range accessible to us. It must be emphasized that the 75° point has not been normalized to the neutral photopion cross section, since the short range of recoil protons from that reaction at this angle prevented them from being observed in our telescope. The observed angular distribution suggests the presence of a $\cos^2\theta$ term, as expected on theoretical grounds.[2]

CONCLUSION

The results presented here are, evidently, to be regarded only as preliminary. The calibration procedure involving the photopion cross section reveals, in particular, that the technique of discriminating between scattering and pion production events by working close to the bremsstrahlung end point is subject to systematic errors whose influence cannot be considered as eliminated until their origin is better understood. At present it appears that a more promising technique would be to discriminate entirely on the basis of the angular correlation peculiar to the scattering; this involves observations in much better geometry, requiring higher beam intensities to maintain an acceptable counting rate.

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Elastic Scattering of Photons by Protons. II*

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The differential cross section for Compton scattering on protons is calculated at various energies in the region of and above the peak corresponding to the 3–3 pion-nucleon resonance. For this, the unitarity of the S matrix and a set of approximate dispersion relations for the scattering amplitudes is used. Small admixtures of electric quadrupole in the radiation that produces the first photopion resonance are shown to affect the angular distribution significantly. In the region of the second photopion peak the resonant behavior is found to be clearly reflected on the proton Compton effect.

I. INTRODUCTION

ONE of the first and most successful applications of dispersion relations and unitarity of the S matrix is on the scattering of photons by protons.[1–5] It has been shown that the basic characteristics of the process in the low-energy region can be explained by assuming that the main contribution to the absorptive part of the process comes from single pion photoproduction in the s state (for π^+) and in the state of the first resonance. Finally, inclusion of the Low amplitude[6] gives very good agreement with the experimental data below 270 Mev.[7,8]

Recently, the experimental group at Cornell[9] has extended the existing data well above the energy of the first peak. The purpose of the present paper is to show that these results can be well accounted for by the basic contributions to single pion photoproduction including that of the second resonance, the contribution of two-pion production estimated with the help of the 3-3 isobar model, and the Low amplitude. Moreover, apart from magnetic dipole, small admixtures of electric quadrupole radiation are considered to be responsible for the photoproduction at the first resonance. Their effect is discussed at the end, where also the results of the calculation are compared with the existing experimental data.

II. APPLICATION OF UNITARITY

Provided that we treat the problem to the lowest order in the fine-structure constant, the amplitude for scattering of photons by particles with spin $\frac{1}{2}$ can be written

$$\begin{aligned}A_{\gamma\to\gamma}=&f_1(\hat{e},\hat{e}')+f_2(\hat{k}\times\hat{e},\,\hat{k}'\times\hat{e}')\\&+if_3(\boldsymbol{\sigma},\,\hat{e}\times\hat{e}')+if_4(\boldsymbol{\sigma},\,[\hat{k}'\times\hat{e}']\times[\hat{k}\times\hat{e}])\\&+if_5\{(\boldsymbol{\sigma},\hat{k})(\hat{k}',\,\hat{e}'\times\hat{e})-(\boldsymbol{\sigma},\hat{k}')(\hat{k},\,\hat{e}\times\hat{e}')\}\\&\quad+if_6\{(\boldsymbol{\sigma},\hat{k}')(\hat{k}',\,\hat{e}'\times\hat{e})-(\boldsymbol{\sigma}\hat{k})(\hat{k},\,\hat{e}\times\hat{e}')\},\end{aligned}\tag{1}$$

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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Here $\hat{e}$, $\hat{e}'$ are the initial and final polarization vectors and $\hat{k}$, $\hat{k}'$ are unit vectors in the direction of the incoming and outgoing photon. The scalar amplitudes $f_i(i=1, 2, \cdots, 6)$ are, in general, functions of two variables, say

$$\nu=(p+p')(k+k')/4M, \quad \Delta^2=(k-k')^2/4, \qquad (2)$$

where p, p' are the 4-vectors of the incoming and outgoing proton momenta. For forward scattering, ν is the energy of the photon in the laboratory system.

