Experiments on Pair and Triplet Production by 5–90-Mev X Rays*

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The energy partition between the positrons and electrons of 5787 pairs and 1872 triplets produced by a hardened beam of x rays of maximum energy 90 Mev, has been investigated using a nuclear emulsion technique. The results agree approximately with the theory of Bethe and Heitler. The relative total cross sections of triplet production agree, within experimental error, with the theory of Vortuba and Wheeler and Lamb with an exchange correction. Similarly, the angular distributions of both pairs and triplets can be explained within experimental error either with the theory of Borsellino or with that of Bethe.

INTRODUCTION

ALTHOUGH pair production from photons has been extensively studied in detail by many workers[1] with photon energies less than 20 Mev, fewer experiments have been done at higher energies. However, Emigh,[2] using a cloud chamber to investigate relative pair cross sections of different elements in the region 50 to 300 Mev, reported results which were systematically lower than values predicted by the theory of Bethe and Heitler but which were in good agreement with theory for the energy partition of pair electrons. Two experiments[3,4] employed an emulsion technique and photons of energy of the order of 100 Mev to investigate the energy partition between the members of pairs. Some discrepancies with theory were reported, although it should be stated that these experiments suffered from lack of sufficient statistics. Furthermore, while the present experiment was in progress, Hart, Cocconi, Cocconi, and Sellen[5] and Gates,[6] respectively, using a diffusion chamber and a bubble chamber, published results on triplet production in hydrogen, with a range of photon energies which included that used in the present experiment. These results gave reasonably good agreement with the theory particularly as regards energy partition.

The aim of this experiment was to investigate some details of pair and triplet production using photons of maximum energy 90 Mev. Since a visual method of detection was particularly well suited to such studies, a nuclear emulsion technique was used to record the events. Although this method is quite laborious it has the advantage of providing a permanent record of events which can be inspected and measured at convenience.

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[2] C. R. Emigh, Phys. Rev. **86**, 1028 (1952).

[3] I. B. McDiarmid, Can. J. Phys. **30**, 670 (1952).

[4] G. Baroni, A. Borsellino, L. Scarsi, and G. Vanderhaeghe, Nuovo cimento **10**, 1653 (1953).

[5] E. L. Hart, G. Cocconi, V. T. Cocconi, and J. M. Sellen, Phys. Rev. **115**, 678 (1959).

[6] D. C. Gates, University of California Radiation Laboratory Report, UCRL-9390, 1960 (unpublished).

MEASUREMENTS AND ERROR

Ilford G-5 nuclear emulsions of size 1 in.×2 in.×400 μ were exposed to a hardened beam of x rays of maximum energy 90 Mev, produced by the electron synchrotron at the National Bureau of Standards. The developed plates were scanned for pair and triplet events with Leitz Ortholux microscopes fitted with 100× objectives under oil immersion and 10× eyepieces. The microscope eyepieces were raised to 16× magnification for the purposes of actual track measurement.

The energy of any particular electron was determined by measuring the multiple scattering of its track using Fowler's coordinate method.[7] To do this, the plates were mounted on the microscope stage in such a way that the direction of tracks was roughly parallel to its lateral mechanical movement. A micrometer scale with rectangular axes was placed in one eyepiece so that it could be observed superimposed on the field of view. The scale was adjusted so that its x axis was actually parallel to the stage lateral movement. For the purposes of measurement, a track was first aligned along the x axis. The stage was then moved laterally 50 μ at a time. The deflection of the track at the end of each of these cell lengths was then read from the y axis of the eyepiece scale which was divided into 0.8-μ divisions. Wherever possible, fourteen such measurements were taken successively on each track corresponding to a total length of 700 μ. In cases where a track left the emulsion in less length, it was accepted with as many measurements as possible although some accuracy had to be sacrificed.

