may not correspond to fully saturated conditions, since it is based[5] on light nuclei. Another apparent example of density dependence has long been recognized in the spin-orbit potential, which seems to increase in strength as a shell fills[13]: For example, the splitting at $A=17$ is only about half as strong as the p-hole splitting at $A=15$.

Such strong density dependences suggest that shell model calculations cannot hope to find general agreement with experiment by using a fixed set of strictly local two-body potentials.

[13] R. W. King, Phys. Rev. 100, 1240 (1955).

PHYSICAL REVIEW VOLUME 124, NUMBER 3 NOVEMBER 1, 1961

Beta-Gamma Directional Correlation in the Decay of Eu^{154}†

L. D. WYLY, E. T. PATRONIS, HARRY DULANEY, AND C. H. BRADEN
School of Physics, Georgia Institute of Technology, Atlanta, Georgia
(Received June 23, 1961)

The directional correlation for the (1855 kev-beta) (123 kev-gamma) cascade in the decay of Eu^{154} has been measured. The directional correlation coefficient A_2 varies from -0.15 at a beta energy of 1100 kev to -0.18 at a beta energy of 1600 kev. In terms of a single nuclear matrix element parameter, ζ_1, the directional correlation suggests $\zeta_1=2.6$ or -0.2 while the beta spectral shape correction suggests $\zeta_1=1.3$. The results suggest that the attenuation of ordinary first-forbidden matrix elements relative to B_{ij} is less marked in Eu^{154} than in the comparable transition in Eu^{152}. Interpretation of the data in terms of a less restrictive formulation of the theory is investigated. Relations between matrix element ratios may be found but no unique set of matrix element ratios is demanded by the experimental data.

INTRODUCTION

THE principal transitions in the decay of Eu^{154} are shown in Fig. 1.[1] The 1855-kev beta transition is believed to be first forbidden with spin change 1. The logft product for the 1855-kev beta group is about 12.4 which is unusually high for a first forbidden transition. A negative anisotropy for the (1855-kev beta) (123-kev gamma) directional correlation has been found[2] which is somewhat lower in magnitude than the directional correlation for the comparable cascade in Eu^{152}.[3] Additional measurements of the directional correlation as a function of beta energy in Eu^{154} have been reported.[4,5] The shape of the spectrum for the 1855-kev beta group has been measured and found to be intermediate between an allowed and a unique shape,[6] but somewhat closer to the allowed shape than the comparable transition in Eu^{152}.[6]

In the present report we present our final data for the directional correlation in Eu^{154}. Comparison is made with the approximate formulation of beta-decay theory in terms of a single nuclear matrix element parameter. Interpretation of the data in terms of a less restrictive formulation of the theory is also discussed.

EXPERIMENTAL PROCEDURE AND RESULTS

The Eu^{154} was produced at the Oak Ridge National Laboratory by irradiation of isotopically enriched Eu^{153} (about 0.6% Eu^{151}). Two different irradiations were

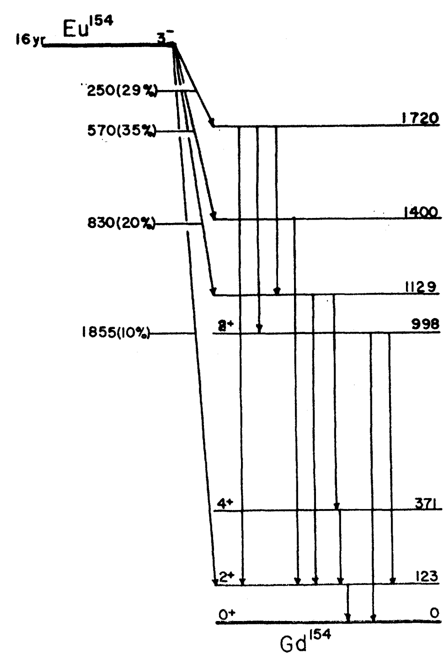

FIG. 1. Principal transitions in the decay of 16-year Eu^{154}.[1]

† Supported in part by a grant from the National Science Foundation.

[1] *Nuclear Data Sheets*, National Academy of Sciences—National Research Council, NRC 59-3-63 (National Research Council, Washington, D. C.).