The contribution of single pion photoproduction to the absorptive part of the amplitudes $f_i(\nu,\Delta^2)$ is easily calculated through unitarity. In the notation of Chew *et al.*[10] we shall consider the following contributions: (a) E_{0+} for the production of charged s-wave pions (due to electric dipole radiation), (b) M_{1+} and E_{1+} for pions produced in the $P_{\frac{3}{2}}{}^+$, $T=\frac{3}{2}$ state (3-3 resonance) by magnetic dipole and electric quadrupole radiation, respectively, and (c) E_{2-} for pions produced in the $D_{\frac{3}{2}}{}^-$, $T=\frac{1}{2}$ state (second resonance) by electric dipole; we neglect any magnetic quadrupole contribution. The amplitude for production of a pion in the direction $\hat{q}$ by a photon $(\hat{k},\hat{e})$ is then

$$\begin{aligned}A_{\gamma\to\pi}=iE_{0+}(\boldsymbol{\sigma},\hat{e})+M_{1+}\{2(\hat{q},\,\hat{k}\times\hat{e})-i(\boldsymbol{\sigma},\,\hat{q}\times(\hat{k}\times\hat{e}))\}\\ +(i/2)E_{1+}\{(\boldsymbol{\sigma},\hat{k})(\hat{e},\hat{q})+(\boldsymbol{\sigma},\hat{e})(\hat{k},\hat{q})\}\\ +iE_{2-}[(\boldsymbol{\sigma},\hat{e})-3(\boldsymbol{\sigma},\hat{q})(\hat{q},\hat{e})]. \qquad (3)\end{aligned}$$

Direct application of the unitarity of the S matrix in the center-of-momentum system gives

$$\begin{aligned}&\mathrm{Im}f_{c1}=k_c(|E_{0+}|^2+2|E_{2-}|^2+\tfrac{1}{3}|E_{1+}|^2\cos\theta_c),\\ &\mathrm{Im}f_{c2}=k_c(2|M_{1+}|^2-\tfrac{1}{6}|E_{1+}|^2),\\ &\mathrm{Im}f_{c3}=k_c(|E_{0+}|^2-|E_{2-}|^2+\tfrac{1}{3}|E_{1+}|^2\cos\theta_c),\\ &\mathrm{Im}f_{c4}=k_c(-|M_{1+}|^2+\tfrac{1}{12}|E_{1+}|^2\\ &\qquad\qquad+\tfrac{1}{2}(E_{1+}{}^*M_{1+}+E_{1+}M_{1+}{}^*)), \qquad (4)\\ &\mathrm{Im}f_{c5}=k_c(-\tfrac{1}{6}|E_{1+}|^2-\tfrac{1}{2}(E_{1+}{}^*M_{1+}+E_{1+}M_{1+}{}^*)),\\ &\mathrm{Im}f_{c6}=0.\end{aligned}$$

Here θ_c is the scattering angle in the c.m. system (denoted by the index c). The last expressions are in agreement with the results of reference 5 as far as common terms are considered.

The photoproduction amplitudes may be determined from the experimental values of the total cross sections for neutral and charged pion production as well as certain simple assumptions. From Eq. (3) and taking into account the corresponding isospin states,

$$\sigma_{\gamma\to\pi^0}=(8\pi/3)(2|M_{1+}|^2+\tfrac{1}{6}|E_{1+}|^2+|E_{2-}|^2), \qquad (5a)$$

and

$$\begin{aligned}\sigma_{\gamma\to\pi^+}=(4\pi/3)\\ \times(3|E_{0+}|^2+2|M_{1+}|^2+\tfrac{1}{6}|E_{1+}|^2+4|E_{2-}|^2). \qquad (5b)\end{aligned}$$

[10] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1345 (1957).