The energy is obtained from the relation

$$p\beta=\frac{K'Z}{\bar{\alpha}(t)}\left(\frac{t}{100}\right)^{\frac{1}{2}}\text{ Mev,} \tag{1}$$

where p is the particle momentum in units of Mev/c, Z is its charge, and $\bar{\alpha}(t)$ is the mean projected deflection of the particle. The latter is obtained from the second differences of the y readings described above with a cell length $t=50\ \mu$. The quantity K', although referred to as the "scattering constant," actually varies slowly[8] with β and t, where β is the ratio of the velocity of the

[7] P. H. Fowler, Phil. Mag. **41**, 169 (1950).

[8] L. Voyvodic and E. Pickup, Phys. Rev. **85**, 91 (1952).

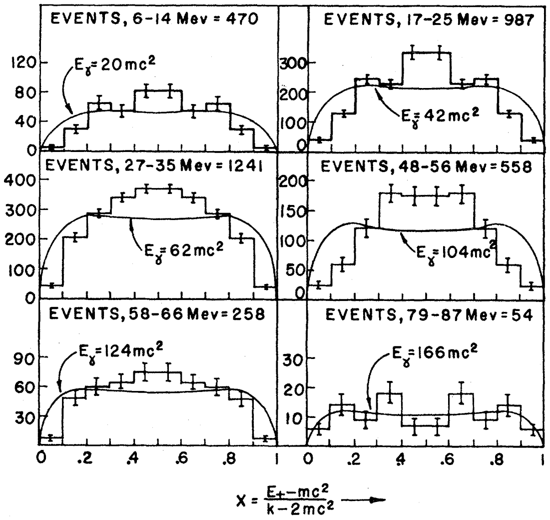

FIG. 1. The Bethe-Heitler theoretical energy partition function for different photon energies with corresponding experimental histogram, plotted after normalization over the whole area with respect to the theoretical curve.

particle to the velocity of light and t is the cell length. The magnitude of the total error involved in the measurement of the mean projected angle $\bar{\alpha}(t)$ was estimated after the calculation of Ekspong[9] and is given by the expression

$$\Delta = [(0.89/n)\bar{\alpha}(t)^2 + (0.073t^{-0.84})^2]^{\frac{1}{2}}, \qquad (2)$$

where n is the number of measurements recorded from each track. Since $\bar{\alpha}(t)$ contains the total error in the denominator of the expression for the energy, the error in the latter is therefore asymmetric. The total error in any particular interval was obtained by dividing the error estimated from the above expression by the number of events observed in the same interval.[8]

The determination of the angle between two members of a pair or between the two high-energy members of a triplet can be ascertained from the same measurements on y readings as were used for energy calculations in addition to the depth readings z at the end of the first cell length. The space angle calculated from such measurements was subject to an inaccuracy due to multiple scattering over the first cell length. The error so introduced was estimated from the average multiple scattering for the whole length of the track and combined with the error due to grain noise, and to reading the eyepiece scale, in the estimation of the total error in the angle. The total error was thus dependent on the track energies and is given in Table I.

RESULTS AND DISCUSSIONS

In a given volume of emulsion 6056 pairs were observed out of which 5787 were measured. Of the remaining pairs, 169 were rejected on account of

[9] A. Gusla Ekspong, Arkiv. Fysik **9**, 59 (1955).

TABLE I. The table indicates the estimated total error in the space angle of divergence ω and in $x = \omega/\omega_0$ at different energies.[a]

k in Mev	ω_0	Total error in radians	Error in $x = \omega/\omega_0$
12.5	0.163	±0.073	±0.45
22.5	0.091	±0.042	±0.46
30.0	0.068	±0.031	±0.46
50.0	0.041	±0.019	±0.47
65.0	0.031	±0.015	±0.51
75.0	0.027	±0.014	±0.52

[a] The notation in the columns is defined below Eq. (3) of the text.

distortion suffered by one or both tracks near the edge of the emulsion and 100 had at least one track too short for measurement with reasonable accuracy due to its being scattered out of the emulsion. In a total of 54 433 fields of view, 1935 triplets were observed out of which 1872 were measured and 63 rejected, and 21 887 pairs were recorded. The volume represented by each field of view was $120\,\mu \times 150\,\mu \times 220\,\mu$. The above data were used to study: (i) the energy partition between the members of the pair and between the high-energy partners of the triplet, (ii) the relative triplet cross sections, (iii) the pair to triplet ratios, and (iv) the angular divergence of pairs and triplets.

