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Pion-Pion Scattering and Low-Energy Pion Production*

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A model for the reaction $\pi+N\to 2\pi+N$ at low energies, with $\pi-\pi$ scattering lengths as parameters, discussed in a previous analysis is extended by removing the assumption that $a_0/a_2=\frac{5}{2}$. The parameters are determined by using the π^+ angular distribution from $\pi^-+p\to\pi^-+\pi^++n$ at 430 Mev and the total cross section for $\pi^++p\to\pi^++\pi^++n$ at 470 Mev. We find two acceptable sets for the $\pi-\pi$ scattering lengths: $a_0=0.50\,\mu^{-1}$, $a_1=0.07\,\mu^{-1}$, $a_2=0.16\,\mu^{-1}$ and $a_0=0.65\,\mu^{-1}$, $a_1=0.07\,\mu^{-1}$, $a_2=-0.14\,\mu^{-1}$. The total cross sections for the various charge channel and for single partial waves are computed. The predictions of this model are in qualitative agreement with experiment.

IN this paper we discuss a reanalysis of low-energy pion production using the recent experiments of Barish *et al.*[1] This treatment is based on a model of low-energy pion production derived from the static theory,[2] containing the single pion exchange and final-state isobar rescattering contributions. The solution of an integral equation indicates the dominance of the single pion exchange contribution at low energies, within the framework of the model, for $\pi+n\to\pi+\pi+N$. The transition matrix derived in I, given by Eq. (4.3) of I, is generalized here to include the scattering lengths a_0, a_1, a_2 (subscripts indicating the isotopic spin of the $\pi-\pi$ system) as independent parameters to be determined by experiment. In what follows, our deductions will be made within the context of this extended model. The discussion is limited to low-energy incident pion energies in order to minimize the effects of the $D_{\frac{3}{2}}$, $T=\frac{1}{2}$ isobar in the initial state and to confine the final $\pi-\pi$ system to a relatively limited energy region so that the scattering length approximation might have at least qualitative validity.

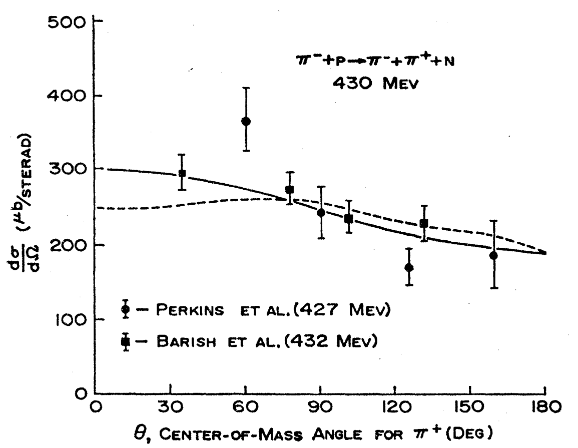

FIG. 1. Angular differential cross section for π^+ meson for incident π^- mesons at 430 Mev. The solid line indicates the prediction for the set with $a_2>0$, the dashed line for $a_2<0$.

The transition amplitude for $\pi^-+p\to\pi^-+\pi^++n$ at low energies is dominated by $\pi-\pi$ scattering in a $T=0$ state, while the π^+ angular distribution depends strongly on the sign of the interference of a_0 and a_1, the $T=0$ and $T=1$ amplitudes. Therefore, by fitting the π^+ angular distribution of Barish *et al.*[1] at 430 Mev, which contains isotropic terms and terms linear in $\cos\theta$, using Eqs. (4.9)–(4.12) of I and our extended transition amplitude, we can expect to be able to estimate a_0, a_1 and determine their relative signs. We have fitted a_1 and $(2a_0+a_2)$, the quantities which enter in $\pi^+-\pi^-$ scattering, to this angular distribution. The terms which depend on a_2 alone are quite small, and thus do not contribute new information.