We adopt a model similar to that of Peierls and Sakurai[11] which has been very successful in explaining the main features of single pion photoproduction up to energies of the third resonance. In the present calculation the ratio

$$\rho=|E_{1+}|/|M_{1+}| \qquad (6)$$

will be used as parameter.

III. DISPERSION RELATIONS AND LOW AMPLITUDE

The behavior of the scalar amplitudes f_i as $k\to 0$ is determined from the low energy theorem.[12] To the first order in the photon energy we have in the lab system

$$\begin{aligned}&f_1\to -e^2/M, && f_2\to 0,\\ &f_3\to -(e^2/2M^2)k, && f_4\to -(e^2/2M^2)(1+g)^2k, \qquad (7)\\ &f_5\to 0, && f_6\to (e^2/2M^2)(1+g)k,\end{aligned}$$

where g is the anomalous magnetic moment of the spin-$\frac{1}{2}$ particle.[13]

It has been proved[5] that for forward scattering the amplitudes $f_1(\nu)+f_2(\nu)$, $f_3(\nu)$, $f_4(\nu)$, and $f_5(\nu)+f_6(\nu)$ satisfy simple symmetry properties (f_1+f_2 symmetric, the rest antisymmetric) as well as dispersion relations. Now, in the c.m. system the scalars f_2 can be written as series of powers of $\cos\theta_c$. The coefficients of the expansion are linear combinations of the partial wave amplitudes, which correspond to states with definite angular momentum and parity and are functions of the c.m. energy only. However, it is easy to see that $\cos\theta_c$ dependence appears in f_i only if quadrupole or higher multipole terms are included in the expansion. Consequently, if we assume that the state corresponding to the 3-3 pion-nucleon resonance is essentially due to magnetic dipole radiation, then the amplitudes f_i are, in a good approximation, functions of the photon energy only.

In the approximation of keeping states with $J\leqq\frac{3}{2}$ it is possible to derive dispersion relations for certain combinations of the partial wave amplitudes.[5] From these it can be seen that approximate dispersion relations hold for f_2 and f_6 separately. Since magnetic quadrupole does not contribute to our model, the partial wave analysis to the order $J\leqq\frac{3}{2}$ shows that f_2 and f_6 are functions of the photon energy only, even if electric quadrupole in significant amount is present. Equations (1) and (7) suggest then the following form

[11] R. F. Peierls, Phys. Rev. Letters **1**, 174 (1958); J. J. Sakurai, Phys. Rev. Letters **1**, 258 (1958); P. C. Stein, *ibid.* **2**, 473 (1959).

[12] F. E. Low, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, *ibid.* **96**, 1433 (1954).

[13] J. D. Walecka (unpublished) points out that in the c.m. system perturbation theory gives an additional term:

$$(e^2/M)(k/M)(\hat{k},\hat{e}')(\hat{k}',\hat{e})$$

of dispersion relations in the forward direction[14]:

$$\mathrm{Re}f_2(\nu)-\mathrm{Re}f_2(0)=\frac{2\nu^2}{\pi}\,\mathrm{P}\int_{\nu_0}^{\infty}\frac{\mathrm{Im}f_2(\nu')d\nu'}{\nu'(\nu'^2-\nu^2)},\qquad(8)$$

$$\mathrm{Re}f_6(\nu)-\nu\mathrm{Re}f_6'(0)=\frac{2\nu^3}{\pi}\,\mathrm{P}\int_{\nu_0}^{\infty}\frac{\mathrm{Im}f_6(\nu')d\nu'}{\nu'^2(\nu'^2-\nu^2)}.\qquad(9)$$

