularly simple form of the matrix element which makes the formulas for the observables more transparent.

Our discussion will be analogous to the phenomenological treatment of nucleon-nucleon scattering experiments[8] which has in fact proven very useful already. In the case of that reaction, experiments of fair complexity (such as spin correlation measurements) have already been carried out. A similar state of experimental skill in photoproduction will make the present discussion useful. In fact, our results might influence the direction of development of experimental techniques.

One more general remark is in order before the detailed calculations are presented. In this paper we will

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[1] H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, Illinois, 1955), Vol. II, p. 20.

[2] Ronald F. Peierls, Phys. Rev. **118**, 325 (1960) and references mentioned therein. For a survey see also E. H. Bellamy, *Progress in Nuclear Physics* (Pergamon Press, New York, 1960), Vol. 8, pp. 239–291.

[3] M. J. Moravcsik, Phys. Rev. Letters **2**, 352 (1959); Y. Nambu and J. J. Sakurai, *ibid.* **6**, 377 (1961).

[4] P. C. Stein, Phys. Rev. Letters **2**, 473 (1959); R. Querzoli, G. Salvini, and A. Silverman, Nuovo cimento **19**, 53 (1961); L. Bertanza, P. Franzini, I. Mannelli, G. V. Silvestrini and V. Z. Peterson, *ibid.* **19**, 953 (1961); J. O. Maloy, G. A. Salandin, A. Manfredini, V. Z. Peterson, J. I. Friedman and H. Kendall, Phys. Rev. **122**, 1338 (1961).

[5] R. E. Taylor and R. F. Mozley, Phys. Rev. **117**, 835 (1960); R. C. Smith and R. F. Mozley, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 22.

[6] For a summary as of the middle of 1960, see M. H. MacGregor, M. J. Moravcsik and H. P. Stapp, Ann. Rev. Nuc. Sci. **10**, 323 (1960).

[7] G. T. Hoff, Phys. Rev. **122**, 665 (1961).

[8] See, e.g., R. J. N. Phillips, Harwell Atomic Energy Research Establishment Report AERE-R3141, 1960 (unpublished), and references cited therein.

deal with the production amplitudes in terms of the coefficients of the invariants and will not make an expansion into angular momentum and multipole states. It is true that in low-energy pion physics such an expansion has proven useful.[1] Firstly, at low energies only a few states are expected to play a role. Secondly, in low-energy pion physics, even these few states are not of equal importance, and in fact most qualitative features can be explained by considering only one state. Furthermore, for low-energy photopions unitarity and time-reversal invariance requires[9,10] that the complex production amplitude in a given angular momentum state be a real number times an imaginary exponential whose exponent is given by the corresponding pion-nucleon scattering phase shift. This cuts in half the number of real quantities to be determined.

This favorable situation, however, disappears rapidly as we go to either higher energy pions or to K mesons. As the energy increases, the number of states contributing to pion photoproduction increases rapidly, and at the higher resonances already many states have to be taken into account. Furthermore, it is questionable whether these higher resonances are caused by one dominant angular momentum state.[11] Finally, at these energies multiple pion production becomes possible, and this eliminates the simplification imposed by unitarity and time-reversal invariance which we discussed above. The same is true for photoproduced K mesons at *all* energies, since multiple pion production is always possible above the threshold for K production. It is unlikely, therefore, that with the advent of multi-Bev electron accelerators the decomposition into angular momentum states will continue to be a useful concept. For this reason we will not use it in this paper but work in terms of the coefficients of the invariants.

II. FORM OF THE MATRIX ELEMENT

One can easily ascertain that the production matrix for photoproduction of a spin-zero boson with spin-$\frac{1}{2}$ fermions in the initial and final states is a linear combination of four invariants. This can be seen either by looking at the covariant forms,[12] or by just listing the possible vectors and constructing from them rotation invariants (dot products) with the proper parity (to make them reflection-invariant).[13]

It might be interesting to remark parenthetically that the fact that four invariants appear in photoproduction as compared to two for the scattering of a spin-zero boson on a spin-$\frac{1}{2}$ fermion makes the former potentially a more powerful tool in probing the boson-fermion interaction. The more coefficients of invariant one is able to determine, the more refined information one obtains about the underlying coupling.

The four invariants mentioned above refer to a given charge state of the photoproduction reaction. If all charge states are considered together, a reduction of the four parameters per charge state may be achieved by the use of isotopic spin. Thus, e.g., in photoproduction of pions from nucleons all four possible charge states of the reaction can be described in terms of 12 parameters[9] instead of the 16 one would expect without the use of isotopic spin. This isotopic spin structure is not only well known but is also independent of the discussion of the invariants themselves or of anything else that takes place in coordinate, momentum, or spin space. In this paper, therefore, we will discuss only one particular (but unspecified) charge state in terms of the four invariants.