(i) The Energy Partition between the Members of the Pair and between the High-Energy Partners of the Triplet

The calculation of the energy distribution between the partners of a pair at different photon energies for Ilford G-5 emulsion was carried out taking into account the weighted contributions of the different elements in the emulsion. After proper normalization, the experi-

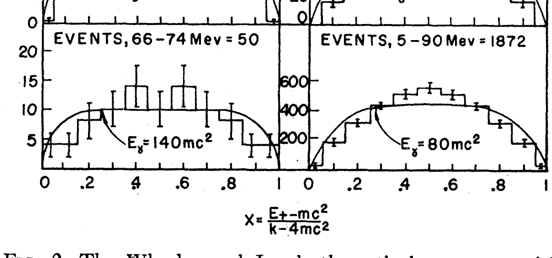

FIG. 2. The Wheeler and Lamb theoretical energy partition function with exchange correction for different photon energies with corresponding experimental histogram, plotted after normalization over the whole area with respect to the theoretical curve.

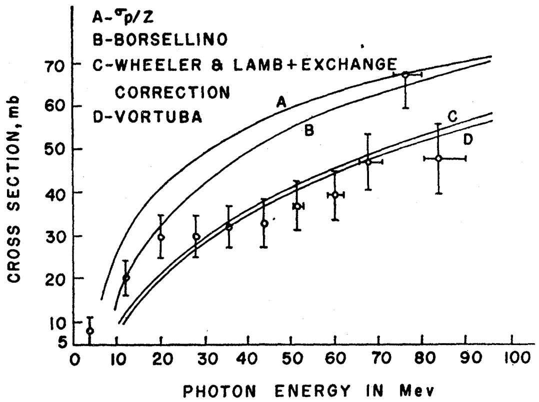

FIG. 3. Relative total triplet cross section. The experimental points have been plotted after normalization with respect to the theory of Wheeler and Lamb with exchange correction.

mental results were compared with those deduced from the Bethe-Heitler formulas[10] for differential cross sections. Some representative curves are given in Fig. 1.

The energy distribution functions for high-energy partners of triplets have been calculated from Wheeler and Lamb's differential formula[11] taking into account the exchange correction.[12] The curves predicted by these calculations were compared with experimental data after proper normalization and several are reproduced in Fig. 2.

(ii) Relative Triplet Cross Sections

The relative total triplet cross section was studied calculating the shape of the incident bremsstrahlung spectrum from the experimental energy distribution of the 5787 pairs measured. The theoretical total pair cross sections of G-5 emulsion needed for this purpose were calculated for photon energies up to 90 Mev by taking into account the weighted contributions from the various elements constituting the emulsion. The individual cross sections for the emulsion components were taken from the calculations of Grodstein[13] who used the Bethe-Heitler theory including a Coulomb correction. Finally the relative total triplet cross section was determined at different energies from the experimental energy distribution of the triplet events and the shape of the bremsstrahlung spectrum deduced by the manner described above.

Theoretical triplet cross sections for photon energies up to 100 Mev have been calculated from four different theories. The calculation based on Borsellino's theory was done by Davisson[14] using Wheeler and Lamb's results as a guide beyond 50 Mev. Grodstein[13] performed calculations based on Vortuba's theory and a set of values was also obtained based on Wheeler and Lamb's theory, including the exchange correction, by numerical integration of the differential formula at various photon energies. The fourth set of theoretical values was calculated by dividing the pair cross-section values by the average Z of the emulsion, i.e., σ_p/Z.

The experimental relative total triplet cross-section curve was normalized to the same area as the curve obtained from the Wheeler and Lamb theory including the exchange correction. All four theoretical curves are shown in Fig. 3, along with the experimental points.