Taking into account Eqs. (4), (5), and (8) we finally obtain the following set:

$$\mathrm{Re}\{f_1(\nu)+f_2(\nu)\}=-\frac{e^2}{M}+\frac{\nu^2}{2\pi^2}P\int_{\nu_0}^{\infty}\frac{\sigma_{\mathrm{total}}(\nu')}{\nu'^2-\nu^2}d\nu',$$

$$\mathrm{Re}f_2(\nu)=\frac{2\nu^2}{\pi}P\int_{\nu_0}^{\infty}\frac{\mathrm{Im}f_2(\nu')}{\nu'(\nu'^2-\nu^2)}d\nu',\qquad(10)$$

and

$$\mathrm{Re}f_j(\nu)=C_j\nu+\frac{2\nu^3}{\pi}P\int_{\nu_0}^{\infty}\frac{\mathrm{Im}f_j(\nu')}{\nu'^2(\nu'^2-\nu^2)}d\nu',$$

for $j=3, 4, 5, 6$; σ_{total} is the total cross section for pion photoproduction. In view of the limiting values of (7), the constants C_j are given by:

$$\begin{array}{ll} C_3=-(e^2/2M^2) & C_4=-(e^2/2M^2)(1+g)^2 \\ C_5=0 & C_6=(e^2/2M^2)(1+g), \end{array}\qquad(10')$$

and ν_0 is the threshold for meson production in the lab system given by

$$\nu_0=\mu(1+\mu/2M);\qquad(11)$$

μ is the pion mass. For small ρ, terms with $|E_{1+}|^2$ may be neglected in the $\mathrm{Im}f_i$ of (10); in the subsequent calculation E_{1+} has been assumed in phase with M_{1+}. For strong $\cos\theta_c$ dependence of the amplitudes f_i, the partial wave dispersion relations and the multipole analysis of reference 5 show how to extend Eqs. (10).

It has been shown[7,8] that above 100-Mev photon lab energy a very significant contribution to Compton scattering may come from the Low amplitude, which corresponds to γ-p scattering through exchange of a virtual neutral pion. Provided that the variation of form factors with momentum transfer is neglected, the corresponding amplitude is[7,15]

$$A_{\mathrm{Low}}=\frac{e^2}{M}\frac{g}{\alpha}\frac{M}{\mu}\frac{1}{(\mu\tau)^{\frac{1}{2}}}\frac{2k_c^3}{W}[\mu^2-(k-k')^2]^{-1}\times i\boldsymbol{\sigma}\cdot(\hat{k}_c-\hat{k}_c')(\hat{k}_c-\hat{k}_c')\cdot(\hat{e}\times\hat{e}'),\qquad(12)$$

where g is the pion-nucleon coupling constant, α the fine-structure constant, W the total c.m. energy, and τ the lifetime of π^0 in the virtual process; in the present calculation $\tau=10^{-16}$ sec has been used, which is in agreement with both the limits set in reference 7 and the existing experimental data on neutral pion decay.[16] From the amplitude (12) it is easily seen that the Low process contributes to f_{c5} and f_{c6} only. As regards to the sign of this contribution, we have, in general, chosen that of reference 7; since some objections have been recently raised,[17] we indicate in Fig. 1 (differential cross section at $\theta_c=90°$) the effect of making the opposite choice.

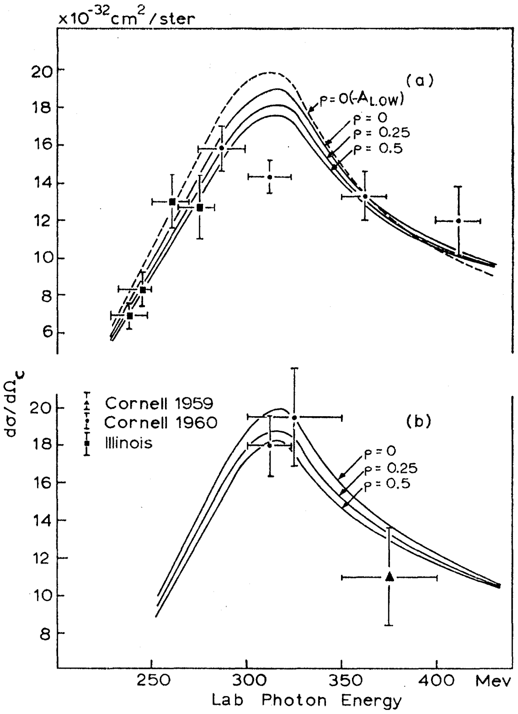

FIG. 1. Differential cross sections for proton Compton scattering at (a) $\theta_c=90°$, (b) $\theta_c=75°$.