In constructing the four invariants in the center-of-mass, system, one has to work with the photon polarization vector $\mathbf{e}$, the photon momentum $\mathbf{k}$, the boson momentum vector $\mathbf{q}$, and the nucleon spin $\boldsymbol{\sigma}$. The corresponding unit vectors in the first three directions will be denoted by $\boldsymbol{\varepsilon}$, $\boldsymbol{\kappa}$, and $\boldsymbol{\xi}$, respectively. The vector $\boldsymbol{\sigma}$ has the usual normalization $\boldsymbol{\sigma}^2=3$, and we use the conventional Pauli matrices, as given, for instance, by Bohm.[14]

In the invariants $\boldsymbol{\varepsilon}$ has to appear once and only once, since we are dealing with a process of first order in the electromagnetic coupling. Furthermore, $\boldsymbol{\sigma}$ will appear once or not at all, because if it appears more often one can reduce it by

$$\boldsymbol{\sigma}\cdot\mathbf{a}\boldsymbol{\sigma}\cdot\mathbf{b}=\mathbf{a}\cdot\mathbf{b}+i\boldsymbol{\sigma}\cdot\mathbf{a}\times\mathbf{b}. \tag{2.1}$$

There is only one way to construct an invariant not containing $\boldsymbol{\sigma}$, since there are only two vectors left in addition to $\boldsymbol{\varepsilon}$, and $\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}=0$. Thus for a scalar boson (S) the invariant is $\boldsymbol{\varepsilon}\cdot\boldsymbol{\xi}$, and for a pseudoscalar boson (PS) it is $\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}\times\boldsymbol{\xi}$.

It is perhaps needless to remark that all invariants are written down *modulo* any power of $\boldsymbol{\xi}\cdot\boldsymbol{\kappa}=\cos\theta$, where θ is the production angle. In fact, the coefficients of the invariants in the M matrix for the production will depend on $\cos\theta$ as well as on the energy of the process, but on no other variable.

The remaining three invariants will depend on $\boldsymbol{\sigma}$. It is important to realize, however, that these three invariants can be any $\boldsymbol{\sigma}\cdot\mathbf{a}$, $\boldsymbol{\sigma}\cdot\mathbf{b}$, and $\boldsymbol{\sigma}\cdot\mathbf{c}$ where $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are three non-coplanar vectors (or pseudovectors, as the case may be). If $\mathbf{a}$, $\mathbf{b}$, or $\mathbf{c}$ do not contain $\boldsymbol{\varepsilon}$, one can always multiply the appropriate invariant by some $\boldsymbol{\varepsilon}\cdot\mathbf{d}$. We can, therefore, select our three invariants on the basis of the following three considerations:

[9] K. M. Watson, Phys. Rev. **95**, 228 (1954). For a more physical exposition of this theorem, see also E. Fermi, Suppl. Nuovo cimento 2, 17 (1955), particularly pp. 57–60.

[10] M. Kawaguchi and S. Minami, Progr. Theoret. Phys. (Kyoto) **12**, 789 (1954).

[11] C. D. Wood, T. J. Devlin, J. A. Helland, M. J. Longo, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. Letters **6**, 481 (1961).

[12] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

[13] See, e.g., M. J. Moravcsik, Brookhaven National Laboratory Report BNL-459, 1957 (unpublished), pp. 15–17.

[14] D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1951), p. 391.

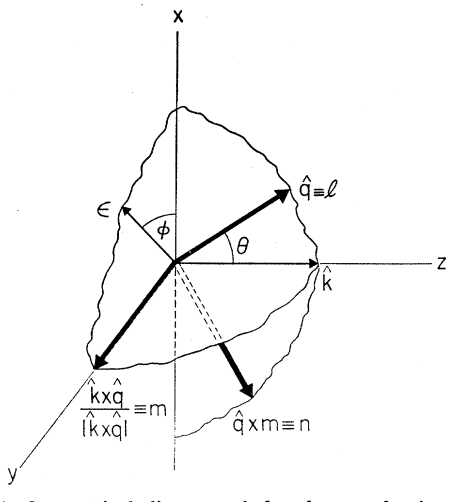

FIG. 1. Geometrical diagram of the photoproduction process. $\boldsymbol{\varepsilon}$ is the photo polarization vector for linear polarization, $\mathbf{k}$ is the photon momentum, and $\mathbf{q}$ is the boson momentum. The vectors $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$ are unit vectors used in the text to construct the invariants. The carets denote unit vectors.

