which symbolic equation has the following explicit meaning as an equal-time commutator:

$$(1/i)[\phi_a{}^0(x),\phi_b{}^k(x')]=f^2\Big[\mathfrak{D}_\phi(\mathbf{x},\mathbf{x}')i\text{‘}t\phi^k(x')\text{’}$$
$$+\int(d\mathbf{x}'')\;\mathfrak{D}_\phi(\mathbf{x},\mathbf{x}'')i\text{‘}t\phi(x'')\text{’}$$
$$\cdot\nabla''\mathfrak{D}_\phi(\mathbf{x}'',\mathbf{x}')(-\partial'^k-i\text{‘}t\phi^k(x')\text{’})\Big]_{ab}.$$

If the expression for ϕ^0 is substituted in the equation for $\partial_0\boldsymbol{\phi}$, one obtains a fundamental transverse equation of motion,

$$-\partial_0\boldsymbol{\phi}=f^2[1+(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})\mathfrak{D}_\phi\nabla]:\mathbf{G},$$

or, equivalently,

$$-\partial_0\boldsymbol{\phi}=f^2\{[1+(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})\mathfrak{D}_\phi\nabla]$$
$$\cdot[1+\nabla\mathfrak{D}_\phi(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})]\}:\mathbf{G}^T$$
$$-f^2[1+(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})\mathfrak{D}_\phi\nabla]\cdot i\text{‘}t\boldsymbol{\phi}\text{’}\mathfrak{D}_\phi k^0.$$

One implication of this equation of motion is the equal-time commutator

$$[i\partial_0\boldsymbol{\phi},\boldsymbol{\phi}]=f^2[1+(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})\mathfrak{D}_\phi\nabla]\cdot[1+\nabla\mathfrak{D}_\phi(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})].$$

If we introduce an arbitrary real numerical transverse vector function $\mathbf{a}_a(\mathbf{x})$ and define the Hermitian operator

$$A(x^0)=\int(d\mathbf{x})\;\mathbf{a}\,.\,\boldsymbol{\phi},$$

this commutator becomes

$$[i\partial_0 A,A]=f^2\int(d\mathbf{x})\;\mathbf{b}\,.\,\mathbf{b},$$

where the Hermitian vector $\mathbf{b}$ is

$$\mathbf{b}=[1+\nabla\mathfrak{D}_\phi(\nabla-i\text{‘}t\boldsymbol{\phi}\text{’})]\cdot\mathbf{a}.$$

But the vacuum expectation value of such a commutator can never be negative,

$$\langle[[A,P^0],A]\rangle=2\langle AP^0A\rangle>0,$$

and therefore

$$f^2>0.$$

Finally, we state the equal-time commutator

$$i[G_a{}^{0k}(x),G_b{}^{0l}(x')]=[\partial^k\mathfrak{D}_\phi(\mathbf{x},\mathbf{x}')\,.\,i\text{‘}tG^{0l}(x')\text{’}$$
$$+i\text{‘}tG^{0k}(x)\text{’}\,.\,\mathfrak{D}_\phi(\mathbf{x},\mathbf{x}')\partial'^l]_{ab},$$

and leave the proof to the reader.

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Strange Particle Production in Proton-Proton Collisions*

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An estimate is made for strange particle production in p-p collisions based on the single pion exchange model. For the three-particle final state (KYN), fair agreement with experiment is achieved in both the total cross sections and the momentum and angular distributions of the final particles. For the four-particle final state ($KYN\pi$), general qualitative agreement is achieved for all total cross sections but one. In this, the case of $\pi^0+p+K^++\Lambda^0(\Sigma^0)$ production, it is suggested that perhaps Y^* production in p-p collisions plays an important role.

I. INTRODUCTION

RECENTLY there has been much interest in the single boson exchange mechanism in high-energy collisions.[1] Ferrari has made a calculation for associated production in proton-proton collisions at incoming energies of 2 to 3 Bev in the lab system.[2] Of the two models he considered (single pion exchange and single K-meson exchange) the pion exchange model seems to fit the experimental data rather well.[3] This result naturally suggests that perhaps single pion exchange plays a dominant role even at the relatively low energies of a few Bev for many other nucleon-nucleon collision processes. In this note we shall concern ourselves only with strange particle productions with or without an additional pion in the final states.

In Sec. II the three-particle final states of the kind $p+p\rightarrow Y+K+N$ are considered. The treatment is essentially that of Ferrari's with some modifications.

* Work supported in part by U. S. Atomic Energy Commission.
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[1] F. Salzman and G. Salzman, Phys. Rev. **120**, 599 (1960). I. M. Dremin and D. S. Chernavskii, Soviet Phys.—JETP **11**, 167 (1960); V. I. Veksler, *Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960* (Interscience Publishers, Inc., New York, 1960).

