Transverse Magnetoresistance of Germanium in the Quantum Limit*

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The transverse magnetoresistance of very pure single-crystal, n-type germanium has been measured as a function of temperature and magnetic field, in pulsed fields up to 194 kgauss, in the temperature range from 11° to 78°K. The results below 20°K are obscured by hot electron effects and non-Ohmic behavior of the crystal, but in the quantum limit and above, the magnetoresistance ratio varies linearly with magnetic field above 40 kgauss and has a T^{-1} temperature dependence. The results of these experiments do not agree with theoretical predictions for various scattering mechanisms.

INTRODUCTION

MAGNETORESISTANCE, that is, the change of electrical resistance in a magnetic field, has been a valuable tool for improving the understanding of the conduction mechanisms of semiconductor crystals. Low-field magnetoresistance measurements, for example, have confirmed the many-valley model of the conduction band of germanium. At very low temperatures and high magnetic fields (referred to as the "quantum limit"), the galvanomagnetic behavior of germanium is best described by a quantum theory.[1] In germanium the energy band structure is characterized by four energy ellipsoids of revolution with their axes oriented along [111] axes. When a magnetic field H is applied to a germanium crystal, the charge carriers of charge e and of cyclotron effective mass m^* are caused to rotate about the field lines with an effective angular frequency ω_0, where $\omega_0=|e|H/m^*c$. Now if $\omega_0\tau\geq 1$, where τ is the time between two successive collisions, the effects of quantization of the carrier orbits must be taken into account in a proper theory. The "quantum limit" is that combination of temperatures and magnetic fields for which

$$\hbar\omega_0\gg kT,\ E_F \quad \text{or} \quad \hbar|e|H/m^*c\gg kT,\ E_F,$$

where E_F = Fermi energy.

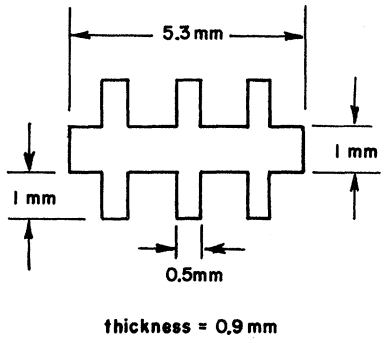

FIG. 1. Geometry and approximate dimensions of experimental sample.

Now the effective mass for a given direction of magnetic field is given by

$$m^*=\left[\frac{m_t^2 m_l}{m_l\cos^2\theta+m_t\sin^2\theta}\right]^{\frac{1}{2}},$$

where m_l and m_t are the longitudinal and transverse masses, with respect to the axes of the energy ellipsoid, and θ is the angle between the magnetic field and the major axis of the ellipsoid. With the magnetic field in the [110] direction, $\theta=90°$ for two of the ellipsoids and $\theta=35.2°$ for the other two. Using $m_l=1.58m$, $m_t=0.082m$,[2] m^* comes out to be about $0.36m$ for the first set and $0.1m$ for the other set. The temperature range for which the assumptions of the quantum limit are valid is found by expressing the relation $\hbar\omega_0\gg kT$ in terms of temperature, i.e., $\hbar|e|H/m^*ck\gg T$. Using $H=1.9(10^5)$ gauss and $m^*=0.36m$ for the first set of ellipsoids, it is required that $70.5°\text{K}\gg T$; for the second set of ellipsoids, using $m^*=0.1m$, then $254°\text{K}\gg T$. From this it is seen that, with these large magnetic fields, the approach to the quantum limit is possible below 70°K. This condition is met in most of the experiments reported in this paper. The purpose of these experiments has been to determine the behavior of the transverse magnetoresistance in and near the quantum limit. Measurements of this type have been attempted but were not reported due to the "pronounced peculiarities of metal-germanium contacts in high magnetic fields."[3]

DESCRIPTION OF APPARATUS

Apparatus similar to that used in these experiments has been described elsewhere.[4–7] It consists essentially of an air-core solenoid through which a capacitor bank is discharged. The magnetic field at the center of the solenoid builds up to a peak value of about 190 kgauss in 7 msec and then decays to zero. A signal proportional to the magnetic field drives the horizontal amplifier of

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[2] C. Kittel, *Introduction to Solid State Physics* (John Wiley & Sons, New York, 1957), 2nd edition, p. 376.
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[4] H. P. Furth and R. W. Waniek, Rev. Sci. Instr. **27**, 195 (1956).
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[7] W. F. Love and W. F. Wei, Phys. Rev. **123**, 67 (1961).

an oscilloscope and the voltage drop across the appropriate electrodes of the crystal is applied to the vertical amplifier. A Polaroid picture is taken of the oscilloscope screen during the time the magnetic field is building up. The magnetic field was calibrated with a pickup coil and an *RC* integrator. Temperatures were measured with a gold-cobalt vs copper thermocouple.