1. The calculation of the observables as well as the interpretation of the experiments are greatly simplified if $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are mutually orthogonal.

2. The vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ should represent as closely as possible experimentally measurable directions.

3. The calculations are simplified if $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ do not contain $\boldsymbol{\varepsilon}$.

On the basis of these considerations we choose the unit vectors $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$, pointing in the $\boldsymbol{\xi}$, $\boldsymbol{\kappa}\times\boldsymbol{\xi}$, and $\boldsymbol{\xi}\times(\boldsymbol{\kappa}\times\boldsymbol{\xi})$ directions, respectively. The invariants then must be multiplied by $\boldsymbol{\varepsilon}\cdot\mathbf{l}$, $\boldsymbol{\varepsilon}\cdot\mathbf{m}$, or $\boldsymbol{\varepsilon}\cdot\mathbf{n}$, depending on the parity requirements ($\mathbf{l}$ and $\mathbf{n}$ are vectors, $\mathbf{m}$ is a pseudovector). The products $\boldsymbol{\varepsilon}\cdot\mathbf{l}$ and $\boldsymbol{\varepsilon}\cdot\mathbf{n}$ are not linearly independent since

$$\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}=0=\boldsymbol{\varepsilon}\cdot\mathbf{l}\cos\theta+\boldsymbol{\varepsilon}\cdot\mathbf{n}\sin\theta, \tag{2.2}$$

so that only one of them can be used. We select arbitrarily $\boldsymbol{\varepsilon}\cdot\mathbf{l}$. The geometrical arrangement of the various vectors is shown in Fig. 1. Thus we finally arrive at the following form of the production matrix:

$$M=A\boldsymbol{\varepsilon}\cdot\mathbf{l}+iB\boldsymbol{\sigma}\cdot\mathbf{l}\boldsymbol{\varepsilon}\cdot\mathbf{m}+iC\boldsymbol{\sigma}\cdot\mathbf{m}\boldsymbol{\varepsilon}\cdot\mathbf{l}+iD\boldsymbol{\sigma}\cdot\mathbf{n}\boldsymbol{\varepsilon}\cdot\mathbf{m}, \quad (S)$$
$$M=A\boldsymbol{\varepsilon}\cdot\mathbf{m}+iB\boldsymbol{\sigma}\cdot\mathbf{l}\boldsymbol{\varepsilon}\cdot\mathbf{l}+iC\boldsymbol{\sigma}\cdot\mathbf{m}\boldsymbol{\varepsilon}\cdot\mathbf{m}+iD\boldsymbol{\sigma}\cdot\mathbf{n}\boldsymbol{\varepsilon}\cdot\mathbf{l}. \quad (PS) \tag{2.3}$$

The three directions we have chosen, $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$, are fairly directly related to experiments, since they denote polarizations for the recoil nucleon longitudinally, transversely perpendicular to the production plane, and transversely parallel to the production plane, respectively, all in the center of mass system. Unfortunately in the laboratory system these directions are not so simple, but this is unavoidably due to the energy dependent and relativistic kinematic relationship between the laboratory and center-of-mass systems. In nucleon-nucleon scattering (at least in the nonrelativistic case) is is possible to define the longitudinal and transverse directions in the laboratory system in terms of quantities in the center-of-mass system simply and once and for all for all energies, since we deal with elastic scattering of equal masses. This is not possible in our case, but the resulting inconvenience is minor.

The question may be raised as to whether the choice of invariants might not be influenced by theoretical considerations. It is conceivable that the structure of the theory might be simpler in one representation than in another. In the framework of dispersion relations, for instance, it would be possible that the position and strength of the singularities naturally suggest a convenient set of invariants. If this is so, it is not apparent at the present time. For instance, Chew *et al.*[12] use a representation different from ours, but for no particular reason, and their expressions are not particularly simple. In fact, they use two sets of amplitudes, one set having simple properties under crossing, the other having a simple form in terms of angular momentum states. In any case, present theories of photo-production hold, at best, only for low energy pions. In the absence of a general theory, therefore, we might as well conform to experimental requirements and to considerations of convenience.

The above invariants have been constructed under the requirement of rotation and reflection invariance. One might think that perhaps other symmetries also bear on the form of the invariants. This is not the case. Simple time reversal invariance arguments like those used for nucleon-nucleon scattering[8] are not applicable here since the initial and final states are different. Unitarity[9,10] coupled with general time reversal invariance, does have a bearing on the M matrix, as we mentioned in Sec. I, by requiring that in the elastic region of the production process the phase of the complex amplitude in a given angular momentum state be the corresponding scattering phase. If a partial wave decomposition is not made, the unitarity condition is in terms of an integral relation on M, which at higher energies also includes inelastic processes. The practical significance of such a relation is therefore slight. Finally, there is crossing symmetry which has been discussed by Chew *et al.*[12] It is clear from their work that crossing affects only the energy dependence of the coefficients of the invariants, and in fact perceptibly so only if one studies the production amplitude in the whole complex plane. This is not usually done in phenomenological considerations.[15] Thus time-reversal invariance, unitarity, or crossing symmetry do not affect our present considerations.