[2] E. Ferrari, Nuovo cimento **15**, 652 (1960); Phys. Rev. **120**, 988 (1960).

[3] R. I. Louttit, T. W. Morris, D. C. Rahm, R. R. Rau, A. M. Thorndike, and W. J. Willis, *Proceedings of the Tenth International Rochester Conference on High-Energy Physics, 1960* (Interscience Publishers, Inc., New York, 1960).

The momentum-energy transfer squared, Δ^2, is given a cutoff at nucleon mass squared, M^2. This energy-momentum cutoff has as a consequence that the interference term in the total cross section is negligibly small. This point has already been made by several authors,[1] and we shall neglect the interference term throughout in our calculation. In order to compare the calculated results with experimental values available,[3] total cross sections for various reactions are computed at 2.85 Bev incident proton lab energy. Furthermore, the momentum and angular distributions of the final-state particles are calculated for a particular reaction, $p+p\to p+K^++\Lambda^0$.

In Sec. III the four-particle final states of the kind $p+p\to Y+K+N+\pi$ are considered. The treatment follows closely that proposed by Salzman and Salzman.[4] It is however emphasized that in the energy range under consideration the virtual pion nucleon scattering cross section may be modified significantly, by the factor $[(W-M)^2+\Delta^2][(W-M)^2-\mu^2]^{-1}$, from the real pion-nucleon scattering cross section. It is found in the course of the calculation that for all reactions except one the calculated cross sections agree with the experimental results fairly well. The one exception is in the production of $\pi^0+p+K^++\Lambda^0(\Sigma^0)$, where the calculated value is much too small in comparison with the experimental cross section. It is suggested that perhaps Y^* and K' formation in p-p collisions plays an important role in this case.

II. THREE-PARTICLE PRODUCTION

The reactions under consideration are

$$p+p\to p+K^++\Lambda^0, \tag{1a}$$

$$p+p\to p+K^0+\Sigma^+, \tag{1b}$$

$$p+p\to n+K^++\Sigma^+, \tag{1c}$$

$$p+p\to p+K^++\Sigma^0. \tag{1d}$$

The processes are represented graphically by the diagrams in Fig. 1. We shall write the scattering cross section for any of the processes considered as follows:

$$d\sigma=d\sigma_A+d\sigma_B. \tag{2}$$

The interference term has been left out explicitly in (2).

$$\begin{aligned} d\sigma_A=&\frac{1}{(2\pi)^5}\frac{U}{4p}|\langle f|T|i\rangle|^2\delta(p_1+p_2-p_n-p_Y-p_K)\\ &\times d^3p_{nU}d^3p_{YU}d^3p_{KU}\frac{M}{E_{nU}}\frac{M_Y}{E_{YU}}\frac{1}{2E_{KU}}\left(\frac{2M}{U}\right)^2\\ =&\frac{1}{(2\pi)^5}\frac{4M^3M_Y}{pU}|\langle f|T|i\rangle|^2\delta(p_1+p_2-p_n-p_Y-p_K)\\ &\times\delta(p_Y^2+M_Y^2)\delta(p_K^2+M_K^2)\delta(p_n^2+M^2)\\ &\times d^4p_nd^4p_Yd^4p_K, \end{aligned} \tag{3}$$

[4] F. Salzman and G. Salzman, Phys. Rev. 121, 1541 (1961).

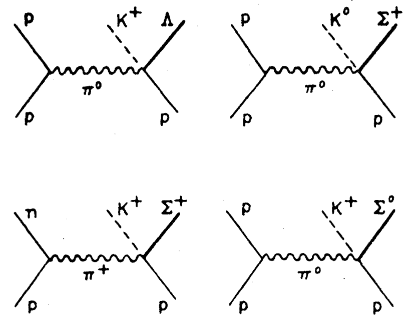

FIG. 1. Diagrams representing processes (1a)–(1d).

where p_1, p_2, and p_n, p_Y, p_K are the 4 momenta of the incoming protons and of the outgoing nucleon, hyperon, and K meson respectively. $U^2=-(p_1+p_2)^2$ is the square of the total energy in the center-of-mass system (the U system) and $p=(\frac{1}{4}U^2-M^2)^{\frac{1}{2}}$ is the center-of-mass momentum of the initial protons. M, M_Y, and M_K are the rest masses of nucleon, hyperon, and K meson, respectively. The subscript U designates the various momenta and energies in the U system.

$$\langle f|T|i\rangle=\langle p_Y,p_K|\Delta,p_2\rangle\langle p_n,\Delta|p_1\rangle(\Delta^2+\mu^2)^{-1}. \tag{4}$$

$\Delta^2=(p_1-p_n)^2$ and μ is the pion mass. $d\sigma_B$ is obtained from $d\sigma_A$ by an interchange of p_1 and p_2. Upon integration over the angles the two terms give equal contributions to the total cross section, and we have $\sigma_{\text{tot}}=2\sigma_A$.

