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$K_1{}^0-K_2{}^0$ Mass Difference*

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To account for the $K_1{}^0-K_2{}^0$ mass difference a direct $K_1{}^0-2\pi$ interaction is introduced which gives $\tau(K_1{}^0)[m(K_1{}^0)-m(K_2{}^0)]$ in terms of the $I=0$, s-wave pion-pion scattering phase shifts.

NOW that the $K_1{}^0-K_2{}^0$ mass difference (hence called Δm) has been measured, experiments to determine its sign are sought.[1,2] There has not been a comparable success in a theoretical prediction of Δm as a consequence of the difficulty of disentangling strong and weak interactions, and in particular because the form of the latter is not known for weak interactions involving a change of strangeness. In order partially to avoid this difficulty we calculate $\tau(K_1{}^0)\Delta m$ which is expected to be less sensitive to the exact form of the weak interactions that give rise to the $K_1{}^0$ decay, and under assumptions stated below it is shown that the s-wave, $I=0$ π-π scattering phase shifts are sufficient for this purpose.

The following facts greatly facilitate the calculation:

(a) $K_1{}^0$, $K_2{}^0$ leptonic decay rates are equal within experimental accuracy.[3] (b) The decay of $K_1{}^0$ into pions in an isotopic spin state $I=2$ is negligible.[4] (a) suggests that the matrix elements $\langle 0|\phi(K_1{}^0)|n\rangle$, $\langle 0|\phi(K_2{}^0)|n\rangle$ will also be equal when $|n\rangle$ represents a state of many particles, since there will not be strong suppressions arising from conservation laws; thus a knowledge of multiparticle transitions will not be necessary in determining Δm. Furthermore the expected slowness of the rates for $K_1{}^0\rightarrow 3\pi$,[5] $K_2{}^0\rightarrow 3\pi$ compared to that of $K_1{}^0\rightarrow 2\pi$ shows that a knowledge of the matrix element $\langle 0|\phi(K_1{}^0)|\pi,\pi\rangle$ alone will suffice for our calculation. Baryon-antibaryon intermediate states will give a much smaller contribution to Δm in view of the large energy denominators that they produce; (b) shows that $I=0$ for the intermediate states of interest. Under these assumptions suggested from (a) and (b) it suffices to introduce a direct $K_1{}^0$-2π interaction to determine $\tau(K_1{}^0)\Delta m$.[6] A lowest-order perturbation calculation using such a direct interaction requires a cutoff of 20 Bev to reproduce the experimental results and predicts a negative $K_1{}^0-K_2{}^0$ mass difference.[7] We shall instead use the experimental π-π $I=0$, s-wave phase shifts and not rely on perturbation theory in determining $\tau(K_1{}^0)\Delta m$.

The Heisenberg equations of motion for our model are

$$(\Box_x-\mu_K{}^2)\phi_{K_1{}^0}=C\phi_\pi\cdot\phi_\pi, \tag{1}$$

$$(\Box_x-\mu^2)\phi_\pi=-J_\pi, \tag{2}$$

where C is of the order of the weak interaction coupling constants. μ_K and μ are the $K_1{}^0$ and π meson masses, respectively. To second order in the weak interactions and exactly for all strong interactions arising from J_π, we can obtain the $K_1{}^0$ propagator, $\Delta_F'(k^2)$, by adding to the Hamiltonian that leads to Eqs. (1) and (2) a term $C\phi_\pi\cdot\phi_\pi\phi_1\phi_2$ and then calculating the amplitude for $\phi(K_1{}^0)\rightarrow\phi_1+\phi_2$ (we use the same symbol for fields and particles). Using dispersion-theoretic techniques,[8] we find

$$\Delta_F'(k^2)=\Delta_F(k^2)+\Delta_F(k^2)\Sigma(k^2)\Delta_F(k^2)$$

for $k^2=-\mu_K{}^2$, where

$$\Sigma(-\mu_K{}^2)=\alpha\int_0^\infty\frac{|F(k)|^2k^2dk}{(k^2+\mu^2)^{\frac{1}{2}}(k^2+\mu^2-\frac{1}{4}\mu_K{}^2-i\epsilon)}.$$