III. NUMBER OF EXPERIMENTS

The number of experiments, their independence, the minimum set of experiments determining the produc-

[15] See, however, M. Cini, R. Gatto, E. L. Goldwasser, and M. Ruderman, Nuovo cimento **10**, 243 (1958).

tion matrix, and similar questions can be discussed in strict analogy to the nucleon-nucleon scattering problem.[8,16] There the M matrix in spin space is a 4×4 matrix since there are two nucleons present in both the initial and final states. In terms of the density matrix formalism one can say that both the initial and the final density matrices are 4×4, thus giving altogether $16\times 16=256$ different experiments. These 256 experiments are different in the sense that the outcome of none of them can be predicted from the others without looking at the formulas giving the observables in terms of the coefficients of the invariants.

In the case of the photoproduction the M matrix is 2×4, since there is only one nucleon in the final state. The initial state consists of a nucleon and a photon. Although the photon is a spin-1 particle, its transverse nature makes it behave in spin space in many ways like a spin-$\frac{1}{2}$ particle.[17] Thus its spin has only two possible z components, and it can be described in terms of four parameters (e.g., the Stokes parameters), just like the spin-$\frac{1}{2}$ particle can be described by a 2×2 matrix. In terms of the polarization vector, we can choose sets of four independent polarization states. One such set is, for instance, the two perpendicular linear polarizations, a 45° linear polarization, and one circular polarization. Thus the dimension of the initial spin space is as if we had two spin-$\frac{1}{2}$ particles. Hence the M matrix has $2\times 4=8$ elements, and, consequently, we have $8\times 8=64$ experiments that are different in the above sense. Alternatively, one can say that the initial density matrix is 4×4, the final one 2×2, and they are connected by $16\times 4=64$ coefficients, each representing an experiment.

We will now list these 64 experimental quantities. They are:

- I_0, differential cross section, unpolarized target;
- P_i, polarization of the recoil fermion, unpolarized target;
- I_0A_i, differential cross section, polarized target;
- T_{ij}, polarization of the recoil fermion, polarized target.

The indices i and j correspond to the three possible nucleon spin directions, in our case **l**, **m**, and **n**.

Each of the above observables can have a superscript according to whether the incident photon is unpolarized (U), polarized parallel (‖), perpendicular (⊥), or at 45° (45°) to the production plane, or is right (R) or left (L) circularly polarized.

The notation suggests the analogy with nucleon-nucleon scattering. P stands for polarization, A for "asymmetry" experiments.[8] The latter is identical to the polarization for nucleon-nucleon scattering on account of simple time reversal arguments[18] which here do not apply, hence P_i and A_i are independent experiments. The T_{ij}'s are analogous to the "triple scattering" experiments in nucleon-nucleon scattering, although neither there nor here is it necessary to carry our three scatterings to perform the experiment.

If our task is to determine the M matrix from experiments, however, many fewer than 64 measurements of the ones discussed above will suffice. In fact, the four complex coefficients in the M matrix appear in the observables in the form of 16 bilinear combinations such as $|A|^2$, $|B|^2$, $\cdots$, $\mathrm{Re}A^*B$, $\mathrm{Im}A^*B$, $\mathrm{Re}A^*C$, $\cdots$. Thus 16 appropriately chosen experiments out of the 64 are, in principle, sufficient to determine the four complex coefficients. In fact, if we do not care about possible ambiguities, not even 16 are necessary: four complex numbers represent 7 parameters (one phase being arbitrary), so that 7 well-chosen experiments should in fact suffice to measure them. The specific sets of experiments accomplishing this will be discussed in Sec. V.