Parameters are found with the same character as in I which are rejected because they give a large $\cos\theta$ term in the angular distribution.[3] The other solution found has $(2a_0+a_2)=1.16$ and $a_1=0.07$, which gives a very small $\cos^2\theta$ term, in agreement with the latest experiments. The total π^++p inelastic cross section at 470 Mev, taken from the curve of Kopp *et al.*,[4] was used to determine a_2, since a_2 makes its most important contribution in $T=\frac{3}{2}$ production. Again two solutions re-

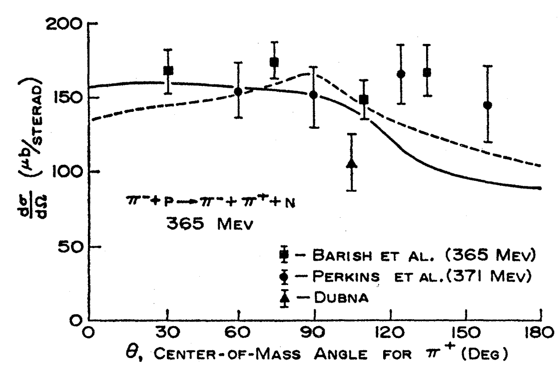

FIG. 2. Angular differential cross section for π^+ meson for incident π^- mesons at 365 Mev. The solid line is for the set with $a_2>0$, the dashed line for $a_2<0$.

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[1] B. C. Barish, R. J. Kurz, P. G. McManigal, V. Perez-Mendez, and J. Solomon, Phys. Rev. Letters **6**, 297 (1961).
[2] C. J. Goebel and H. J. Schnitzer, Phys. Rev. **123**, 1021 (1961). This paper will be referred to as I. In I, the S-wave dominant situation $a_0/a_2=\frac{5}{2}$ was examined. C. J. Goebel and H. J. Schnitzer, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 298.
[3] In I, we found $a_0=-0.28\,\mu^{-1}$, $a_1=0.12\,\mu^{-1}$, and $a_2/a_0=\frac{2}{5}$.
[4] J. K. Kopp, A. M. Shapiro, and A. R. Erwin, Phys. Rev. **123**, 301 (1961).

TABLE I. Production cross sections (mb).

Lab energy (Mev)	$\sigma(\pi^-+p)\rightarrow$ $\pi^-\pi^+n$	$\pi^0\pi^-p$	$\pi^0\pi^0n$	$\sigma(\pi^++p)\rightarrow$ $\pi^+\pi^+n$	$\pi^+\pi^0p$	$\sigma_{T=\frac{1}{2}}$	$\sigma_{T=\frac{3}{2}}$	σ_{π^-}
171	0	0	0	0	0	0	0	0
265	0.55	0.08	0.09	0.16	0.06	0.97	0.22	0.72
364	2.06	0.29	0.34	0.48	0.49	3.55	0.97	2.69
468	3.95	1.50	0.50	0.80	0.95	8.08	1.75	6.01
578				1.12	1.72		2.84	

TABLE II. Single partial-wave cross sections (mb).

Lab energy (Mev)	$P_{\frac{1}{2}}\rightarrow ss$ $T=\frac{1}{2}$	$T=\frac{3}{2}$	$S_{\frac{1}{2}}\rightarrow sp$ $T=\frac{1}{2}$	$T=\frac{3}{2}$	$D_{\frac{3}{2}}\rightarrow sp$ $T=\frac{1}{2}$	$T=\frac{3}{2}$	$2\pi\lambda\!\!\!^{-2}$
171	0	0	0	0	0	0	53.5
265	0.71	0.17	0.06	0.01	0.20	0.04	40.0
364	1.79	0.49	0.38	0.11	1.38	0.37	21.9
468	3.24	0.76	1.41	0.34	3.43	0.65	16.2
578	4.30	1.12	3.19	0.85	5.97	0.87	12.6
690	5.60	1.37	6.17	1.52	8.54	1.27	10.2

sult, which differ little except for the sign of a_2. They are

$$a_0=0.50,\quad a_1=0.07,\quad a_2=0.16,$$

or

$$a_0=0.65,\quad a_1=0.07,\quad a_2=-0.14.$$

The computed π^+ angular distributions for 430 Mev and 365 Mev are shown in Fig. 1 and 2,[5] (with $a_2>0$ in solid line and $a_2<0$ in dashed line). From these angular distributions there is no clear distinction between the two cases, although the calculations of Bransden and Moffat[6] favor $a_2>0$. This ambiguity in the sign of a_2 is not surprising since we have no experimental information that depends on the interference of the a_2 and a_1, such as the π^+ angular distribution in $\pi^++p\rightarrow\pi^++\pi^0+p$. We also note that our solutions indicate that the π^- angular distributions in $\pi^-+p\rightarrow\pi^-+\pi^++n$ should be peaked forward in the energy region below 500 Mev. It is interesting that we have found a large positive a_0, which lends support to the interpretation of Truong[7] of the experiments of Booth *et al.*[8]