(iii) Pair to Triplet Ratio

In order to take advantage of all 1872 triplets actually measured, it was assumed that the distribution of the 5787 pairs measured was representative of all 21 887 pairs recorded. Justification for this was provided by the fact that 3000 pairs produced an essentially identical spectrum to that given by all 5787 pairs. The ratio of pairs to triplets was determined for events in a series of 8-Mev intervals, the number of triplets being that actually measured in an interval and the number of pairs being 3.78 times the number measured in the same interval. Figure 4 shows the results obtained for the ratio as a function of energy, and it can be seen that, within statistical accuracy, the pair to triplet ratio does not vary considerably over the entire range of photon energy.

(iv) Angular Divergence of Pairs and Triplets

Borsellino's expression for the distribution of the angle of divergence is[15]

$$d\sigma = 16a(1-a)F\frac{x}{(1+x^2)^2}dx, \qquad (3)$$

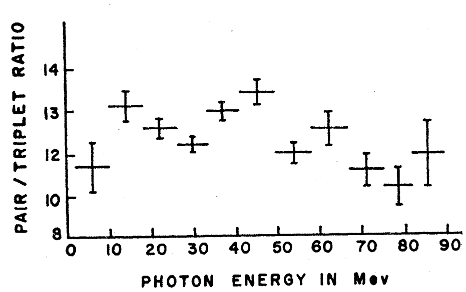

FIG. 4. Ratio of pairs to triplets. The horizontal bar represents the energy interval (8 Mev) at each photon energy.

[10] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A166**, 83 (1934); W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1934), 3rd ed.; H. A. Bethe and J. Ashkin, *Experimental Nuclear Physics*, edited by E. Segré (John Wiley & Sons, Inc., New York, 1953), Vol. 1.

[11] J. A. Wheeler and W. Lamb, Phys. Rev. **55**, 858 (1939); **101**, 1836 (1956).

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[13] G. W. Grodstein, *X-ray Attenuation Coefficients from 10 kev to 100 Mev*, National Bureau of Standards Circular No. 583 (U. S. Government Printing Office, Washington, D. C., 1957).

[14] C. M. Davisson, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955), Chap. 2.

[15] A. Borsellino, Phys. Rev. **89**, 1023 (1953).