EXPERIMENTAL PROCEDURE

The germanium crystal used in these experiments was obtained from Bell Telephone Laboratories. It is *n*-type with a room temperature resistivity of 34 ohm-cm and an impurity concentration of 8×10^{13} per cm^3. The growth axis of the crystal is in a [111] crystal direction. The crystal was oriented by means of a Laue back-reflection x-ray technique. Several x-rays were taken in different crystal directions in order to verify that the orientation was correct. It is believed that the accuracy of the orientation of the sample used in these experiments is within $\pm2°$.

The crystal was cut into the bridge shape, as shown in Fig. 1, by means of a crystal milling apparatus assembled in the laboratory. The current flow in the crystal was in the [100] direction, and the magnetic field, perpendicular to the plane surface of the crystal, was in the [110] crystal direction. The actual cutting of the crystal was done with a diamond saw blade.

The electrode surfaces were lapped, before soldering, with 600 grit silicon carbide abrasive, and Divco 335 flux was used to facilitate "tinning" of the electrode surfaces. Electrodes were attached to the eight terminals of the crystal by means of a tin-lead solder containing two percent antimony, which is an *n*-type impurity. The solder alloys with the germanium and forms a good electrical contact and a strong mechanical bond. The antimony impurity assures that the alloyed portion of the crystal is strongly *n* type, thereby preventing the formation of a *p-n* junction, which might give hole injection. Pictures were taken of the current-voltage characteristic of each of the electrodes, displayed on the oscilloscope. Electrodes exhibiting non-Ohmic behavior were resoldered until a linear characteristic was observed at liquid nitrogen temperature. Current-voltage characteristics were also measured at lower temperatures, and the current electrodes were found to be essentially Ohmic at 20°K. After the wires were attached to the electrodes, the crystal was given a quick dip in CP4 etchant.

EXPERIMENTAL RESULTS

Several anomalous effects were observed in these experiments. At 13°K it was noticed that the Hall voltage, as a function of magnetic field, has a gradually increasing slope. This is the magnetically induced "freeze-out" of carriers first observed in InSb by Frederikse and Hosler[8] and Sladek.[9] At high magnetic fields the energy gap between the impurity levels and the conduction band is increased. The increasing slope of the Hall voltage corresponds to a smaller number of impurities being ionized at high magnetic fields. At higher current densities, "hot electron" effects were observed. Figure 2 shows the behavior of the magnetoresistance voltage (vertical) vs magnetic field (horizontal) at 16.6°K. The departure from linear behavior is accompanied by vertical noise-like oscillations, similar to those observed before in InSb.[10] It is thought that these oscillations are due to impact ionization of bound electrons, giving an avalanche-type electrical breakdown. Except in the anomalous cases mentioned above, the Hall voltage was linearly proportional to the magnetic field, yielding a field-independent Hall coefficient.

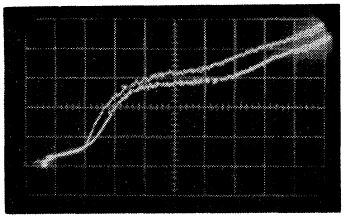

FIG. 2. Magnetoresistance voltage (vertical) vs magnetic field (horizontal, increasing from left to right) at 16.6°K, illustrating "hot electron" effects. Two sweeps were photographed.

Figure 3 shows the results of the measurements of the resistance of the crystal as a function of magnetic field. The results are presented as the ratio of $\rho(H)/\rho(0)$ versus field, where $\rho(H)$ is the value of the resistivity at a given value of H, and $\rho(0)$ is the zero field resistivity. It should be noted that the transverse magnetoresistance increases linearly with field, except for an anomalous behavior at 23.1°K. With the exception of the curves labeled 0.1 ma, all the data shown in Fig. 3 were obtained with 1 ma flowing in the crystal. At temperatures below about 30°K, the magnetoresistance becomes sensitive to the current density, being higher for lower currents. The curve marked 30°K (0.1 ma) is about where one would expect from the two curves on either side of it, whereas the curve marked 23.1°K (0.1 ma) is noticeably different from its neighbor, marked 23°K. The crystal becomes non-Ohmic at lower temperatures. At 11°K the resistance of the crystal was 2000 ohm with 1 ma flowing, and about 1 megohm with 1 μamp flowing. This is interpreted as a hot-electron effect.