To proceed further, we arbitrarily choose the solution with $a_2>0$. With the parameters of the model fixed, we can extend our predictions. Some of the results[1,5,9,10] are given in Figs. 3, 4, and Tables I and II. Although we will not discuss them in detail, the total cross sections are in qualitative agreement with experiment. One might hope for better agreement with the very low

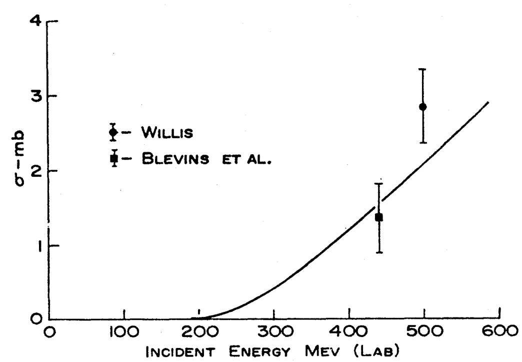

FIG. 3. Total π^+-p production cross section.

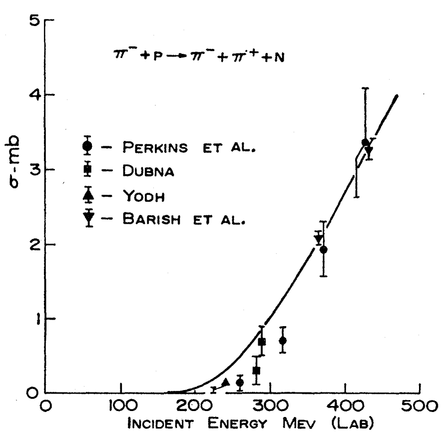

FIG. 4. Total $\pi^-+p\rightarrow\pi^-+\pi^++n$ cross section.

[5] W. Perkins, J. C. Caris, R. W. Kenny, and V. Perez-Mendez, Phys. Rev. **118**, 1364 (1960), reference 1; and V. G. Zinov and S. M. Korenchenko, Zhur Eksptl. i Teoret. Fiz. **34**, 301 (1958) [Translation: Soviet Phys.—JETP **34** (7), 210 (1958)].

[6] B. R. Desai, Phys. Rev. Letters **6**, 497 (1961); B. H. Bransden and J. W. Moffat, Phys. Rev. Letters **6**, 708 (1961); B. H. Bransden and J. W. Moffat, Nuovo cimento **21**, 505 (1961). Both of these calculations give a_0 large and positive for solutions which yield a P-wave $\pi-\pi$ resonance.

[7] T. N. Truong, Phys. Rev. Letters **6**, 308 (1961).

[8] The most recent results are given in N. E. Booth, A. Abashian, and K. M. Crows, Phys. Rev. Letters **7**, 35 (1961).

[9] M. E. Blevins, M. M. Block, and J. Leitner, Phys. Rev. **112**, 1287 (1958); W. J. Willis, Phys. Rev. **116**, 753 (1959); reference 4, J. Kirz, J. Schwartz, and R. D. Tripp, University of California (Berkeley) Report, UCRL—9941 find $\sigma=0.12\pm0.01$ mb for $\pi^++p\rightarrow\pi^++\pi^++n$ at 357 Mev.