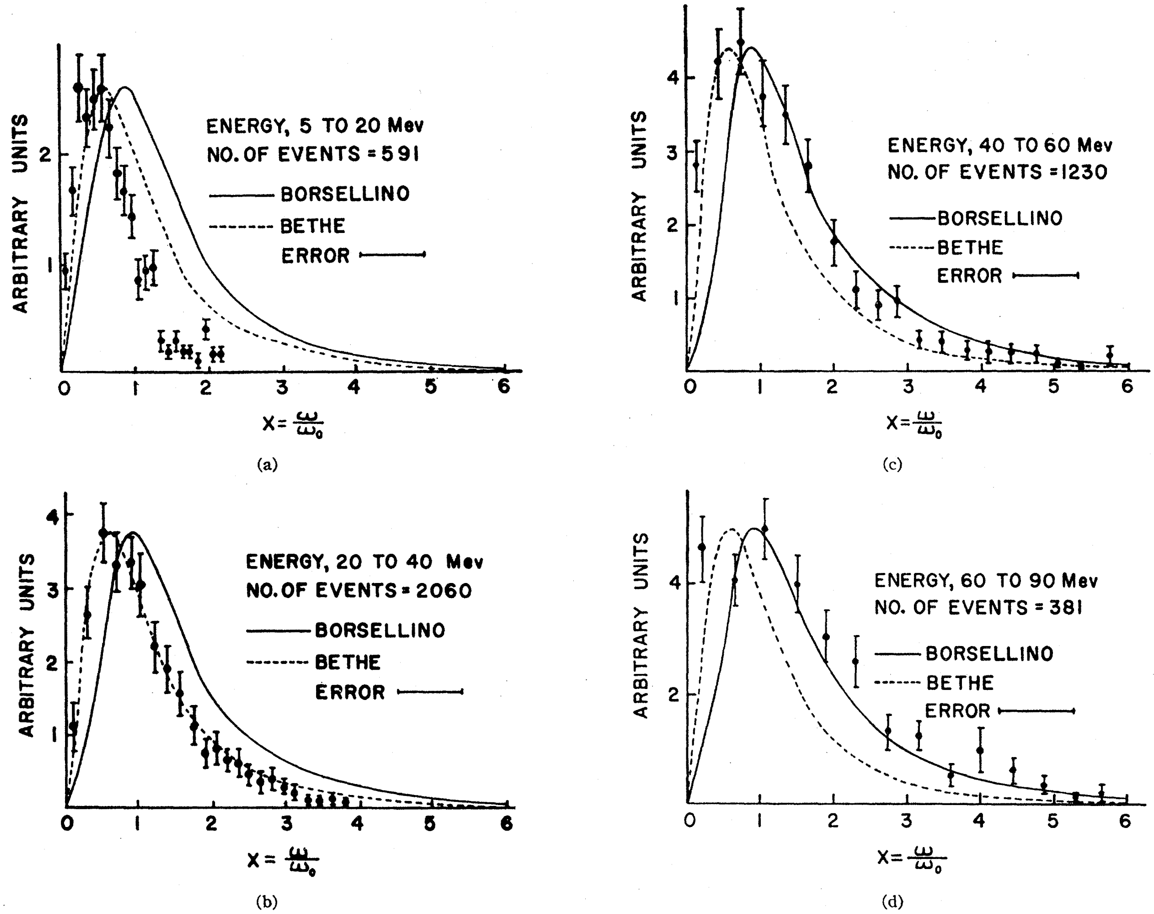

FIG. 5. Distribution of the opening angle ω between the two partners of the pairs at different energy intervals. k is the photon energy for which the theoretical curves have been drawn. The abscissa represents the ratio $x=\omega/\omega_0$.

where $a=E_+/k$, F is a function of x and a, E_-, E_+, and k are the energies of the electron, the positron, and the photon, respectively, E_- and k are the energies of the electron and photon, respectively, and $x=\omega/\omega_0$, where ω is the angle of divergence and $\omega_0=kmc^2/E_+E_-$. If equipartition of energy between the two members of the pair is assumed, $E_+=E_-=k/2$ and $\omega_0=4mc^2/k$. With Borsellino's notation, Bethe's expression[16] for the distribution of angle of divergence is given by $x/(1+x^2)^2$, where F has been assumed to be constant.

Out of 5787 pairs and 1872 triplets, only those events which had a value $a=E_+/k$ between 0.2 and 0.8 were taken into consideration. In addition, if all three partners of a triplet had comparable energy the event was rejected for the final analysis of results. In conclusion, 4262 pairs and 1657 triplets were taken into account.

The experimental data for pairs and for triplets were grouped into four and two energy intervals, respectively. The interval of energy chosen for triplets was large because of the data available for analysis were limited. The choice of such large intervals was acceptable because the shape of the theoretical curve including the position of the maximum varied very little with the energy. For each group of energies, a grouped frequency distribution of the space angle of divergence ω was made. The experimental curves for pairs, Fig. 5 (a)–(d), and for triplets, Fig. 6 (a) and (b), were normalized in such a way as to make the maxima of the theoretical and experimental curves the same.

CONCLUSION

The experimental results for the partition of energy between pair partners shown in Fig. 1 and between high-energy triplet partners shown in Fig. 2 indicate only approximate agreement with the theory. In order to search for the discrepancy, a series of experiments was performed to evaluate detection efficiency, personal error, systematic error, and the influence of cell length on the estimation of energies. The results obtained in the present experiment were corrected after the method of McDiarmid[3] for efficiency in the detection of a pair with extreme asymmetric energy partition. The correc-

[16] H. A. Bethe, Proc. Can. Soc. 30, 524 (1934).

tion did not influence the results. In order to ascertain personal error due to failure to observe an event in a given volume of the emulsion, a series of independent experiments was performed. The test showed that the different microscopists agreed to within less than a fraction of one percent. Out of the 6056 pairs, 100 were completely rejected for the purposes of energy measurement for having insufficient length in the emulsion for reasonable accuracy in the final estimation of energy. Even if it is assumed that these 100 events representing approximately 1.6% of the total belong to extreme asymmetric energy partition, the discrepancies between experimental and theoretical results still fall beyond statistical error. The choice of 50 μ for the cell length in the measurement of multiple scattering was made after consideration of the various sources of error, including stage noise, involved in the measurements. However it did not seem to play an important part in the estimation of energies as the measurement of some 2000 events with 100-μ overlapping cells gave results nearly identical with those obtained using a 50-μ cell length.