IV. CALCULATION AND DISCUSSION

To include the effect of two-pion photoproduction in the present calculation we have made use of the 3-3 isobar model. According to this, one of the final pions and the final nucleon emerge in such a way that they are always in a resonant 3-3 state, while the remaining meson is produced in an s state with respect to the "isobar." Since the 3-3 state has even parity and the pion is pseudoscalar, the three-body system under consideration will have odd parity (and total angular

[14] One might object to this form of dispersion relation for $f_2(\nu)$. However, in the approximation of keeping states with $J\leq\frac{3}{2}$ and in the notation of reference 5, $\mathcal{E}_2$ and $\mathfrak{M}_2$ are not uniquely determined in terms of R_i. In this approximation, a dispersion relation of the form of Eq. (8) may be derived for $\mathcal{E}_2+\mathfrak{M}_2$; this gives additional support to Eq. (8).

[15] G. Bernardini *et al.*, Nuovo cimento **18**, 1203 (1960).

[16] G. Harris, J. Orear, and S. Taylor, Phys. Rev. **106**, 327 (1957); R. Blackie, A. Engler, and T. Mulvey, Phys. Rev. Letters **5**, 384 (1960); R. Glasser, N. Seeman, and B. Stiller, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 30; A. Tollestrup, S. Berman, R. Gomez, H. Ruderman, *ibid.*, p. 27.

[17] L. I. Lapidus and Chou Kuang-chao (to be published).

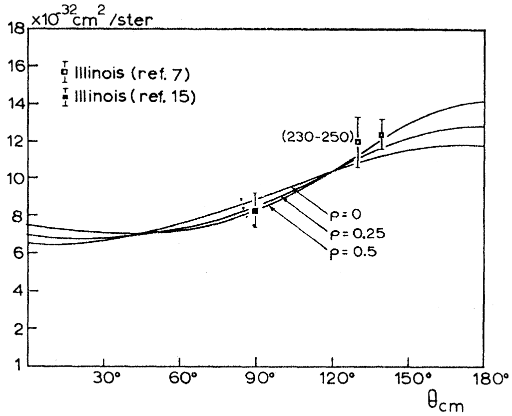

FIG. 2. Angular distributions at 245 Mev.

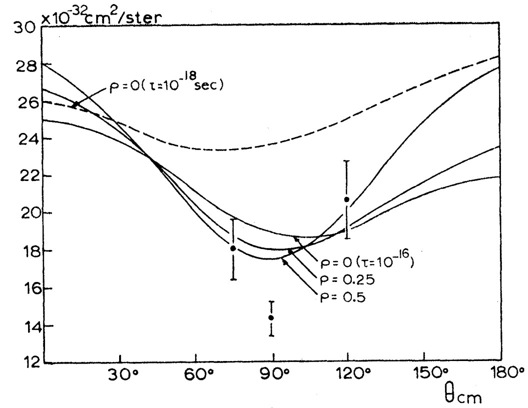

FIG. 3. Angular distributions at 310 Mev.

momentum $J=\frac{3}{2}$). Hence, it is most likely produced by electric dipole radiation, contributing thus to the absorptive parts of f_{c1} and f_{c3}. In the present work it is assumed that

$$\sigma_{\gamma\to\pi\pi}=1.5\sigma_{\gamma\to\pi^+\pi^-};\tag{13}$$

for $\sigma_{\gamma\to\pi^+\pi^-}$ the experimental results of Chasan *et al.*[18] have been used.