IV. CALCULATION OF THE OBSERVABLES

The calculation of the observables in terms of the four coefficients (which correspond to the Wolfenstein parameters in nucleon-nucleon scattering) is straightforward. We have

$$\begin{aligned} I_0&=\tfrac{1}{2}\,\mathrm{Tr}(M^\dagger M),\\ I_0P_i&=\tfrac{1}{2}\,\mathrm{Tr}(M^\dagger\sigma_i M),\\ I_0A_i&=\tfrac{1}{2}\,\mathrm{Tr}(M^\dagger M\sigma_i),\\ I_0T_{ij}&=\tfrac{1}{2}\,\mathrm{Tr}(M^\dagger\sigma_i M\sigma_j).\end{aligned}\qquad(4.1)$$

The evaluation of these traces requires the derivation of a few simple trace formulas, the most complex of which is

$$\tfrac{1}{2}\,\mathrm{Tr}(\boldsymbol{\sigma}\cdot\mathbf{a}\sigma_i\boldsymbol{\sigma}\cdot\mathbf{b}\sigma_j)=-\mathbf{a}\cdot\mathbf{b}\delta_{ij}+a_ib_j+a_jb_i.\qquad(4.2)$$

The calculation involves the evaluation of scalar products of $\boldsymbol{\varepsilon}$ with other vectors. For linearly polarized photons we have (see Fig. 1)

$$\boldsymbol{\varepsilon}\cdot\mathbf{l}=\sin\theta\cos\phi,\quad \boldsymbol{\varepsilon}\cdot\mathbf{m}=\sin\phi,\qquad(4.3)$$

where $\phi=0$ corresponds to the (‖) case and $\phi=90°$ to the (⊥) case. For circular polarization we have

$$\boldsymbol{\varepsilon}_{\pm}=2^{-\frac{1}{2}}(\mathbf{x}\pm i\mathbf{y}),\qquad(4.4)$$

where **x** and **y** are unit vectors in the x and y directions (see Fig. 1). The + corresponds to (R) and the − to (L). In this case we have

$$\boldsymbol{\varepsilon}_{\pm}\cdot\mathbf{l}=2^{-\frac{1}{2}}\sin\theta,\quad \boldsymbol{\varepsilon}_{\pm}\cdot\mathbf{m}=\pm 2^{-\frac{1}{2}}i.\qquad(4.5)$$

[16] See also reference 6, pp. 292–303.

[17] For an illuminating survey of polarized light see U. Fano, J. Opt. Soc. Am. **39**, 859 (1949). See also U. Fano, Revs. Modern Phys. **29**, 74 (1957). I am also indebted to Professor Fano for some illuminating private communication.

[18] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952). It is perhaps of some interest to remark in this connection that in pion-nucleon scattering the equality of P and A follows merely from rotation and reflection invariance. In fact, there is no pion-nucleon scattering experiment which could test the validity of time reversal invariance.

TABLE I. Photoproduction observables for linear photon polarization. For the definition of θ and ϕ, see Fig. 1.