[2] H. Dulaney, C. H. Braden, and L. D. Wyly, Bull. Am. Phys. Soc. 5, 450 (1960).

[3] H. Dulaney, C. H. Braden, and L. D. Wyly, Phys. Rev. 117, 1092 (1960).

[4] R. G. Wilkinson, K. S. R. Sastry, and R. F. Petry, Bull. Am. Phys. Soc. 6, 72 (1961).

[5] H. Dulaney, L. D. Wyly, and C. H. Braden, Bull. Am. Phys. Soc. 6 (to be published, 1961).

[6] L. M. Langer and D. R. Smith, Phys. Rev. 119, 1308 (1960).

used. The results reported here are from the last irradiation which produced sources of high specific activity. The results are in essential agreement with those obtained over a two-year period from the thicker sources of lower specific activity. We observed an effect on the beta counting rate shortly after receipt of the last irradiation which suggests the presence of a short-lived activity, perhaps Eu^{156}. After disappearance of this effect, no further significant change in intensity of the 123-kev gamma relative to the beta spectrum above 1100-kev energy was noted over a period of two months. Measurements made in the presence of the short-lived activity are not reported although there was no effect noted on the directional correlation. Sources were made by evaporation of a nitric acid solution on 0.2-mil aluminum foil. Two sources of different strength were used.

The coincidence circuit was of the fast-slow type[7] and operated at a resolving time of about 10^{-7} sec. The gamma detector used a $1\frac{1}{2}$-in. diam by 1-in. high NaI(Tl) crystal. The beta detector used a $1\frac{1}{2}$-in. diam by 1.1-cm high anthracene crystal which, along with the source, was mounted inside a thin-wall aluminum vacuum chamber. The fast pulses to the coincidence circuit were taken from the amplifier integral discriminator outputs. Slow coincidences were between the output of the fast coincidence circuit and a pulse-height analyzer on the gamma counting channel. The slow coincidence output gated a twenty-channel pulse-height analyzer which was attached to the beta counting channel. Fast coincidences and the gamma counting rate were monitored. The beta counting rate was checked at hourly intervals. Accidental coincidences were determined periodically by insertion of a 0.5-μsec delay in one of the counting channels. An estimate of the gamma-gamma coincidences was obtained by insertion of a 0.95-cm thick Lucite absorber between the source and the beta counter. This method cannot determine the gamma-gamma coincidence rate with precision because of electrons knocked out of the absorber by gamma rays. It is reasonable to assume that the correct gamma-gamma coincidence rate is no higher than the rate determined in this manner.

The directional correlation experiments were performed with the source 7.5 cm from each detector. The gamma spectrum exhibited a strong peak at about 123 kev and the gamma pulse-height analyzer was set on this peak with a window width of approximately 12 kev. The beta 20-channel analyzer was calibrated with the 976-kev conversion electron of Bi^{207} and the 633-kev conversion electron of Cs^{137}. Additional calibration work was done to ensure the validity of the extrapolation of the energy calibration above 976 kev. The reported mean beta energies are believed accurate to ±30 kev. Measurements were made at angles of 90°, 180°, and 270° between the beta and gamma counters. Beta-gamma coincidence data were normalized to the product of the beta and gamma counting rates.

[7] F. K. McGowan, Phys. Rev. **79**, 404 (1950).

TABLE I. Experimental results. A_2 is the coefficient of $P_2(\cos\theta)$ in the expansion of the directional correlation in terms of even-order Legendre polynomials where we assume $A_4=0$. R is the number of real beta-gamma coincidences at 180°. R/C is the ratio of real coincidences to accidentals. $R/\gamma\gamma$ is the ratio of real beta-gamma coincidences to the estimated gamma-gamma coincidences.