The differential cross section for Compton scattering on particles with spin $\frac{1}{2}$ has in the c.m. system the form[7,19]

$$\begin{aligned}
d\sigma/d\Omega_c &\\
=\tfrac{1}{2}[&|f_1|^2+|f_2|^2+3|f_3|^2+3|f_4|^2\\
&+2|f_5|^2+2|f_6|^2+4\,\mathrm{Re}(f_3^*f_6+f_4^*f_5)\\
&+2\cos\theta_c\,\mathrm{Re}(f_1^*f_2+f_3^*f_4+2f_3^*f_5+2f_4^*f_6+3f_5^*f_6)\\
&+\cos^2\theta_c(\tfrac{1}{2}|f_1|^2+\tfrac{1}{2}|f_2|^2-\tfrac{1}{2}|f_3|^2-\tfrac{1}{2}|f_4|^2\\
&+3|f_5|^2+3|f_6|^2+2\,\mathrm{Re}(f_3^*f_6+f_4^*f_5)]\\
&+2\cos^3\theta_c\,\mathrm{Re}(f_5^*f_6),
\end{aligned}\tag{14}$$

provided that f_i have been reduced to c.m.[19a] The results of our calculation for $\rho=0$, 0.25, and 0.50 [20] are presented in Figs. 1–4. Apart from the experimental point at 3.12 Mev and $\theta_c=90°$, which seems to be too low, there is fairly good agreement with the existing data as well as with certain previous calculations.[4] At low energies our predictions at $\theta_c=90°$ are slightly above those of reference 7. Figures 2–4 demonstrate a clear forward-backward asymmetry at energies before and after the peak which is very characteristic of resonant behavior and is essentially due to electric dipole-magnetic dipole interference.

The effect of electric quadrupole on the calculated cross sections is much more prominent in the region of the resonance, as one should expect from the model used. The angular distribution in that region appears to be very sensitive to the quadrupole percentage (more sensitive than for photoproduction). Note that the $\cos^2\theta_c$ dependence is not affected significantly from changes in the π^0 lifetime (Fig. 3). Hence it seems that the region of the first resonance in the proton Compton effect is very appropriate to determine the exact amount of electric quadrupole present in single-pion photoproduction. The existing data favor a ratio $\rho=0.3$–0.4, although not in a conclusive way. From Eqs. (5), this ratio corresponds to about 1% quadrupole contribution to the total photoproduction cross sections; it is very probable, however, that ρ increases with energy. Additional information may in principle be obtained from the polarization P of the recoil proton, along the direction $\hat{k}\times\hat{k}'$. At 310 Mev, $\theta_c=90°$, we find $P\sim35\%$ for $\rho=0$, decreasing by only 8% for $\rho=0.5$; however, at $\theta_c=135°$, $P\sim20\%$ for $\rho=0$, decreasing by 50% for

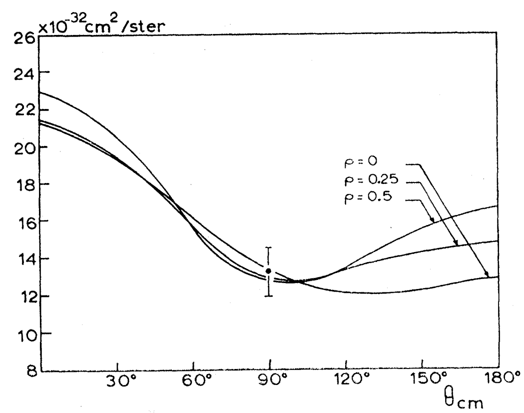

FIG. 4. Angular distributions at 362 Mev.

[18] B. M. Chasan, G. Cocconi, V. T. Cocconi, R. M. Schectman, and D. H. White, Phys. Rev. **119**, 811 (1960).

[19] L. I. Lapidus and Chou Kuang-chao, Zhur. Eksptl' i Teoret. Fiz. **38**, 201 (1960) (translation: Soviet Phys. JETP **11**, 147 (1960).

[19a] The dispersion relations in the form of Eqs. (10) are expected to be valid in a rather wide range of energies, provided that f_i are the Breit system amplitudes. However, within error of order $(k_c/M)^2$, a similar form holds for f_{ci} as well[2–4]; this has been utilized in the calculations presented in Figs. 1–4.

