Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. II*

Y. NAMBU AND G. JONA-LASINIO†
Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois
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Continuing the program developed in a previous paper, a "superconductive" solution describing the proton-neutron doublet is obtained from a nonlinear spinor field Lagrangian. We find the pions of finite mass as nucleon-antinucleon bound states by introducing a small bare mass into the Lagrangian which otherwise possesses a certain type of the γ_5 invariance. In addition, heavier mesons and two-nucleon bound states are obtained in the same approximation. On the basis of numerical mass relations, it is suggested that the bare nucleon field is similar to the electron-neutrino field, and further speculations are made concerning the complete description of the baryons and leptons.

I. INTRODUCTION

IN Part I of this paper[1] we have proposed a model of strong interactions based on an analogy with the BCS-Bogoliubov theory of superconductivity. It is characterized by a nonlinear spinor field possessing γ_5 invariance, and simulates some important features of the meson-nucleon system. The basic principle underlying the model is the idea that field theory may admit, as a result of dynamical instability, extraordinary (nontrivial) solutions that have less symmetries than are built into the Lagrangian.[2] In fact we have obtained as an extraordinary solution a massive fermion and a massless pseudoscalar boson as idealized proton and pion, together with other heavy mesons.

If we now try to make our model more realistic, a number of problems spring up naturally. First of all, we would have to account for the isospin and strangeness quantum numbers. It seems rather obvious that these degrees of freedom have to be built into the theory from the beginning, although there may be some possibility of utilizing both the ordinary and extraordinary solutions to enlarge the Hilbert space, as will be discussed later.

These quantum numbers will not yet be enough to determine our theory satisfactorily, as we expect to have more additional symmetries which are at least approximately satisfied. Among other things, we have postulated the γ_5 invariance as a cornerstone of our previous model. What would be the proper generalization of the γ_5 invariance? Then there also arises the inevitable question of any possible symmetry among baryons of different strangenesses. Since such a symmetry is at any rate only approximate, the test of the theory will depend on its ability to account for the violation of the symmetry as well.

Finally, we face the problem of the baryon versus the lepton, the electromagnetism, and the weak processes. Here our theory creates a particular incentive for speculation concerning the baryon-lepton problem, since the ordinary and extraordinary solutions immediately remind us of these two families of particles.

We do not profess to have any clear-cut answers to these problems. In the present paper we shall again content ourselves with a rather modest task. We will first discuss a generalization of our model which incorporates the isospin for the nucleon and guarantees the existence of the pion. This can be done by demanding a $\gamma_5\times$isospin gauge group with a slight violation so as to give the pion its finite mass. We find that the bare mass necessary to achieve the latter end is at most several Mev. On this basis a suggestion is made that the bare nucleon field is essentially the same as the electron-neutrino field.

The complete picture of the baryon symmetries and the baryon-lepton problem is largely beyond the scope of the present paper, but some relevant discussions on this subject will also be presented, especially those concerned with the Sakata model and the general γ_5 symmetry.

II. MODEL LAGRANGIAN FOR THE NUCLEON

First we would like to observe that the nonlinear spinor field adopted in I is not an essential element of our theory, as is the case with the Heisenberg theory[3] but is rather a model adopted to study our dynamical principles. At least in the present stage of the game, the controlling factors are the symmetry properties and qualitative dynamical characteristics of the basic fermion-fermion interaction, and whether the interaction is due to some fundamental boson, or fundamental nonlinearity (or something entirely new) is of secondary importance. Nevertheless, we have to choose

* This work was supported by the U. S. Atomic Energy Commission.
† Present address: Istituto di Fisica dell'Universita, Roma, and Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy.
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[2] See also J. Goldstone, Nuovo cimento **19**, 154 (1961), N. N. Bogoliubov (to be published), V. G. Vaks and A. I. Larkin, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 871.
[3] H. P. Duerr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, Z. Naturforsch. 14, 441 (1959); W. Heisenberg, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., 1960), p. 851.

some model, and naturally there will arise certain predictions specific to the particular model. We take notice of the fact that the pion, the lightest of the meson family, is pseudoscalar and isovector, whereas its isoscalar counterpart of comparable mass does not seem to exist.[4] If the pion is to be intimately related to a symmetry property as in our previous model, this would imply that the model of nucleons should allow an (approximate) invariance under the $\gamma_5\times$isospin gauge group of Gürsey,[5] but not under the simple (Touschek) γ_5 gauge group, at least not so well as in the former case. For this reason, we would altogether consider the following gauge groups:

$$\psi \to e^{i\alpha}\psi, \qquad \bar{\psi} \to \bar{\psi}e^{-i\alpha}, \tag{2.1a}$$

$$\psi \to \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}')\psi, \qquad \bar{\psi} \to \bar{\psi}\exp(-i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}'), \tag{2.1b}$$

$$\psi \to \exp(i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\alpha}'')\psi, \quad \bar{\psi} \to \bar{\psi}\exp(i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\alpha}''), \tag{2.1c}$$

where τ denotes the nucleon isospin matrices.