Here $F(k)$ is proportional to the matrix element $\langle 0|\phi(K_1{}^0)|\pi,\pi\rangle$ and is given by

$$F(k)=\exp\left[\frac{2k^2}{\pi}P\int_0^\infty\frac{\phi_r(k')dk'}{k'(k'^2-k^2)}\right]$$

where ϕ_r is the real part of the π-π scattering s-phase shift for $I=0$. In the usual way this yields[9]

$$2\tau(K_1{}^0)\Delta m=-\mathrm{Re}\Sigma(-\mu_K{}^2)/\mathrm{Im}\Sigma(-\mu_K{}^2).$$

Since the π-π scattering phase shifts are not yet known, we have made two choices for ϕ_r in order to determine the dependence of $\tau(K_1{}^0)\Delta m$ on the over-all characteristics of the π-π interaction; they are

$$\phi_r=\mathrm{arc\ tan}ka, \tag{3}$$

$$\phi_r=\frac{ck(k-a)}{1+bk^2}, \tag{4}$$

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respectively. (It must be emphasized that if experimentally ϕ_r turns out to have a great deal more structure than is present in our choices, then the qualitative conclusions that we arrive at need have no bearing.) Equation (3) yields $-1.6<\tau(K_1^0)\Delta m<+0.4$, where the lower and higher limits are reached as the scattering length goes to zero and infinity, respectively. With this choice we are limited to positive scattering lengths since for the convergence of our integrals ϕ_r has to be finite and positive at high energy.[10] We utilized the additional freedom of Eq. (4) to fit the various $I=0$ scattering lengths which have been proposed so far[11–14] and at the same time yield $\tau(K_1^0)|\Delta m|=1.5$.[1] For negative scattering lengths the numerical search yields the following conclusions: (a) We can obtain $\tau(K_1^0)\Delta m=-1.5$ for a scattering length $a_0=-0.8\ \mu^{-1}$. The result is sensitive to the high-energy behavior of ϕ_r; 70% of the mass difference comes from center-of-mass pion momenta above 1.3 Bev/c. (b) It is impossible to obtain a possible mass difference for negative scattering lengths. For positive scattering lengths, however, the conclusions are: (a) We can obtain a negative mass difference which is again sensitive to the high-energy behavior, 50–90% of the mass difference comes from center-of-mass pion momenta above 1.3 Bev/c. (b) A positive mass difference can be obtained for $a_0=2.81\ \mu^{-1}$ but not for $a_0=1.96\ \mu^{-1}$; in this case 99% of the mass difference comes from pion momenta less than 0.5 Bev/c. Both choices for ϕ_r support the following qualitative conclusion that negative and small positive scattering lengths will give a negative Δm and a positive large scattering length can yield positive mass difference if ϕ_r is in the range of 0.5–1.8 for very large momenta (values appreciably less than this will give a negative mass difference).

All numerical work was performed on the IBM 650 at the Computation Center of the Pennsylvania State University.

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Two Pictures of the Strong-Coupling Method*

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In the strong-coupling method of the meson theory two different pictures have been used. One picture exhibits the isobaric nature of the meson-nucleon interaction by expressing the Hamiltonian in terms of the integrals of motion of the total system. It may be called the rotation picture. In the other picture the isobaric dependency comes out by splitting the total system into a free field system and a compound nucleon system, such that the interaction between them vanishes for infinite g. It may be called the splitting picture. These two pictures are compared with each other. The difference between them with regard to the scheme of the strong-coupling approximation method, especially with regard to the calculations of isobaric energy corrections and resonance scattering, is investigated.

1. INTRODUCTION

THE strong-coupling approximation method of the meson theory, introduced a long time ago by Wentzel,[1] Oppenheimer and Schwinger,[2] and Pauli and Dancoff,[3] has recently been extended to a somewhat higher degree of completeness. In course of these investigations two pictures have been introduced to carry through this line of approximation.

One of these two pictures has already been introduced by Wentzel[1] and has lately been extended by Serber and Dancoff,[4] Miyazima, Tati, and Tomonaga,[5] Kaufman,[6] Pais and Serber,[7] Nickle and Serber,[8] and Chun.[9] The main characteristic of this picture is to express the Hamiltonian by the operators of the important total integrals of motion of the system, such as the total isospin operator in the charged and the symmetric scalar fixed source theories and the total isospin and angular momentum operator together in

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