[10] J. Deahl, M. Derrick, M. Fetkovich, T. Fields, and G. B. Yodh, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 185.

energy behavior of the $\pi^- + p \rightarrow \pi^- + n^+ + n$ cross section. The contribution from single partial waves indicates strong absorption in the $T=\frac{1}{2}$, $P_{\frac{1}{2}}$, $S_{\frac{1}{2}}$, and $D_{\frac{3}{2}}$ states. Ball and Frazer[11] have discussed in more detail the relationship, conjectured in I, between rapidly rising, strong absorption and elastic resonances, which relates these results to the second $\pi-N$ resonance.[12] The experimental situation on inelastic production in single partial waves is quite meager,[13] but there appears to be at least qualitative agreement with these results.

We conclude by remarking that the other results mentioned above[14] all seem to indicate the same qualitative behavior of low-energy $\pi-\pi$ scattering, while the analysis of the τ decay[15] does not seem to be consistent with these results. Since the other analyses of the $T=0$ scattering length indicate $a_0>0$, this should cast some doubt on the validity of the assumptions made in the above analysis of τ decay.

[11] J. S. Ball and W. R. Frazer, Phys. Rev. Letters **7**, 204 (1961).

[12] See the remarks of G. F. Chew, Revs. Modern Phys. **33**, 366 (1961).

[13] W. D. Walker, J. Davis, and W. D. Shephard, Phys. Rev. **118**, 1612 (1961); B. J. Moyer, Revs. Modern Phys. **33**, 367 (1961), particularly Table I.

[14] Also of interest is Y. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 79.

[15] N. N. Khuri and S. B. Treiman, Phys. Rev. **119**, 1115 (1960); R. F. Sawyer and K. C. Wali, *ibid.* **119**, 1429 (1960); E. Lomon, S. Morris, E. J. Irwin, Jr., and T. Truong, Ann. Phys. **13**, 317 (1961).

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Relativistic Lee Model

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A relativistic version of the well-known exactly soluble Lee model of quantum field theory is constructed, which—unlike the original nonrelativistic Lee model—fulfills also the condition of microcausality (field operators commuting or anticommuting for space-like distances). Microcausality is, however, found compatible with solubility only if an indefinite metric is introduced on a much larger scale than was necessary in the nonrelativistic case. Notwithstanding the fact that a formalism of this type would seem to bear still less resemblance to physical reality than does the original Lee model, we arrive at a theory almost identical with the usual version of relativistic charged scalar (or pseudoscalar) meson field theory. From a certain point of view these two formalisms can even be considered identical. In fact, the difference between the realistic theory and the present Lee model hinges merely on whether one uses, for the representation of the Dirac standard ket $|0\rangle$ a physical vacuum or a suitably defined "completely empty space." The form of the presentation follows closely that of the well-known Pauli-Källén description of the nonrelativistic Lee model, up to the point where it is shown that some of the main results can also be obtained with help of a corresponding Chew-Low equation.

I. INTRODUCTION

IT goes almost without saying that when exactly soluble models of quantum field theory are constructed, some of the necessary characteristics of a realistic theory must be sacrificed. Such models may nevertheless be useful for the purpose of detailed study of some mathematical properties, equally true in realistic theories, but whose clear meaning may somehow be lost due to the complexity of the problem. Common properties of this type may eventually be of great help in developing new approximation methods to be used even if a solution in a closed form does not exist. Also they may be useful in clarifying some obscure points concerning the already existing calculational techniques, such as for example uniqueness of various types of one-meson approximations, etc. It seems certain however that the physical importance of such generally valid properties is bound to depend strongly on the physical importance of those necessary characteristics of a realistic theory that we have managed to retain in the model. Since we seem to be unable to retain all of them, it must remain a question of "intuition" and also of taste which of them to regard as the most important.

It is the aim of this paper to explore one such possibility, where relativistic covariance and microcausality (in the usual sense of the field operators commuting or anticommuting for space-like distances) are considered the physical characteristics that are most worthwhile retaining, but where we shall be ready to resort to an indefinite metric for the sake of practical solubility.

We shall first of all proceed to show that we are indeed forced to sacrifice the definiteness of the metric of the Hilbert space, but that such a step is then already sufficient for our purpose.

We first recall that the formal reason for the solubility of the original Lee model is that the mathematical structure of its interaction Hamiltonian does not permit the number of particles to increase indefinitely, and thus enables one to have only a limited number of

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