	(S)	(PS)
I_0	$(\lvert A\rvert^2+\lvert C\rvert^2)\sin^2\theta\cos^2\phi+(\lvert B\rvert^2+\lvert D\rvert^2)\sin^2\phi$	$(\lvert A\rvert^2+\lvert C\rvert^2)\sin^2\phi+(\lvert B\rvert^2+\lvert D\rvert^2)\sin^2\theta\cos^2\phi$
I_0P_l	$2\,\mathrm{Im}(C^*D-A^*B)\sin\theta\sin\phi\cos\phi$	$2\,\mathrm{Im}(C^*D-A^*B)\sin\theta\sin\phi\cos\phi$
I_0P_m	$-2\,\mathrm{Im}A^*C\sin^2\theta\cos^2\phi-2\,\mathrm{Im}B^*D\sin^2\phi$	$-2\,\mathrm{Im}A^*C\sin^2\phi-2\,\mathrm{Im}B^*D\sin^2\theta\cos^2\phi$
I_0P_n	$2\,\mathrm{Im}(B^*C-A^*D)\sin\theta\sin\phi\cos\phi$	$2\,\mathrm{Im}(B^*C-A^*D)\sin\theta\sin\phi\cos\phi$
I_0A_l	$-2\,\mathrm{Im}(C^*D+A^*B)\sin\theta\sin\phi\cos\phi$	$-2\,\mathrm{Im}(C^*D+A^*B)\sin\theta\sin\phi\cos\phi$
I_0A_m	$-2\,\mathrm{Im}A^*C\sin^2\theta\cos^2\phi+2\,\mathrm{Im}B^*D\sin^2\phi$	$-2\,\mathrm{Im}A^*C\sin^2\phi+2\,\mathrm{Im}B^*D\sin^2\theta\cos^2\phi$
I_0A_n	$-2\,\mathrm{Im}(B^*C+A^*D)\sin\theta\sin\phi\cos\phi$	$-2\,\mathrm{Im}(B^*C+A^*D)\sin\theta\sin\phi\cos\phi$
I_0T_{ll}	$(\lvert A\rvert^2-\lvert C\rvert^2)\sin^2\theta\cos^2\phi+(\lvert B\rvert^2-\lvert D\rvert^2)\sin^2\phi$	$(\lvert A\rvert^2-\lvert C\rvert^2)\sin^2\phi+(\lvert B\rvert^2-\lvert D\rvert^2)\sin^2\theta\cos^2\phi$
I_0T_{mm}	$(\lvert A\rvert^2+\lvert C\rvert^2)\sin^2\theta\cos^2\phi-(\lvert B\rvert^2+\lvert D\rvert^2)\sin^2\phi$	$(\lvert A\rvert^2+\lvert C\rvert^2)\sin^2\phi-(\lvert B\rvert^2+\lvert D\rvert^2)\sin^2\theta\cos^2\phi$
I_0T_{nn}	$(\lvert A\rvert^2-\lvert C\rvert^2)\sin^2\theta\cos^2\phi+(\lvert D\rvert^2-\lvert B\rvert^2)\sin^2\phi$	$(\lvert A\rvert^2-\lvert C\rvert^2)\sin^2\phi+(\lvert D\rvert^2-\lvert B\rvert^2)\sin^2\theta\cos^2\phi$
I_0T_{lm}	$2\,\mathrm{Re}(A^*D+B^*C)\sin\theta\sin\phi\cos\phi$	$2\,\mathrm{Re}(A^*D+B^*C)\sin\theta\sin\phi\cos\phi$
I_0T_{ml}	$2\,\mathrm{Re}(B^*C-A^*D)\sin\theta\sin\phi\cos\phi$	$2\,\mathrm{Re}(B^*C-A^*D)\sin\theta\sin\phi\cos\phi$
I_0T_{ln}	$-2\,\mathrm{Re}A^*C\sin^2\theta\cos^2\phi+2\,\mathrm{Re}B^*D\sin^2\phi$	$-2\,\mathrm{Re}A^*C\sin^2\phi+2\,\mathrm{Re}B^*D\sin^2\theta\cos^2\phi$
I_0T_{nl}	$2\,\mathrm{Re}A^*C\sin^2\theta\cos^2\phi+2\,\mathrm{Re}B^*D\sin^2\phi$	$2\,\mathrm{Re}A^*C\sin^2\phi+2\,\mathrm{Re}B^*D\sin^2\theta\cos^2\phi$
I_0T_{mn}	$2\,\mathrm{Re}(A^*B+C^*D)\sin\theta\sin\phi\cos\phi$	$2\,\mathrm{Re}(A^*B+C^*D)\sin\theta\sin\phi\cos\phi$
I_0T_{nm}	$2\,\mathrm{Re}(C^*D-A^*B)\sin\theta\sin\phi\cos\phi$	$2\,\mathrm{Re}(C^*D-A^*B)\sin\theta\sin\phi\cos\phi$

The rest of the calculation is trivial. Results are given in Tables I and II for linear and circular polarizations. The quantities for unpolarized photons are readily obtained from the results in the tables by averaging over ϕ the quantities pertaining to linear polarization.

V. DISCUSSION OF THE EXPERIMENTS

The measurement of the various experimental quantities will now be discussed with two aims in mind. First we will try to find experiments which can determine the parity of the boson produced. Then we will list sets of experiments that are sufficient to determine the coefficients of the four invariants.

Parity experiments are in the center of interest today as the parity of the K meson remains undetermined.[19] The difficulty of experiments leading to parity assignment is illustrated here also. The structure of invariants, as given by Eq. (2.3) is very similar for the (S) and (PS) cases. Furthermore, since A, B, C, and D are unknown in the absence of a dynamical theory, and they depend on $\cos\theta$, the θ dependence alone cannot be used for parity determination. Thus the parity experiments have to utilize the ϕ dependence and hence have to use polarized photons. Furthermore, the experiments suggested below also involve polarized fermion targets and/or the measurement of the polarization of the recoil fermion. These requirements place the experiments proposed below considerably beyond present day techniques certainly for K mesons and probably even for pions.

