Odd Σ-Λ Parity and Pion-Hyperon Scattering

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The static dispersion relations for the S-wave $\pi-\Lambda$ and $\pi-\Sigma$ scattering amplitudes are investigated under the assumptions that the dominant forces result from a scalar $\pi\Lambda\Sigma$ interaction, and that multiple meson processes may be ignored. If it is required that the $\pi-\Lambda$ effective range is small, or alternately, that no subtractions be made in the dispersion relation for the denominator function of the $\pi-\Lambda$ amplitude, the $\pi\Lambda\Sigma$ coupling constant is determined to be $F^2\sim1.6$. However, such a strong interaction leads to an attractive force in the isotopic-spin 2, $\pi+\Sigma$ S state that is more than sufficient to produce a bound state. A brief discussion is given of the meaning of this result, and of a possible related mechanism for the 1380-Mev $\pi-\Lambda$ resonance.

RECENTLY, various authors have proposed that the $\Sigma-\Lambda$ parity is odd and that the Σ particle be regarded as a $\Lambda-\pi$ bound state.[1,2] In this note static dispersion relations will be used to study the $\pi-\Lambda$ and $\pi-\Sigma$ S-wave scattering amplitudes, under the assumption of a scalar $\pi\Lambda\Sigma$ interaction. The principal result is the prediction of either a $\pi-\Sigma$ bound state of isotopic spin two, or a very large $I=2$ scattering length.

If one writes the dispersion relation for the S-wave $\pi-\Lambda$ scattering amplitude in the static limit, and neglects all singularities other than the Σ pole, the unitary branch cut, the crossed pole, and the crossed unitary cut, the solution to the equation is of the form,

$$T=\frac{e^{i\delta}\sin\delta}{k}=\frac{-2F^2\Delta}{(\omega^2-\Delta^2)D},$$
$$D=1+\frac{F^2\Delta}{\kappa_0}\left(1-\frac{2k^2}{\omega^2-\Delta^2}\right)+\frac{i2F^2\Delta k}{(\omega^2-\Delta^2)}\quad\text{for}\quad|\omega|>\mu,\tag{1}$$
$$D=1+\frac{F^2\Delta}{\kappa_0}\left(1-\frac{2\kappa}{\kappa_0+\kappa}\right)\quad\text{for}\quad|\omega|<\mu,$$

where F^2 is the scalar $\pi\Lambda\Sigma$ coupling constant, Δ is the $\Sigma-\Lambda$ mass difference, μ, ω and $k=(\omega^2-\mu^2)^{\frac{1}{2}}$ are the pion mass, energy and momentum, and κ_0 and κ are given by $\kappa_0=(\mu^2-\Delta^2)^{\frac{1}{2}}$, $\kappa=(\mu^2-\omega^2)^{\frac{1}{2}}$. It is assumed throughout the paper that the denominators of the various amplitudes do not contain poles of the Castillejo, Dalitz, Dyson type. Since the $\bar{K}N$ branch cut has been neglected, Eq. (1) cannot be accurate at energies close to the $\bar{K}+N$ rest mass, especially if the $\pi-\Lambda$ resonance of Alston *et al.*[3] exists in the S state. Reasonable accuracy is expected in the neighborhood of the $\pi-\Lambda$ threshold, however, since this threshold is about twice as far from the resonance as from the Σ pole.

Nambu and Sakurai have pointed out that if the $\pi-\Lambda$ effective range is small, the coupling constant F^2 can be determined from the masses of the π, Λ, and Σ.[2] In the static limit, this determination may be made from Eq. (1) in the following manner. In the effective range approximation, κ_0 (the inverse "radius of the bound $\pi-\Lambda$ in the Σ state") is related to the scattering length $T(\mu)$ and effective range r_0 by the well-known formula, $\kappa_0=-T^{-1}(\mu)+\frac{1}{2}r_0\kappa_0^2$. If $T(\mu)$ and r_0 are determined from Eq. (1), the equation for κ_0 becomes an identity, i.e.,

$$\kappa_0\equiv\kappa_0(\kappa_0+F^2\Delta)(2F^2\Delta)^{-1}-\kappa_0(\kappa_0-F^2\Delta)(2F^2\Delta)^{-1}.$$