Obviously, the first two are generators of the nucleon number gauge and the isospin transformation, respectively. The second and third transformations combined form a four-dimensional rotation group on the four components composed by the proton and neutron of both handednesses.[5] Thus we may also replace Eqs. (2.1a) and (2.1b) by the following transformations

$$\begin{aligned}&\psi_R \to \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}_R)\psi_R, \quad \psi_R^\dagger \to \psi_R^\dagger\exp(-i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}_R),\\ &\psi_L \to \exp(i\boldsymbol{\tau}1\boldsymbol{\alpha}_L)\psi_L, \quad \psi_L^\dagger \to \psi_L^\dagger\exp(-i\boldsymbol{\tau}\cdot\boldsymbol{\alpha}_L),\end{aligned} \tag{2.2}$$

where ψ_R and ψ_L are the right- and left-handed components.

As the simplest Lagrangian that meets our requirements, we adopt the form

$$L=-\bar{\psi}\gamma_\mu\partial_\mu\psi-\bar{\psi}M^0\psi+g_0\Big[\bar{\psi}\psi\bar{\psi}\psi-\sum_{i=1}^{3}\bar{\psi}\gamma_5\tau_i\psi\bar{\psi}\gamma_5\tau_i\psi\Big]. \tag{2.3}$$

If the bare mass operator $M^0=0$, this Lagrangian possesses, in addition to Eq. (2.1), an invariance under the discrete "mass reversal" group:

$$\psi \to \gamma_5\psi, \quad \bar{\psi} \to -\bar{\psi}\gamma_5. \tag{2.4}$$

The bare mass operator M^0 is a possible agent for the breakdown of the Gürsey group, and will be related to the finite pion mass.[6] For the moment, we will assume $M^0=0$. Before going to solve the self-consistent equation for the mass, we give the result of the Fierz transformation on Eq. (2.3): The interaction becomes

$$\begin{aligned}L_{\text{int}}=&\tfrac{1}{4}g_0[\bar{\psi}\psi\bar{\psi}\psi-\bar{\psi}\gamma_5\tau_i\psi\bar{\psi}\gamma_5\tau_i\psi]\\ &+\tfrac{1}{4}g_0[\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi-\bar{\psi}\tau_i\psi\bar{\psi}\tau_i\psi]\\ &-\tfrac{1}{2}g_0[\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma_\mu\psi-\bar{\psi}\gamma_\mu\gamma_5\psi\bar{\psi}\gamma_\mu\gamma_5\psi]\\ &+\tfrac{1}{8}g_0[\bar{\psi}\sigma_{\mu\nu}\psi\bar{\psi}\sigma_{\mu\nu}\psi-\bar{\psi}\sigma_{\mu\nu}\tau_i\psi\bar{\psi}\sigma_{\mu\nu}\tau_i\psi],\end{aligned} \tag{2.5}$$

which is a rather complicated combination of all kinds of terms.

We now apply the linearization procedure of I to Eqs. (2.3) and (2.4), and obtain the self-energy

$$\begin{aligned}m&=(1+\tfrac{1}{4})g_0\,\mathrm{Tr}S_F{}^{(m)}(0)\\ &=-i\frac{10g_0}{(2\pi)^4}\int\frac{d^4pm}{p^2+m^2}F(p,\Lambda).\end{aligned} \tag{2.6}$$

Note that the trace refers to both spin and isospin variables. This differs from Eq. (3.6) of I only by the change of the effective coupling $g_0 \to 5g_0/2 \equiv g_0'$. So we can simply take over the previous formulas, namely,

$$1=\frac{g_0'}{4\pi^2}\int_{4m^2}^{\Lambda^2}d\kappa^2\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}}, \tag{2.7}$$

for the nontrivial solution if the dispersion integral (4.7) of I is used.

III. DETERMINATION OF MESON STATES

Since the interaction Lagrangian in Eqs. (1.1) and (1.3) contains a number of different couplings, we expect to get various kinds of "mesons" as bound nucleon-antinucleon pairs in our simple ladder approximation. As was explained in I, this is the proper approximation to match our self-energy equation at least for the pseudoscalar meson which is expected to have zero mass; moreover, even for other types of bound states we may reasonably trust its qualitative validity in predicting the existence and level ordering of possible bound states to the extent that our interaction is regarded basically as a short-range potential between spinor particles.