There are some parity experiments involving only linearly polarized photons, in which the ϕ dependence of a certain combination of experimental quantities is measured. A few of these are

$$I_0P_m(\mathrm{lin})+I_0A_m(\mathrm{lin})\propto\begin{cases}\cos^2\phi, & (S)\\ \sin^2\phi, & (PS)\end{cases}\qquad\text{(i)}$$

$$I_0T_{ln}(\mathrm{lin})+I_0T_{nl}(\mathrm{lin})\propto\begin{cases}\sin^2\phi, & (S)\\ \cos^2\phi, & (PS)\end{cases}\qquad\text{(ii)}$$

$$I_0T_{ll}(\mathrm{lin})-I_0T_{nn}(\mathrm{lin})\propto\begin{cases}\sin^2\phi, & (S)\\ \cos^2\phi, & (PS)\end{cases}\qquad\text{(iii)}$$

$$I_0(\mathrm{lin})-I_0T_{mm}(\mathrm{lin})\propto\begin{cases}\sin^2\phi, & (S)\\ \cos^2\phi, & (PS)\end{cases}\qquad\text{(iv)}$$

$$I_0(\mathrm{lin})+I_0T_{ll}(\mathrm{lin})+I_0T_{mm}(\mathrm{lin})+I_0T_{nn}(\mathrm{lin})\propto\begin{cases}\sin^2\phi, & (S)\\ \cos^2\phi. & (PS)\end{cases}\qquad\text{(v)}$$

The first of these is the most feasible since it involves none of the T_{ij}'s.

Other experiments also involve circularly polarized photons and are based on determining a certain combination of coefficients from the linear polarization measurement and then measuring the sign of the same combination in the circularly polarized case which then gives the parity. The linear polarization measurement usually has to be carried out at $\phi\neq0°$ or $90°$. We have, for instance $I_0T_{nm}(\mathrm{lin})$ measuring $2\,\mathrm{Re}(C^*D-A^*B)$, then the sign of $I_0P_l(\mathrm{circ})$ gives the parity.

Other pairs of experiments of this sort are

$$I_0T_{lm}(\mathrm{lin})\quad\text{and}\quad I_0P_n(\mathrm{circ}),\qquad\text{(vii)}$$

$$I_0T_{mn}(\mathrm{lin})\quad\text{and}\quad I_0A_l(\mathrm{circ}),\qquad\text{(viii)}$$

$$I_0T_{ml}(\mathrm{lin})\quad\text{and}\quad I_0A_n(\mathrm{circ}),\qquad\text{(ix)}$$

[19] P. T. Matthews, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 700.

TABLE II. Photoproduction observables for circular photon polarization. The upper and lower signs correspond to right and left polarization, respectively. For the definition of θ_0, see Fig. 1.

	(S)	(PS)
I_0	$\frac{1}{2}(\lvert A\rvert^2+\lvert C\rvert^2)\sin^2\theta+\frac{1}{2}(\lvert B\rvert^2+\lvert D\rvert^2)$	$\frac{1}{2}(\lvert A\rvert^2+\lvert C\rvert^2)+\frac{1}{2}(\lvert B\rvert^2+\lvert D\rvert^2)\sin^2\theta$
I_0P_l	$\pm\text{Re}(C^*D-A^*B)\sin\theta$	$\mp\text{Re}(C^*D-A^*B)\sin\theta$
I_0P_m	$-\text{Im}A^*C\sin^2\theta-\text{Im}B^*D$	$-\text{Im}A^*C-\text{Im}B^*D\sin^2\theta$
I_0P_n	$\mp\text{Re}(A^*D+B^*C)\sin\theta$	$\pm\text{Re}(A^*D+B^*C)\sin\theta$
I_0A_l	$\mp\text{Re}(C^*D+A^*B)\sin\theta$	$\pm\text{Re}(C^*D+A^*B)\sin\theta$
I_0A_m	$-\text{Im}A^*C\sin^2\theta+\text{Im}B^*D$	$-\text{Im}A^*C+\text{Im}B^*D\sin^2\theta$
I_0A_n	$\mp\text{Re}(A^*D-B^*C)\sin\theta$	$\pm\text{Re}(A^*D-B^*C)\sin\theta$
I_0T_{ll}	$\frac{1}{2}(\lvert A\rvert^2-\lvert C\rvert^2)\sin^2\theta+\frac{1}{2}(\lvert B\rvert^2-\lvert D\rvert^2)$	$\frac{1}{2}(\lvert A\rvert^2-\lvert C\rvert^2)+\frac{1}{2}(\lvert B\rvert^2-\lvert D\rvert^2)\sin^2\theta$
I_0T_{mm}	$\frac{1}{2}(\lvert A\rvert^2+\lvert C\rvert^2)\sin^2\theta-\frac{1}{2}(\lvert B\rvert^2+\lvert D\rvert^2)$	$\frac{1}{2}(\lvert A\rvert^2+\lvert C\rvert^2)-\frac{1}{2}(\lvert B\rvert^2+\lvert D\rvert^2)\sin^2\theta$
I_0T_{nn}	$\frac{1}{2}(\lvert A\rvert^2-\lvert C\rvert^2)\sin^2\theta+\frac{1}{2}(\lvert D\rvert^2-\lvert B\rvert^2)$	$\frac{1}{2}(\lvert A\rvert^2-\lvert C\rvert^2)+\frac{1}{2}(\lvert D\rvert^2-\lvert B\rvert^2)\sin^2\theta$
I_0T_{lm}	$\mp\text{Im}(A^*D-B^*C)\sin\theta$	$\pm\text{Im}(A^*D-B^*C)\sin\theta$
I_0T_{ml}	$\pm\text{Im}(A^*D+B^*C)\sin\theta$	$\mp\text{Im}(A^*D+B^*C)\sin\theta$
I_0T_{ln}	$-\text{Re}A^*C\sin^2\theta+\text{Re}B^*D$	$-\text{Re}A^*C+\text{Re}B^*D\sin^2\theta$
I_0T_{nl}	$\text{Re}A^*C\sin^2\theta+\text{Re}B^*D$	$\text{Re}A^*C+\text{Re}B^*D\sin^2\theta$
I_0T_{mn}	$\mp\text{Im}(A^*B+C^*D)\sin\theta$	$\pm\text{Im}(A^*B+C^*D)\sin\theta$
I_0T_{nm}	$\pm\text{Im}(A^*B-C^*D)\sin\theta$	$\mp\text{Im}(A^*B-C^*D)\sin\theta$