Mean beta energy (kev)	A_2	R	R/C	$R/\gamma\gamma$
1090	-0.151 ± 0.015	11 700	2.1	2.5
1150	-0.147 ± 0.012	15 100	2.2	4.4
1230	-0.150 ± 0.012	12 100	2.1	8.2
1300	-0.158 ± 0.012	17 100	2.5	18
1380	-0.165 ± 0.012	13 000	2.3	19
1450	-0.165 ± 0.012	11 200	2.4	41
1530	-0.185 ± 0.012	9000	2.6	43
1610	-0.187 ± 0.025	2500	2.4	44
1650	-0.176 ± 0.014	3100	···	45

The differential directional correlation was examined in approximately 80-kev increments of beta energy over the range 1090 to 1610 kev. An additional measurement with a different source and using an automated counting system with fast coincidence resolving time was made at a beta energy of about 1650 kev. The experimental results are listed in Table I. The indicated errors are approximately probable errors due to statistical effects. The data reported at the two lowest energies should not be given as much weight as data at higher energies because of the relatively large correction for gamma-gamma coincidences. A_2 is the usual coefficient of $P_2(\cos\theta)$ in the expansion of the directional correlation in terms of even-order Legendre polynomials where we assume $A_4=0$. Some measurements were made at angles of 135° and 225° in order to substantiate the neglect of the A_4 coefficient. Correction for the finite angular resolution of the detectors was made according to the method of Rose[8] with the aid of the calculations of Stanford and Rivers.[9] No correction for the beta energy spread was made in view of the slow and relatively uniform variation of A_2 with beta energy.

ONE-PARAMETER INTERPRETATION

The theory of first-forbidden beta decay has been given explicit formulation by Morita and Morita[10] and by Kotani.[11] The formulations are similar except that Kotani includes Coulomb corrections. Interpretation of the beta-gamma directional correlation and the corresponding beta spectral shape can, under certain approximations, be made in terms of a single nuclear

[8] M. E. Rose, Phys. Rev. **91**, 610 (1953).

[9] A. L. Stanford and W. K. Rivers, Rev. Sci. Instr. **30**, 719 (1959).

[10] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).

[11] T. Kotani, Phys. Rev. **114**, 795 (1959). We redefine ζ_1 of T. Kotani to be $\zeta_1=Y+(u-x)(W_0/3)$ which brings the results into agreement with Morita and Morita.[10]

matrix element parameter, denoted as ζ_1. The single-parameter analysis has been found useful in interpretation of the decay of Eu^{152}.[3] Inclusion of the Coulomb corrections in the theoretical formulas, which amount to about 20% in the case of Eu, leads to very satisfactory agreement between the measurements and the one-parameter interpretation.[12] A value for ζ_1 of about 0.8 is obtained. Similar analysis can be applied to the decay of Sb^{124}.[10] Using recent data for the directional correlation measured by Steffen[13] and the most recent spectral shape measured by Langer and Smith,[6] a consistent value of ζ_1 approximately equal to 0.8 results. In this instance it happens that the directional correlation is rather insensitive to variation in ζ_1 for values of ζ_1 near 0.8.

It is reasonable then to attempt a one-parameter interpretation of the decay of Eu^{154}. Here we employ the results given by Kotani[11] and generally follow his notation. For a spin change of 1 in the beta decay the formulas for the spectral shape correction, $C(W)$, and the modified directional correlation coefficient, $A_2^* = (W/\lambda_2 p^2)A_2$, are, in terms of the single nuclear matrix element parameter approximation,

$$C(W)=\zeta_1^2+(1/12)(q^2+\lambda_1 p^2), \quad (1)$$

$$\begin{aligned} A_2^* &\equiv WA_2/(\lambda_2 p^2) \\ &= [1/C(W)](2J_1+1)^{\frac{1}{2}}F[W(J_1J_112;2J_0)\zeta_1 \\ &\quad +(1/12)(7/2)^{\frac{1}{2}}W(J_1J_122;2J_0)(\lambda_1/\lambda_2)W]. \quad (2) \end{aligned}$$

The symbols have the following meanings: q is the neutrino momentum in mc units, p is the electron momentum in mc units, W is the total electron energy in mc^2 units, the beta transition is from a state of spin J_0 to a state of spin J_1, $W(J_1J_1\lambda\lambda';nJ_0)$ is a Racah coefficient which is readily calculable from formulas given by G. Racah,[14] and λ_1 and λ_2 are Coulomb factors tabulated