Next we introduce the "cross section" for the virtual process $\pi+p\to K+Y$,

$$\begin{aligned} d\sigma_0(W;\Delta^2)&\\ =&\frac{1}{(2\pi)^2}\frac{MM_Y}{kW}|\langle p_Y,p_K|\Delta,p_2\rangle|^2\delta(p_2+\Delta-p_Y-p_K)\\ &\times\delta(p_Y^2+M_Y^2)\delta(p_K^2+M_K^2)d^4p_Yd^4p_K, \end{aligned} \tag{5}$$

where $W^2=-(p_Y+p_K)^2$ is the square of the total energy in the K-Y center-of-mass system (the W system), and k is defined as

$$W=(k^2+M^2)^{\frac{1}{2}}+(k^2+\mu^2)^{\frac{1}{2}}.$$

Substituting Eqs. (4) and (5) into Eq. (3) and integrating, we obtain

$$\begin{aligned} \sigma_A(U)=&\frac{1}{(2\pi)^3}\frac{4M^2}{pU}\int\int kW|\langle p_n,\Delta|p_1\rangle|^2\frac{1}{(\Delta^2+\mu^2)^2}\\ &\times\delta(\Delta+p_n-p_1)\delta(p_n^2+M^2)\sigma_0(W;\Delta^2)d^4\Delta d^4p_n. \end{aligned} \tag{6}$$

The vertex function $|\langle p_n,\Delta|p_1\rangle|^2$ can be rewritten as

$$|\langle p_n,\Delta|p_1\rangle|^2=4\pi f^2\frac{\Delta^2}{\mu^2}\times\begin{cases}1 & \text{for}\quad \pi^0\\ 2 & \text{for}\quad \pi^\pm,\end{cases} \tag{7}$$

a result expected from p-wave static nucleon model and invariance considerations.

TABLE I. Total cross sections (in mb) for three-particle production. (We have taken $\bar{\sigma}_{\pi^- p\to\Lambda^0 K^0}=0.5$ mb, $\bar{\sigma}_{\pi^+ p\to\Sigma^+ K^+}=0.2$ mb, $\bar{\sigma}_{\pi^0 p\to\Sigma^+ K^0}=\bar{\sigma}_{\pi^- p\to\Sigma^0 K^0}=0.25$ mb, $\bar{\sigma}_{\pi^0 p\to\Sigma^0 K^+}=\frac{1}{2}[\bar{\sigma}_{\pi^+ p\to\Sigma^+ K^+}+\bar{\sigma}_{\pi^- p\to\Sigma^- K^+}-\bar{\sigma}_{\pi^- p\to\Sigma^0 K^0}]=0.1$ mb.)

	Calculated	Experimental
$\Lambda^0 K^+ p$	0.049	0.051±0.011
$\Sigma^+ K^+ n$	0.058	0.047±0.012
$\Sigma^+ K^0 p$	0.036	0.030±0.010
$\Sigma^0 K^+ p$	0.015	0.013±0.007

Since Δ^2 and W^2 are two invariant scalars in the problem, we shall use them as integration variables. The Jacobian for the transformation can be obtained in a straightforward manner, and the result is

$$\sigma_A(U)=\frac{1}{(2\pi)^2}\frac{1}{(pU)^2}\frac{M^2}{2}\frac{4\pi f^2}{\mu^2}\times\binom{1}{2}\int d(W^2)kW\int d(\Delta^2)\frac{\Delta^2}{(\Delta^2+\mu^2)^2}\sigma_0(W;\Delta^2). \quad (8)$$

It is now assumed that

$$\sigma_0(W;\Delta^2)\approx\sigma_0(W;-\mu^2)\equiv\sigma_0(W),$$

where $\sigma_0(W)$ is the real cross section for $\pi+p\to K+Y$ at the c.m. energy W. Therefore,

$$\sigma_A(U)=\frac{1}{8\pi}\left(\frac{G^2}{4\pi}\right)\times\binom{1}{2}\frac{1}{(pU)^2}\int_{W_{\min}}^{W_{\max}}dW\times 2kW^2\sigma_0(W)\int d(\Delta^2)\frac{\Delta^2}{(\Delta^2+\mu^2)^2}. \quad (9)$$