$$I_0P_n(\text{lin}) \quad \text{and} \quad I_0T_{lm}(\text{circ}), \tag{x}$$

$$I_0A_n(\text{lin}) \quad \text{and} \quad I_0T_{ml}(\text{circ}), \tag{xi}$$

$$I_0P_l(\text{lin}) \quad \text{and} \quad I_0T_{mn}(\text{circ}), \tag{xii}$$

$$I_0A_l(\text{lin}) \quad \text{and} \quad I_0T_{nm}(\text{circ}). \tag{xiii}$$

Various other combinations are also possible.

It is evident that if the parity of the boson is known from other experiments, the above measurements test the conservation of parity in photoproduction processes.

Now let us turn to the determination of the coefficients in the M matrix. It is well to note first that of the various polarization states possible for each of the I_0, I_0P_i, I_0A_i, or I_0T_{ij}'s, two and only two supply independent information about the 16 bilinear combinations of the coefficients. For I_0, I_0P_m, I_0A_m, the three I_0T_{ii}'s, I_0T_{ln}, and I_0T_{nl}, where only $\cos^2\phi$ and $\sin^2\phi$ appear, the circular polarization gives no new information and also agrees with the unpolarized case, while the linear polarization gives two combinations of the coefficients. For the other observables the unpolarized measurement is identically zero, and the linear polarization gives only one combination, but the circular polarization yields an additional independent combination.

In view of this we can make the following statements about the sets of seven experiments needed to determine the coefficients at a given production angle.

(1) If we want to use only unpolarized photons, the simplest set, in addition to $I_0(\text{U})$, $I_0P_m(\text{U})$, and $I_0A_m(U)$, has to include any four of the nine $I_0T_{ij}(\text{U})$'s.

(2) If we want to use only linearly polarized photons, the simplest set, in addition to $I_0(\|)$ and $I_0(\perp)$ includes five of the quantities $I_0P_l(45°)$, $I_0P_m(\|)$, $I_0P_m(\perp)$, $I_0P_n(45°)$, $I_0A_l(45°)$, $I_0A_m(\|)$, $I_0A_m(\perp)$, and $I_0A_n(45°)$, except that only two of the four quantities $I_0P_m(\|)$, $I_0P_m(\perp)$, $I_0A_m(\|)$, and $I_0A_m(\perp)$ can appear in the set.

(3) If we have both linearly and circularly polarized photons available, we can use for the simplest set any seven of the following eight quantities: $I_0(\|)$; $I_0(\perp)$; $P_l(\|)$ or $P_l(\perp)$; $P_l(R)$ or $P_l(L)$; two of $P_m(\|)$, $P_m(\perp)$, $P_m(R)$, and $P_m(L)$; $P_n(\|)$ or $P_n(\perp)$; $P_n(R)$ or $P_n(L)$. Thus, in this case, no polarized targets are needed.

Since the set of seven measurements give bilinear combinations of the seven real quantities to be determined, ambiguities are possible. These can be avoided by 16 experiments giving all the 16 bilinear combinations. Even if both linearly and circularly polarized photons are available, the simplest of these sets, in addition to I_0's, I_0P_i's, and I_0A_i's, includes two I_0T_{ij} measurements.

It is evident from the above discussion that an experimental determination of the photoproduction M matrix is not just around the corner. Neither is the corresponding basic theory, however. In the meantime, phenomenological considerations might serve as a stimulant for further experimental efforts.

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