The experimental points for the total relative triplet cross section agree with the curves obtained from the theories both of Vortuba and of Wheeler and Lamb with the exchange correction, within the limits of experimental error. The greatest source of error was due to difficulties involved in the identification of a triplet event when the recoil electron had a very small energy. Triplets could be identified with only one grain developed by the recoil electron corresponding to an energy of the latter amounting to a few hundred electron volts. Since uncertainty could have arisen through the chance occurrence of a background grain near the origin of the event in question, only those with the single grain larger than the average background grain were actually accepted as triplets. Furthermore, cases could exist where the recoil electron failed to ionize entirely giving rise to the classification of such an event as a pair. Bearing these difficulties in mind, agreement between the experimental results and each of the two theories is satisfactory.

It should be noted that the position of the maxima of the curves given by Bethe's expression, $x/(1+x^2)^2$, for angular divergence do not change as a function of energy whereas the maxima of Borsellino's curves shift slightly toward the right as the photon energy increases.

Referring to the pair distribution, it is observed that at the lower photon energies 5–20 Mev and 20–40 Mev, agreement seems to be better with the curve obtained from Bethe's formula than with that obtained from Borsellino's. However, as the energy increases, the experimental points fall nearer to Borsellino's curve than to Bethe's. It should be remarked, however, that at 5–20 Mev the experimental points are more peaked than the two theories predict. For the triplet distribution between 5–40 Mev, the experimental points are better fitted to Bethe's expression than to that of Borsellino. However, in the energy region 40–90 Mev, the agreement with the theories is only approximate. This discrepancy may be due to the small number of triplet events taken into consideration in this energy interval. In conclusion, the angular distribution of pairs and of triplets is well explained within experimental error with the theories of either Borsellino or Bethe.

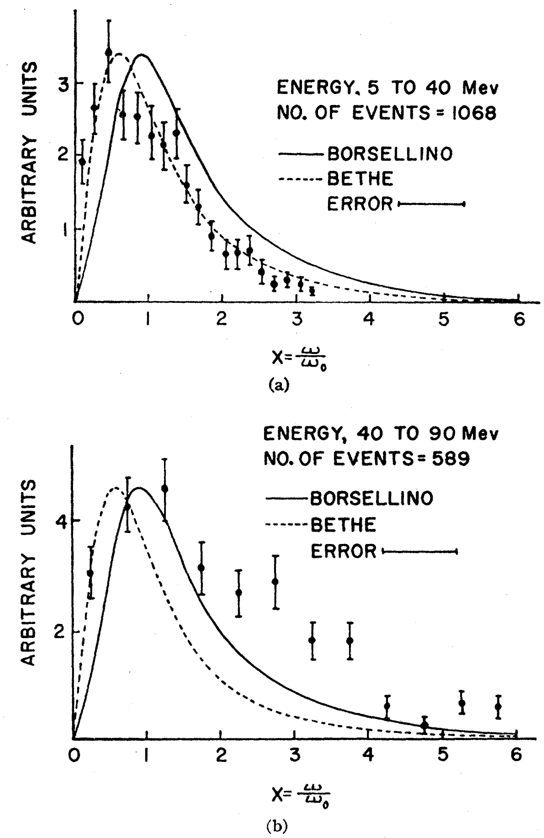

FIG. 6. Distribution of the opening angle ω between the two high-energy partners of the triplets at different energy intervals. k is the photon energy for which the theoretical curves have been drawn. The abscissa represents the ratio $x=\omega/\omega_0$.