This is essentially Eq. (13) in Ferrari's paper. His $I(W)$ is defined as

$$I(W)=\int_{\Delta_{\min}^2(W)}^{\Delta_{\max}^2(W)}d(\Delta^2)\frac{\Delta^2}{(\Delta^2+\mu^2)^2}. \quad (10)$$

However, in the energy range under consideration, incident proton energy at 2.85 Bev in the lab system corresponds to W below 2.1 Bev, and $\Delta_{\max}^2$ is always

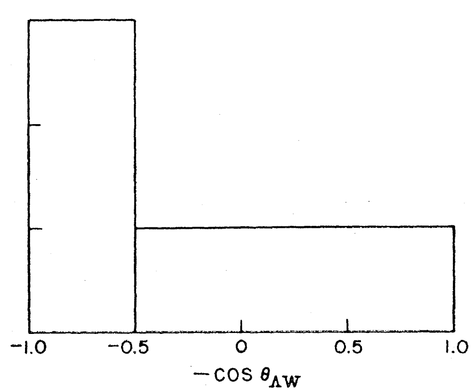

FIG. 2. Angular distribution of Λ^0 in the W system.

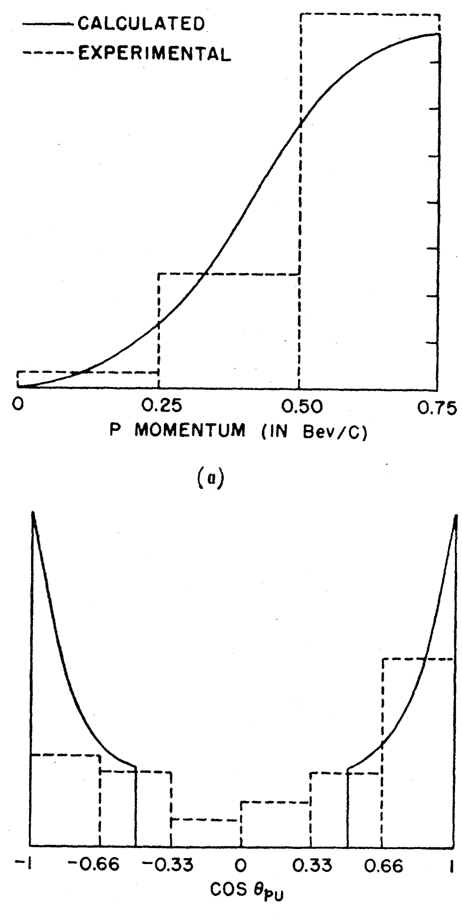

FIG. 3. (a) Momentum distribution and (b) angular distribution of p in the c.m. system. Full curves are calculated values; dashed curves are experimental.

greater than M^2. If we make a cutoff for the energy-momentum transfer squared at M^2, and define

$$J(W)=\int_{\Delta_{\min}^2(W)}^{M^2}d(\Delta^2)\frac{\Delta^2}{(\Delta^2+\mu^2)^2}, \quad (11)$$

Eq. (9) becomes

$$\sigma_A(U)=\frac{1}{8\pi}\left(\frac{G^2}{4\pi}\right)\times\binom{1}{2}\frac{1}{(pU)^2}\int_{W_{\min}}^{W_{\max}}dW\, 2kW^2\sigma_0(W)J(W). \quad (12)$$

A numerical integration of Eq. (12) is now carried out, where the values for $\sigma_0(W)$ are taken from experiments.[5]

[5] *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958); *Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960* (Interscience Publishers, Inc., New York, 1960); F. S. Crawford, Jr., R. L. Douglas, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959).