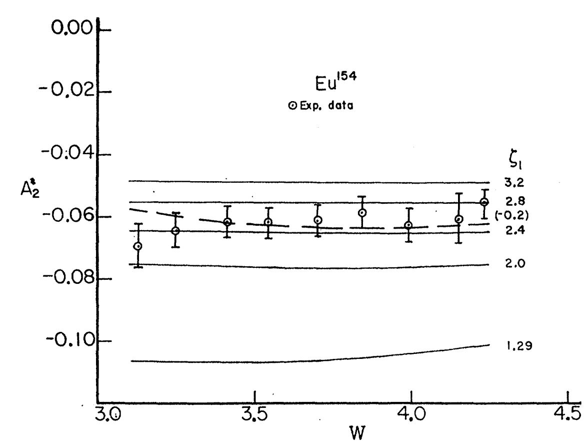

FIG. 2. Plot of modified directional correlation coefficient, $A_2^*=(W/\lambda_2 p^2)A_2$, versus beta energy for the (1855-kev beta) (123-kev gamma) cascade in Eu^{154}.

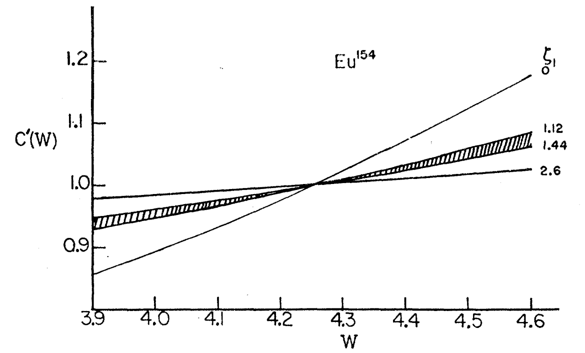

FIG. 3. Plot of normalized spectral shape correction factor, $C'(W)$, versus beta energy for the 1855-kev beta group in Eu^{154}. The shaded area denotes the experimental limits as determined by Langer and Smith.[6] The curves are normalized so $C'(W)=1$ for $W=4.25$.

by Kotani and Ross.[15] The function F characterizes the gamma transition. For a pure multipole transition of order L from a state of spin J_1 to a state of spin J_2 we have $F=F_2(LJ_2J_1)$ where the latter coefficients are tabulated by Biedenharn and Rose.[16]

The beta-decay matrix element parameter ζ_1 is defined as[17]

$$\zeta_1=-(\xi-W_0/3)u+\xi'y-(\xi+W_0/3)x, \quad (3)$$

where

$$u=i\int \boldsymbol{\sigma}\times\mathbf{r} \Big/ \int \mathbf{B}_{ij},$$

$$\xi'y=-iC_v\int \boldsymbol{\alpha} \Big/ C_A\int \mathbf{B}_{ij}, \quad (4)$$

$$x=-C_v\int \mathbf{r} \Big/ C_A\int \mathbf{B}_{ij}.$$

The parameter ξ is the expansion parameter used by Morita and Morita[10] (denoted V there) and by Kotani.[11] For Eu, ξ is about 13.

This single-parameter approximation is valid[10] if the matrix element ratios u and x are very small compared to unity. We also note that a one-parameter approximation obviously results if one retains, in addition to B_{ij}, a single ordinary matrix element. If the relativistic matrix element, α, is retained formulas (1) and (2) are obtained with $\zeta_1=\xi'y$. For retention of one of the non-relativistic matrix elements, slightly different formulas are found[18] but they lead, in most instances, to results not very different from the results given by formulas (1) and (2).

[12] H. J. Fischbeck and R. G. Wilkinson, Phys. Rev. **120**, 1762 (1960).
[13] R. M. Steffen, Phys. Rev. Letters **4**, 290 (1960).
[14] G. Racah, Phys. Rev. **62**, 438 (1942).
[15] T. Kotani and M. Ross, Phys. Rev. **113**, 622 (1959).
[16] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).
[17] For conditions under which the one-parameter formulation is expected to be valid the parameter ζ_1 may be identified with $Y=-\xi u+\xi'y-\xi x$.
[18] Harry Dulaney, thesis, Georgia Institute of Technology, 1961 (unpublished).

In Fig. 2 we plot the directional correlation data given in Table I versus beta energy. We also plot theoretical curves based on the one-parameter formulas (1) and (2) for various values of the matrix element parameter ζ_1. For a spin sequence $3(\beta)2(\gamma)0$ formula (2) reduces to

$$A_2^* = [-1/C(W)][(\zeta_1/7)+(\lambda_1/\lambda_2)(W/42)]. \quad (5)$$

Reasonable agreement with the experimental data is obtained for values of $\zeta_1=2.6(\pm0.2)$ and $\zeta_1=-0.2(\pm0.1)$.