In Fig. 4 is shown the T^{-1} behavior of the magnetoresistance ratio at 180 kgauss.

Results of similar measurements were reported by Karasik,[11] who also observed a linear behavior of the magnetoresistance ratio, although with a different temperature dependence. His measurements, however, were made with higher current densities for the low-temperature curves than those presented in Fig. 3, and were just in the range where the authors have observed both a strong dependence of the magnetoresistance ratio on

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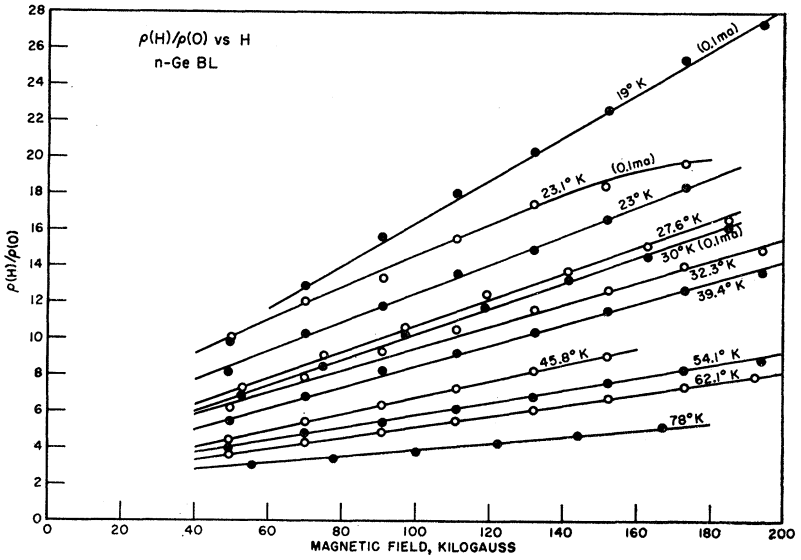

FIG. 3. Magnetoresistance ratio vs magnetic field, with temperature as a parameter.

current density and erratic behavior due to hot-electron effects.

CONCLUSIONS

Adams and Holstein have developed a theory for the behavior of transverse resistivity in the extreme quantum limit.[1] They predict that the transverse resistivity should vary as follows: For low-temperature acoustical scattering, as $H^{\frac{3}{2}}T^{-\frac{3}{2}}$; for high-temperature acoustical scattering, as $H^2T^{-\frac{1}{2}}$; and for ionized impurity scattering, as $H^0T^{-\frac{3}{2}}$. The results of these experiments indicate a behavior that goes as HT^{-1} above 40 kgauss, a result that is not adequately explained by any scattering mechanism thus far suggested. Herring, however, has shown[12] that fluctuations of carrier concentration, even with a random distribution of impurities, can give rise to linear terms in the field dependence of magnetoresistance. This is a possible explanation for the observed experimental behavior.

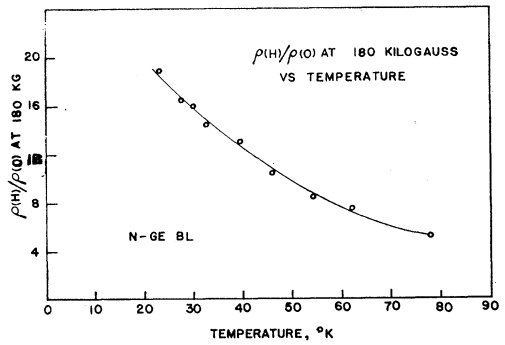

FIG. 4. Magnetoresistance ratio at 180 kgauss vs temperature.

ACKNOWLEDGMENTS

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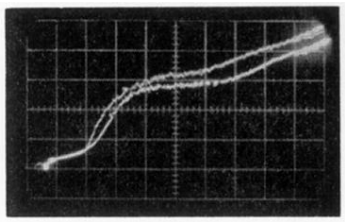

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