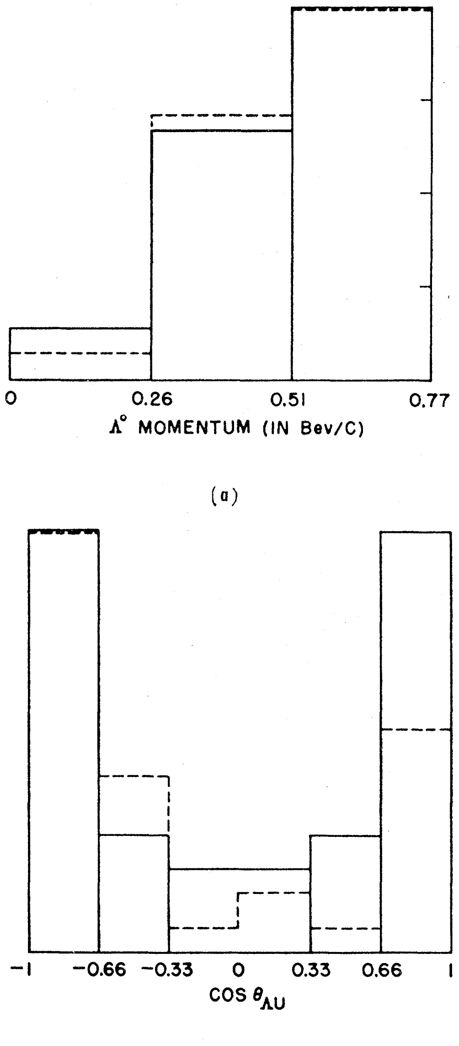

FIG. 4. (a) Momentum distribution and (b) angular distribution of Λ^0 in the c.m. system. Full curves are calculated values, dashed curves are experimental.

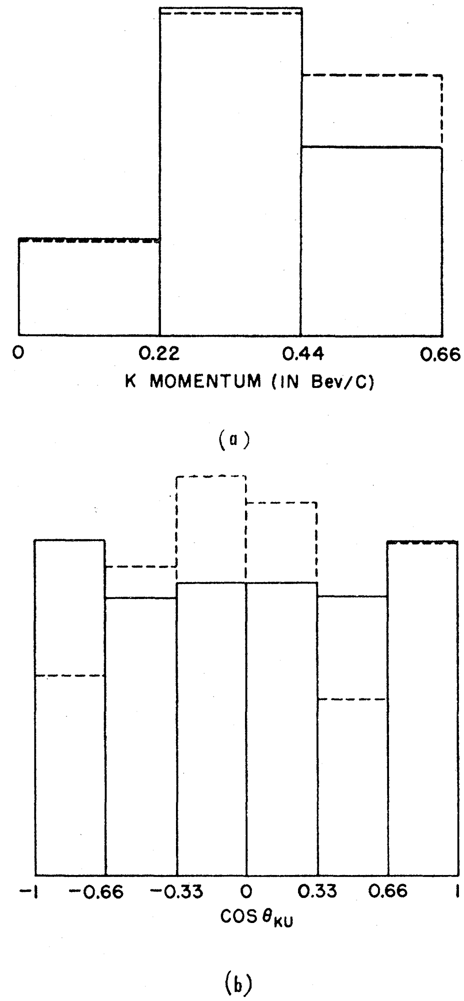

FIG. 5. (a) Momentum distribution and (b) angular distribution of K^+ in the c.m. system. Full curves are calculated values; dashed curves are experimental.

The results are collected in Table I together with experimental cross sections. Fairly good agreement is obtained in all four cases.

In order to have a better comparison between theory and experiment, a detailed calculation of the momentum and angular distributions of the nucleon, hyperon, and K meson in the final states is desirable. An attempt is made here to give a rough calculation in the case of Λ^0 production.

The calculation is in principle straightforward but rather tedious. The momentum and angular distributions of the final proton in the U system are easily done, but that of the hyperon and K meson involve the transformation from the W system to the U system. The information available from $\pi^-+p\rightarrow\Lambda^0+K^0$ interaction is in the W system with no dependence on the azimuthal angle φ whatsoever. However, in the transformation to the U system the angle φ plays an important role and complicates the calculation.

The procedure adopted in calculating the Λ^0 and K^+ distributions in the U system is as follows: The angular distribution of Λ^0 in the W system is approximated by regarding half of the Λ^0's coming out in the backward cone, $-1\leqslant\cos\theta_{\Lambda W}\leqslant-0.5$, and the rest in the interval $-0.5\leqslant\cos\theta_{\Lambda W}\leqslant 1$. An uniform distribution is assumed for both regions. Figure 2 gives this angular distribution. From the experimental evidence available,[5] it seems that this approximation is not too bad in the energy region under consideration. The W integral is then divided into several intervals, and within each interval the Δ^2 integral is again subdivided. The contributions from the various $W-\Delta^2$ regions with the same $p_{\Lambda U}$ or $\cos\theta_{\Lambda U}$ are then collected with the appropriate factors attached. Similar treatment is also applied to the K^+ distribution.

Figures 3–5 give the momentum and angular spectra of the final p, Λ^0, and K^+ in the U system together with the experimental data.[6] Good qualitative agreements can be seen in all the distributions, adding further support to the single pion exchange hypothesis. The unrealistic sharp drop at $\cos\theta_{nU}=\pm 0.5$ in the proton angular distribution is the result of our cutoff at $\Delta^2=M^2$, and should not be taken too seriously. The necessary kinematic relations between the W system and the U system are collected in the Appendix.