In Fig. 3 we plot the range of the experimentally determined spectral shape correction as given by Langer and Smith,[6] $C(W)=(1/12)(q^2+\lambda_1 p^2)+1.67(\pm0.42)$, over the beta energy range $W=3.9$ to $W=4.6$. This shape correction corresponds to $\zeta_1=1.29(\pm0.16)$ in terms of formula (1). We also plot the theoretical curves, based on formula (1), for $\zeta_1=2.6$ and for $\zeta_1=0$. The curves are normalized so $C'(W)=1$ for $W=4.25$. The curve for $\zeta_1=-0.2$ is indistinguishable from the unique shape on this plot and is in disagreement with the results of Langer and Smith. The curve for $\zeta_1=2.6$ is in better agreement with the measured shape but is closer to the allowed shape.

The extent of the disagreement between the one-parameter formulas and the data can best be judged by examination of the various curves plotted on Figs. 2 and 3. We believe that it is perhaps wise not to overstress the disagreement. Experimental uncertainties in the directional correlation data make slight trends in the behavior of A_2^* with beta energy of uncertain validity. The energy dependence of the shape correction is also a relatively small effect and the measurements of the directional correlation and the spectral shape correction[6] encompass somewhat different energy ranges. Furthermore, the theoretical formulas contain many approximations other than those leading to a single matrix element parameter formulation.[10] For example, the formulas are the leading terms only in an expansion in the parameter ξ, and the energy-dependent Coulomb corrections, λ_1 and λ_2,[15] are not exact.

The qualitative conclusion that emerges from the one-parameter interpretation of the data is that the parameter ζ_1 which roughly characterizes the 1855-kev beta transition in Eu^{154} is probably significantly larger than in the case of the comparable 1480-kev beta transition in Eu^{152} or the 2320-kev beta transition in Sb^{124}. The definition of ζ_1 indicates the physical interpretation of this parameter: It measures the relative importance of the ordinary and the B_{ij} beta-decay matrix elements. The decays of Sb^{124} and Eu^{152} have been discussed in terms of attenuation of the ordinary matrix elements due to selection rules characteristic of the collective model of the nucleus, K forbiddenness, and the shell model, j forbiddenness.[10,11] We may infer, in the spirit of these discussions, that the selection rule, perhaps K forbiddenness, is somewhat relaxed in Eu^{154} as compared with Eu^{152}. The validity of the one-parameter formulas is then worsened since the inequalities that are required for the parameters u and x are met to a lesser degree.

THREE-PARAMETER INTERPRETATION

The lack of good agreement between the directional correlation data and the spectral shape data,[6] when interpreted in terms of a single matrix element parameter, suggests that the more general formulas given by Kotani[11] be used. The beta decay may then be characterized by the three matrix element parameters ζ_1, u, and x. In effecting a comparison of the data and the theoretical formulas it has proved convenient to characterize the experimental data by the following conditions:

(A) The spectral shape measurement[6] determines the ratio of the shape correction near the extremities of the energy range covered, i.e., $C(W=4.6)/C(W=3.9)=1.145\pm0.023$, approximately. (The magnitude of the spectral shape correction, which enters into the directional correlation formula, is, of course, not determined from the measurement of the shape correction. It is possible to fit the observed shape with various choices for the matrix element parameters, which then give widely varying values for the magnitude of the shape correction.)

(B) The directional correlation data determine the magnitude of the coefficient A_2^* at an energy near the midpoint of the energy range covered, i.e., $A_2^*(W=3.9)=-0.060\pm0.003$.

A fairly systematic, but tedious, analysis of the data has been developed for location of acceptable sets of the matrix parameters ζ_1, x, and u. For a selected value of ζ_1, each of the conditions (A) and (B) leads to a characteristic curve, an ellipse in this instance, relating the parameters x and u. The points of intersection of the two ellipses give the values of x and u that satisfy conditions (A) and (B) for the particular choice of ζ_1. After matrix element parameters are found that are consistent with conditions (A) and (B), plots of the theoretical formulas for $C(W)$ and A_2^* versus W are made to establish whether satisfactory agreement with the experimental data is maintained over the observed energy range. (The method may be extended to take advantage of additional experimental information interpretable in terms of the beta-decay matrix element parameters.) In view of the considerable computational problem entailed in a concise presentation of a search for all acceptable sets of matrix element parameters, we simply present results for typical sets of parameters.