The assumption of a small effective range implies that the last term in this equation may be neglected, and leads to the relation,

$$F^2\cong\kappa_0\Delta^{-1}\cong1.6.\tag{2}$$

A similar approach has been used recently by Bernstein and Oehme,[4] who estimate that $F^2\cong1.8$. These authors use a fully relativistic dispersion relation, but neglect the singularities imposed by crossing symmetry. A different estimate of $F^2\cong1.47$ has been made by Liu, using the relativistic dispersion relation for the $\pi\Lambda\Sigma$ vertex function.[5]

An alternate derivation of Eq. (2) in the static model follows from the assumption that a subtraction should not be made in the dispersion relation for a scattering denominator function unless necessary to obtain a convergent integral. The form of the denominator function D given in Eq. (1) was obtained from a once-subtracted dispersion relation, together with the unitarity requirement and the condition $D(\pm\Delta)=1$ (implied by the definition of the coupling constant F^2). However, if no subtractions are made, the equation for D still involves a convergent integral, i.e.,

$$D(\omega)=\frac{4F^2\Delta}{\pi}\int_\mu^\infty\frac{d\omega'k'\omega'}{[\omega'^2-(\omega-i\epsilon)^2](\omega'^2-\Delta^2)}.\tag{3}$$

It can be shown by direct integration that the condition $D(\pm\Delta)=1$, when applied to Eq. (3), leads to the coupling constant relation $F^2=\kappa_0\Delta^{-1}$. The two expressions for D, Eqs. (1) and (3), are identical if $F^2=\kappa_0\Delta^{-1}$.

The corresponding static equations for S-wave $\pi-\Sigma$ scattering are more complicated, because the crossing relations couple the amplitudes of different isotopic spins. For simplicity, we neglect the "crossed unitary cut," and also neglect the effects of the pseudoscalar

[1] S. Barshay, Phys. Rev. Letters 1, 97 (1958); S. Barshay and H. Pendleton, III, Phys. Rev. Letters 6, 421 (1961).
[2] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 6, 377 (1961).
[3] M. Alston *et al.*, Phys. Rev. Letters 5, 520 (1960).
[4] J. Bernstein and R. Oehme, Phys. Rev. Letters 6, 639 (1961)
[5] L. S. Liu (private communication; and to be published).

$\pi\Sigma\Sigma$ interaction. The equations may be written,

$$
\begin{aligned}
T_i &= F^2 N_i(\omega)/D_i(\omega), \\
N_i &= \frac{A_i}{\omega-\Delta} + \frac{B_i D_i(-\Delta)}{\omega+\Delta}, \qquad (4) \\
D_i &= 1 - \frac{F^2(\omega-\Delta)}{\pi} \int_\mu^\infty \frac{d\omega'(\omega'^2-\mu^2)^{\frac{3}{2}} N_i(\omega')}{(\omega'-\Delta)(\omega'-\omega-i\epsilon)},
\end{aligned}
$$

where the values of the constants A_i and B_i for the different isotopic spin states are, $A_0=A_2=1$, $A_1=-1$, $B_1=B_2=0$, $B_0=-3$. The integrals in this equation are easily evaluated and are of the type encountered in the Lee model. In the case of isotopic spin 2, there is only the "attractive" pole at $\omega=\Delta$, and it may be shown that for $F^2>0.9$, the denominator function D vanishes at an energy in the interval $\Delta<\omega<\mu$, implying the existence of a $\pi-\Sigma$ bound state. A rough calculation indicates that this conclusion is insensitive to the presence of the crossed unitary cut. The effect of including the $\Lambda+2\pi$ branch cut would be to increase the effective attraction, and decrease the value of F^2 needed to produce binding.[6]

It is unlikely that a bound $\pi-\Sigma$ state of $I=2$ actually exists, since it has not been observed. It may be that the effects of interactions omitted in the various determinations of F^2, and in Eq. (4) ($\pi-\pi$ interaction, $\bar{K}-N$ interaction, etc.), weaken the attraction sufficiently that no bound state is predicted. If this is the case, however, it seems likely that the state is almost bound, so that the scattering length $T_2(\mu)$ is large and positive. [However, no resonance is predicted; it can be shown from Eq. (4) that if F^2 is small enough so that D_2 fails to vanish for $\omega<\mu$, then $\mathrm{Re}D_2$ fails to vanish for $\omega>\mu$.] If $T_2(\mu)$ is large and positive, the number of correlated $\pi^+\Sigma^+$ and $\pi^-\Sigma^-$ pairs occurring in the final states of various high-energy processes should exceed the number predicted from phase-space considerations. One should look for pairs of kinetic energy less than 50 Mev in the $\pi-\Sigma$ center-of-mass system.