III. FOUR-PARTICLE PRODUCTION

The reactions under study are

$$p+p \to \pi^+ + p + K^0 + \Lambda^0, \tag{13a}$$

$$p+p \to \pi^+ + n + K^+ + \Lambda^0, \tag{13b}$$

$$p+p \to \pi^0 + p + K^+ + \Lambda^0, \tag{13c}$$

$$p+p \to \pi^+ + p + K^0 + \Sigma^0, \tag{13d}$$

$$p+p \to \pi^+ + n + K^+ + \Sigma^0, \tag{13e}$$

$$p+p \to \pi^0 + p + K^+ + \Sigma^0, \tag{13f}$$

$$p+p \to \pi^- + p + K^+ + \Sigma^+, \tag{13g}$$

$$p+p \to \pi^+ + p + K^+ + \Sigma^-, \tag{13h}$$

$$p+p \to \pi^+ + n + K^0 + \Sigma^+, \tag{13i}$$

$$p+p \to \pi^0 + p + K^0 + \Sigma^+, \tag{13j}$$

$$p+p \to \pi^0 + n + K^+ + \Sigma^+. \tag{13k}$$

The analysis of (13) is handicapped by the possibility of the final-state pion being produced at either vertex. Figures 6 and 7 represent the two possible types of graphs.

Since detailed information on the interaction $\pi+p \to \pi+K+Y$ in the energy region relevant here (pion kinetic energy in the lab system between 1 and 1.7 Bev) is not yet available, we shall not concern ourselves with this type of graph here. Of course it will be necessary to reevaluate our results when the relevant information becomes available and the $\pi+p\to\pi+K+Y$ cross sections are shown to be important. In that case an analogous treatment to that given in Sec. II will be applicable.

Here we shall restrict ourselves only to the graphs of the type in Fig. 6. At one vertex we have the pion-proton elastic or charge exchange scattering. The energy of interest lies in the $\frac{3}{2}$-$\frac{3}{2}$ resonance region, although one of the pions involved is a virtual one. The other vertex involves the associated production of strange particles in π-p collisions as in Sec. II.

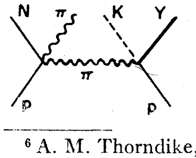

FIG. 6. Diagram representing processes (13).

[6] A. M. Thorndike, Bull. Am. Phys. Soc. 6, 17 (1961), and private communication from A. M. Thorndike and R. R. Rau. I am indebted to Dr. Thorndike and Dr. Rau for this information.

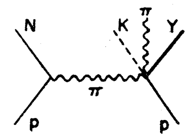

FIG. 7. Diagram representing processes (13).

The expression for the nucleon-nucleon scattering cross section has been given by several authors.[1] Here we shall follow Salzman and Salzman closely.[4]

$$\sigma_{\text{tot}}=2\sigma_A, \tag{14}$$

$$\sigma_A(U)=\frac{2}{(2\pi)^3}\frac{1}{(pU)^2}\int d(\Delta^2)\frac{1}{(\Delta^2+\mu^2)^2}\int\int dW dW' \times kW^2\sigma(W;\Delta^2)k'W'^2\sigma(W';\Delta^2). \tag{15}$$

Again the interference term is explicitly neglected in Eq. (14). The notation used is similar to that used in Sec. II with $U^2=-(p_1+p_2)^2$, the total energy squared in the over-all center-of-mass system (the U system), $W^2=-(p_Y+p_K)^2$, the total energy squared in the K-Y c.m. system (the W system), $W'^2=-(p_n+p_\pi)^2$, the total energy squared in the final π-N system (the W' system).

$$\left.\begin{aligned} U&=2(p^2+M^2)^{\frac{1}{2}},\\ W&=(k^2+M^2)^{\frac{1}{2}}+(k^2+\mu^2)^{\frac{1}{2}},\\ W'&=(k'^2+M^2)^{\frac{1}{2}}+(k'^2+\mu^2)^{\frac{1}{2}},\\ \Delta^2&=(p_1-P_1)^2=(P_2-p_2)^2,\\ P_1&=p_Y+p_K,\\ P_2&=p_\pi+p_n.\end{aligned}\right\} \tag{16}$$