[20] Large electric quadrupole contribution is excluded from the angular distribution of π^0 photoproduction. See D. R. Corson, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1956).

$\rho=0.5$. If it is confirmed by further experimental work, the presence of electric quadrupole may affect the analysis of the data at lower energies and the limits of virtual π^0 lifetime as determined in reference 7. Moreover, it will help to understand the asymmetry in the angular distribution of π^+ photoproduction at high energy.

In a recent paper Minami,[21] by describing the proton Compton effect in terms of shadow scattering due to photoproduction of pions, predicts a strong and broad peak in the cross section corresponding to the second photopion resonance. Although the dispersion relations of (10) contain many theoretical uncertainties at these energies, we have applied them at 760 Mev. For $\rho=0$ and $\rho=0.25$ the differential cross-section in units of 10^{-32} cm²/sterad is

$$\frac{d\sigma}{d\Omega_c}(90°)=12.75,\ 14.95,$$

correspondingly. This estimation agrees qualitatively with the predictions of reference 21 as well as with the value $(d\sigma/d\Omega_c)(90°)=(13.0\pm6.0)\times10^{-32}$ cm²/sr reported from Frascati.[22] Since the resonant behavior seems to be clearly reflected on the proton Compton effect, further experimental work in that region will add very useful information regarding the character and the details of the second resonance.

In the present calculation the effect of the third photopion resonance has been entirely neglected; it is not difficult, however, to include it in the calculation of Sec. 2. This effect, as well as that of the $T=1$, $J=1$ pion-pion resonance (which enters through double pion photoproduction) is certainly negligible in the region of the first resonance, but could be very significant in that of the second. Even more significant might be the two-pion exchange contribution in a Low-type process. A practical form of this contribution is now under investigation.

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[21] Shigeo Minami (to be published).

[22] G. Cortellessa, A. Reale, and P. Salvadori, Rend. ist. super. sanità (to be published).

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Neutron Gas*

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We assume that the neutron-neutron potential is well-behaved and velocity-dependent. We can then apply perturbation theory to find the energy per particle of a neutron gas, in the range of Fermi wave numbers $0.5<k_f<2$ f^{-1}. The energy through first order is found in closed form, or by a single numerical integration. We use two different velocity-dependent potentials adjusted to fit observed nucleon-nucleon ^1S and ^1D phase shifts. In the range of densities $0.5<k_f<1$ f^{-1}, our two potentials give nearly the same energy/particle (within 0.5 Mev); our values tend to run an Mev below values found by Brueckner *et al.*, for the Gammel-Thaler potential. Wider divergences appear at higher densities. Our values, and Brueckner's are higher than those found by Salpeter by a semiempirical approach. A crude estimate of the second-order energy for our potentials indicates that perturbation theory converges rapidly in the density range considered. Our results suggest that at moderately low densities the energy/particle in a many-body system is insensitive to the shape or nonlocal character of the assumed two-body potential.

INTRODUCTION

THE neutron gas is a good proving ground for many-body calculations for two reasons. First, the neutron-neutron potentials are rather well known, since they must fit the accurately determined proton-proton phase shifts.[1] (We assume charge independence, and have not corrected for Coulomb effects.) Second, there is no experimental data on the neutron gas, to prejudice us for or against any special calculation.

We are not comparing with experiment, but we have two interesting comparisons to make. First, certain terms are neglected in any calculational method for a many-body problem. We need estimates to show that the neglected terms are small in comparison with the terms considered. Second, Bég[2] has discussed whether two potentials (one static and the other velocity-dependent) which give the same phase shifts in the

* Supported by the National Science Foundation.

† Now at the Department of Physics, Cornell University.

[1] G. Breit, M. H. Hull, K. Lassila, and K. D. Pyatt, Phys. Rev. **120**, 2227 (1960).

[2] M. A. B. Bég, Ann. Phys. **13**, 110 (1961).