In the case of $I=0$, $\pi-\Sigma$ scattering, the negative pole term at $\omega=-\Delta$ (which is associated with scattering through a virtual Λ state), decreases the attraction so that even for $F^2=1.6$, the predicted scattering length is not much larger than that given in Born approximation.

If $F^2>\kappa_0\Delta^{-1}$ a ghost pole occurs in Eq. (1) at a negative value of $(\omega^2-\Delta^2)$, and a "reverse resonance" (phase shift decreases through 90°) occurs at a positive value of $(k^2-\kappa_0^2)$. We do not regard this as significant, since for such large values of F^2, the $\pi-\Lambda$ state must be strongly coupled to the state of a pion plus the $\pi-\Sigma$ bound state, or nearly bound state, predicted by Eq. (4). The neglect of this coupling is expected to limit the validity of Eq. (1) to energies close to the $\pi-\Lambda$ threshold. Even for small values of F^2, a ghost pole and "reverse resonance" occur at large values of $|\omega|$ in the expression for the $I=1$, $\pi-\Sigma$ amplitude. However, because of the neglect of crossing and of states involving more than one meson, Eq. (4) would not be valid for large $|\omega|$ even if the hyperon masses were infinite and the $\pi\Lambda\Sigma$ interaction were the only interaction existing in nature.

The above results are analogous to those that may be deduced from the simple model of the Σ as a $\pi-\Lambda$ bound state. Franklin has pointed out that in this model, one would expect a state of a Λ plus two pions to be bound (relative to the $\pi+\Sigma$ rest mass) provided that there is no strong pion-pion repulsion.[7] The two pions, being bosons, would exist in the same state relative to the Λ. The $I=2$ bound state predicted by Eqs. (2) and (4) can be thought of as such a $\Lambda+2\pi$ state. As discussed above, such a bound state would also be predicted for $I=0$ were it not for the "repulsive" effect of the Λ-particle pole.

We now turn our attention to the $\pi-\Lambda$ resonance, assuming that it exists in the S-state, and that the $\bar{K}\Lambda N$ parity and $\Sigma-\Lambda$ parity are odd. It can be shown by the matrix N/D dispersion relations that the poles in the coupled $\pi-\Lambda$ and $\bar{K}-N$ S-wave channels cannot produce the resonance by themselves.[8] However, if a $\pi-\Sigma$ bound state (which we shall call the Λ') did exist, and the binding energy were sufficiently large, the resonance would be predicted by the static dispersion relations for the coupled S-wave $\pi-\Lambda$, $\pi-\Lambda'$, and $\bar{K}-N$ channels. The resonance would be roughly analogous to a state of a $\Lambda'+\pi$ or a $\Lambda+3\pi$. If the Λ' were of isotopic spin 0, it might have escaped experimental detection. Unfortunately, the static dispersion relations predict that the attractive $\pi-\Sigma$ force is much stronger for $I=2$. In order for the $\pi+\Sigma$ to be bound in the $I=0$ state and unbound in the $I=2$ state, some strong mechanism would be necessary, such as an S-wave $\pi-\pi$ interaction that is very strong and isospin dependent. Hence, this possible mechanism for the $\pi-\Lambda$ resonance appears to be unlikely.

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[6] This may be seen from the results of R. D. Amado, Phys. Rev. 122, 696 (1961). [See particularly Eq. (52).] Amado derives the exact equation for $\theta-V$ scattering in the Lee model, in which the state $2\theta+N$ is the only coupled inelastic channel. In this model an increase in the coupling constant g^2 leads to a $\theta+V$ bound-state (with no $\theta-N$ ghost present) only if the cutoff function is sufficiently rapid.

[7] J. Franklin (private communication to Y. Nambu). We are indebted to Professor Nambu for this information.

[8] K. C. Wali, T. Fulton, and G. Feldman, Phys. Rev. Letters 6, 644 (1961), find that the static dispersion relations for the $I=1$, $\pi-\Lambda$, $\pi-\Sigma$, and $\bar{K}-N$ states are able to predict a $j=\frac{1}{2}$ resonance only if the pseudoscalar interaction constants $g_{\pi\Sigma\Sigma}^2$ and $g_{KN\Lambda}^2$ are nonzero. This implies that the coupling of the $P_{\frac{1}{2}}$, $\pi-\Sigma$ state to the S-wave $\pi-\Lambda$ and $\bar{K}-N$ states is crucial in their model.