$\sigma(W;\Delta^2)$ is the "cross section" for K-Y production, and is set equal to $\sigma(W)$ as in Sec. II. $\sigma(W';\Delta^2)$ is the "cross section" for π-p elastic or charge exchange scattering, and we set

$$\begin{aligned}\sigma(W';\Delta^2)&=\left(\frac{\Delta_{W'}}{k'}\right)^2\sigma(W';-\mu^2)=\left(\frac{\Delta_{W'}}{k'}\right)^2\sigma(W')\\ &=\left[\frac{(W'-M)^2+\Delta^2}{(W'-M)^2-\mu^2}\frac{(W'+M)^2+\Delta^2}{(W'+M)^2-\mu^2}\right]\sigma(W')\\ &\approx\frac{(W'-M)^2+\Delta^2}{(W'-M)^2-\mu^2}\sigma(W'),\end{aligned} \tag{17}$$

where $\Delta_{W'}$ is the three-momentum transfer in the W' system, or the momentum of the virtual pion in that system, and k' is the momentum of the corresponding real pion. This momentum square dependence for the "scattering cross section" is suggested by the p-wave static nucleon model, and is probably correct qualitatively in the low-energy $\frac{3}{2}$-$\frac{3}{2}$ resonance region. It might be emphasized that this correction factor deviates from unity appreciably for $\Delta^2\leqslant M^2$ when W' is in the resonance region, $1.2M\leqslant W'\leqslant 1.45M$ (the region of

TABLE II. Total cross sections (in μb) for four-particle production.

	Calculated		Experimental
$\pi^++p+K^0+\begin{cases}\Lambda^0\\ \Sigma^0\end{cases}$	9.6, 2.8	12.4	14±4
$\pi^++n+K^++\begin{cases}\Lambda^0\\ \Sigma^0\end{cases}$	1.1, 0.25	1.35	2±1
$\pi^0+p+K^++\begin{cases}\Lambda^0\\ \Sigma^0\end{cases}$	2.2, 0.5	2.70	11±4
$\pi^++p+K^++\Sigma^-$	2.8		3±2
$\pi^-+p+K^++\Sigma^+$	0.25	2.60	4±3
$\pi^++n+K^0+\Sigma^+$	0.62		
$\pi^0+p+K^0+\Sigma^+$	1.24		
$\pi^0+n+K^++\Sigma^+$	0.50		

interest here). The net result of including this correction is that Eq. (15) is approximately increased by a factor of ten. In fact, even at very high energies $U=14M$ (proton lab energy of 100 Bev) the contribution from low W and W' regions may play an important role in the nucleon-nucleon production cross sections both because of the large $\frac{3}{2}$-$\frac{3}{2}$ resonance cross section and also because of the large correction factor involved.

It is now a straightforward task to calculate the integral (15) by numerical integration keeping in mind the relationship among the limits of the three integration variables,[4]

$$\Delta^2U^2\left[1-\frac{W^2+W'^2+2M^2+\Delta^2}{U^2}\right]=(W^2-M^2)(W'^2-M^2)+(W'^2-W^2)^2\frac{M^2}{U^2}. \quad (18)$$

From the present experimental data[5,7] on $\pi+p\rightarrow\pi+N$ and $\pi+p\rightarrow K+Y$ we have evaluated the total cross sections for the processes (13a)–(13k). Isotopic spin invariance has been invoked to calculate π^0-p interactions. The results are collected in Table II together with experimental data. It is seen that there are fair agreements between calculated values and the experimental results in all cases but one. In the case of $\pi^0+p+K^++\Lambda^0(\Sigma^0)$ production the calculated value is much too small in comparison with the experimental result.

It is true that perhaps one should not attach too much importance to the absolute values of the calculated cross sections, since this is only a first attempt at an order of magnitude estimate. However, the relative cross sections should be more meaningful, because they reflect directly the result of isotopic spin invariance apart from minor phase-space considerations. In this sense the discrepancy between the calculated cross section and the experimental value for $\pi^0+p+K^++\Lambda^0(\Sigma^0)$ production is perhaps a real one.

As pointed out at the beginning of this section, the possibility of having the pion produced at the K-Y vertex is not necessarily small. The recent discovery of a hyperon resonance Y^* may enhance the contribution from this type of graphs, at least in cases where Λ^0 is involved. Indeed, experimentally several probable Y^*'s were found in p-p collisions.[6] Since the number of observed events is small in all four-particle final-state strange particle productions, it is not yet statistically significant enough to say whether the single pion exchange model gives an adequate description of this type of interactions.

IV. DISCUSSION

Originally the idea of an intermediate virtual pion exchange was introduced for very high energy interactions where the collisions are mostly peripheral. The treatment considered here seems to indicate that it may also play a dominant role at the relatively low energy of 3 Bev. Whether our agreements with experiment may be purely fortuitous is not clear. The various approximations made are plausible but not established. In the case of three-particle production one has a stronger reason to expect the model to be essentially correct. On the other hand, four-particle productions involve more uncertainties both theoretically and experimentally. Information on $KY\pi$ production in π-p collisions and especially Y^* formation will be most helpful in clarifying many aspects of the problem here.