We find that a fit to the observed spectral shape[6] cannot be found for values of ζ_1 less than about unity. For smaller values of ζ_1 the theoretical shape correction shows too great an energy dependence which cannot be compensated by adjustment of u or x. For values of ζ_1 greater than roughly 3.5 a fit to the spectral shape precludes a fit to the directional correlation.

In Fig. 4 we present plots of A_2^* versus W for various sets of matrix element parameters. The information is presented as pairs of curves for each value of ζ_1. Each pair of curves has been chosen to roughly straddle the

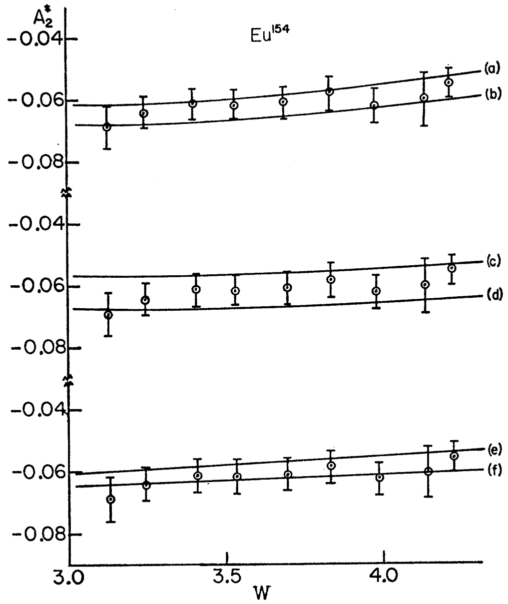

FIG. 4. Plot of theoretical curves of A_2^* versus the total beta energy, W, for selected sets of the beta decay matrix element parameters ζ_1, u, and x. The experimental data are also indicated. The values of the matrix element parameters for the various curves are (a) $\zeta_1=1.3$, $u=0$, $x=-0.3$; (b) $\zeta_1=1.3$, $u=0$, $x=-0.26$; (c) $\zeta_1=2$, $u=-0.2$, $x=-0.25$; (d) $\zeta_1=2$, $u=-0.2$, $x=-0.1$; (e) $\zeta_1=3$, $u=-0.6$, $x=0.5$; and (f) $\zeta_1=3$, $u=-0.3$, $x=0.5$.

experimental data. Plots of the normalized shape correction versus W are not presented since, for the parameters chosen, such plots show no significant variation from the shaded area in Fig. 3.

We may now summarize the qualitative nature of the conclusions arising from the three-parameter analysis. The experimental data do not demand a unique set of matrix element parameters. However, for a choice of a parameter ζ_1, for example, the data are rather restrictive in regard to acceptable choices for the parameters u and x. In the spirit of the interpretation of the decay of Eu^{154} in terms of a selection rule effect, magnitudes for x or u of the order 0.2 and larger are consistent with less attenuation of the ordinary matrix elements relative to B_{ij} in the beta decay of Eu^{154} than in the comparable decay of Eu^{152}.

The three-parameter analysis of the data is particularly suggestive of the need of experimental results for another type of measurement interpretable in terms of the matrix element ratios ζ_1, x, and u. For example, knowledge of any departure of the longitudinal polarization of the beta rays from the value v/c should prove valuable. It is believed that an additional type of measurement would be of more value in adducing a unique set of matrix element parameters than higher precision in the present data. This point of view is much emphasized by recognition of the inexact nature of the theoretical expressions.[10,11] Determination of precise values of the matrix element parameters in this decay should also include consideration of possible attenuation of the directional correlation by perturbation of the 123-kev level in Gd^{154}.[19] We make no allowance for this effect, hence the results cited here are associated with an unperturbed correlation.

ACKNOWLEDGMENTS

The authors wish to thank Mr. Nisbet S. Kendrick for invaluable assistance with many phases of this work and Mr. R. E. Wood for taking part of the data.

[19] R. Stiening and M. Deutsch, Phys. Rev. **121**, 1484 (1961).