APPENDIX: KINEMATIC RELATIONS FOR PROCESSES (1)

$$W_{\min}=M_Y+M_K,$$
$$W_{\max}=U-M,$$
$$P=p_Y+p_K,$$
$$P^2=-W^2.$$

In the W system we have (see Fig. 8):

$$p_{YW}=p_{KW}=[W^2-(M_Y+M_K)^2]^{\frac{1}{2}}\times[W^2-(M_Y-M_K)^2]^{\frac{1}{2}}/2W,$$

the momenta of the hyperon and K meson in the W

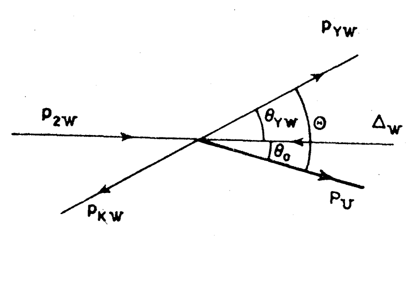

FIG. 8. Kinematics in the W system.

[7] G. Puppi and A. Stanghellini, Nuovo cimento 5, 1305 (1957); J. Ashkin, J. P. Blaser, F. Feiner, and M. O. Stern, Phys. Rev. **105**, 724 (1957); W. D. Walker, W. D. Shephard, and J. Davis, Phys. Rev. **118**, 1612 (1960).

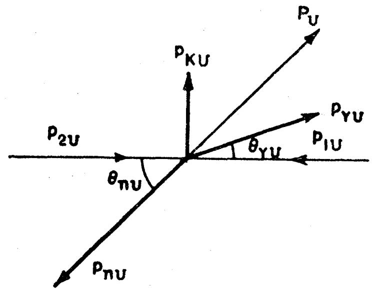

FIG. 9. Kinematics in the U system.

system;

$$E_{YW}=(W^2+M_Y^2-M_K^2)/2W,$$
$$E_{KW}=(W^2-M_Y^2+M_K^2)/2W,$$

the energies of the hyperon and K meson, respectively;

$$E_{2W}=(p_{2W}^2+M^2)^{\frac{1}{2}}=(W^2+M^2+\Delta^2)/2W,$$

the energy of the incident proton; and

$$W=(p_{YW}^2+M_Y^2)^{\frac{1}{2}}+(p_{KW}^2+M_K^2)^{\frac{1}{2}},$$

the total energy in the W system.

In the U system we have (see Fig. 9):

$$P_{nU}=P_U=[U^2-(W+M)^2]^{\frac{1}{2}}[U^2-(W-M)^2]^{\frac{1}{2}}/2U,$$

the momenta of the recoil nucleon and the W system in the over-all c.m. system; and

$$E_{nU}=(U^2-W^2+M^2)/2U,$$
$$W_U=(U^2+W^2-M^2)/2U=(W^2+P_U^2)^{\frac{1}{2}},$$

the energies of the recoil nucleon and the W system, respectively.

The kinematic relations between the W system and the U system are given by

$$\beta=P_U/W_U,$$
$$(1-\beta^2)^{-\frac{1}{2}}=(U^2+W^2-M^2)/2UW,$$

where β is the velocity of the W system relative to the U system.

The various angles appearing in Figs. 8 and 9 are defined as follows:

$$\mathbf{p}_{1U}\cdot\mathbf{p}_{nU}=\mathbf{p}_{2U}\cdot\mathbf{P}_U=pp_{nU}\cos\theta_{nU},$$
$$\mathbf{p}_{2W}\cdot\mathbf{P}_U=p_{2W}P_U\cos\theta_0,$$
$$\mathbf{p}_{2U}\cdot\mathbf{p}_{YU}=pp_{YU}\cos\theta_{YU},$$
$$\mathbf{p}_{2W}\cdot\mathbf{p}_{YW}=p_{2W}p_{YW}\cos\theta_{YW},$$
$$\mathbf{p}_{YW}\cdot\mathbf{P}_U=p_{YW}P_U\cos\Theta,$$

and we also have the relations

$$\cos\Theta=\cos\theta_{YW}\cos\theta_0+\sin\theta_{YW}\sin\theta_0\cos\varphi,$$
$$\tan\theta_0=p\sin\theta_{nU}(1-\beta^2)^{\frac{1}{2}}/(p\cos\theta_{nU}-\tfrac{1}{2}U\beta).$$

The energies of the hyperon and the K meson in the U system are given by

$$E_{YU}=(E_{YW}+\beta p_{YW}\cos\Theta)(1-\beta^2)^{-\frac{1}{2}},$$
$$E_{KU}=(E_{KW}-\beta p_{KW}\cos\Theta)(1-\beta^2)^{-\frac{1}{2}}.$$

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