For general discussion, it is convenient to follow the procedure given in the Appendix of I. The basic equation to be considered is of the type

$$\begin{aligned}\Gamma(p+\tfrac{1}{2}q,\,p-\tfrac{1}{2}q)=\gamma+i\textstyle\sum_n g_nO_n\\ \times\int\mathrm{Tr}[O_nS_F(p'+\tfrac{1}{2}q)\Gamma(p'+\tfrac{1}{2}q,\,p'-\tfrac{1}{2}q)\\ \times S_F(p'-\tfrac{1}{2}q)]d^4p',\end{aligned} \tag{3.1}$$

where the summation on the right-hand side is over the various tensor forms in the interaction Lagrangian. The "vertex function" $\Gamma(p+\frac{1}{2}q,\,p-\frac{1}{2}q)$ reduces to a bound state wave function when it becomes a homogeneous solution ($\gamma=0$) for a particular value of $q^2=-\mu^2$. We will briefly discuss those two-nucleon states for which there is a possibility of binding.

A. Pseudoscalar, Isovector Meson

Unlike the case in I, only the pseudoscalar interaction $\sim\bar{\psi}\gamma_5\tau_i\psi\bar{\psi}\gamma_5\tau_i\psi$ contributes to this state. Assuming

[4] It may not be impossible that the ordinary γ_5 invariance is violated more strongly than the Gürsey γ_5 invariance so that the mass of the $\pi_0{}^0$ meson may come sufficiently high. But to achieve this end by means of a bare mass does not seem to be feasible.

[5] F. Gürsey, Nuovo cimento **16**, 230 (1960).

[6] For its possible origin, see Sec. V.

$\Gamma_i^P=\gamma_5\tau_i\Gamma^P$, we obtain

$$\Gamma^P=\gamma^P+\Gamma^P[1-q^2I^P(q^2)],$$

$$I^P(q^2)=\frac{g_0'}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{d\kappa^2}{q^2+\kappa^2}\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}},\tag{3.2}$$

where, of course, Eq. (2.7) was utilized. This has a homogeneous solution for $q^2=0$, corresponding to the zero-mass "pion." This pion-nucleon coupling is of pure pseudoscalar type, which can be calculated from the inhomogeneous equation with $\gamma^P=g_0'\gamma_5\tau_i$, as was done in the Appendix of I. We get, namely,[7]

$$G_P^2/4\pi=g_0'[4\pi I^P(0)]^{-1}$$
$$=\pi\left[\int_{4m^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2}\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}}\right]^{-1}.\tag{3.3}$$

B. Scalar, Isoscalar Meson

With the ansatz $\Gamma=\Gamma^S$ we have

$$\Gamma^S=\gamma^S+\Gamma^SI^S(q^2),$$

$$I^S(q^2)=\frac{g_0'}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{\kappa^2-4m^2}{q^2+\kappa^2}d\kappa^2\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}}\tag{3.4}$$
$$=1-(q^2+4m^2)I^P(q^2).$$

This leads to a zero-binding state: $q^2=-4m^2$ with the scalar nucleon coupling constant

$$G_S^2/4\pi=g_0'[4\pi I^P(-4m^2)]^{-1}.\tag{3.5}$$

C. Vector Mesons

There are two vector mesons, with isospin 1 and 0. The isovector meson arises from the tensor interaction $\sim\bar\psi\sigma_{\mu\nu}\tau_i\psi\bar\psi\sigma_{\mu\nu}\tau_i\psi$ with the wave function of the type

$$\Gamma_{\mu i}{}^V=\sigma_{\mu\nu}q_\nu\tau_i\Gamma^V.$$

The mass is determined from[8]

$$1=-\frac{g_0'}{60\pi^2}\int_{4m^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2-\mu^2}\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}}\times\left[\kappa^2-4m^2-\mu^2\left(2+\frac{4m^2}{\kappa^2}\right)\right],$$

which has a solution (for sufficiently small Λ^2)

$$\mu^2\geqslant 10m^2/3.$$

The coupling of this meson to the nucleon is necessarily of the derivative type.

To the isoscalar meson, both vector and tensor interactions contribute, the former being attractive and the latter repulsive. The wave function will have the form

$$\Gamma_\mu{}^V=\gamma_\mu\Gamma_1{}^V+\sigma_{\mu\nu}q_\nu\Gamma_2{}^V,$$

which yields a coupled equation for Γ_1 and Γ_2. This coupling, however, is rather small, so that we get a solution by neglecting Γ_2:

$$1=\frac{g_0'}{15\pi^2}\mu^2\int_{4m^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2-\mu^2}\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}}\left(1+\frac{2m^2}{\kappa^2}\right),$$
$$\mu^2\geqslant 20m^2/7.$$

The nuclear coupling will be predominantly nonderivative.

D. The "Deuteron" States

As in I, we can discuss the nucleon-nucleon states in parallel with the meson states. The interaction may be written conveniently in the form

$$L_{\text{int}}=\tfrac{1}{4}g_0[\bar\psi\gamma_\mu\psi^c\bar\psi^c\gamma_\mu\psi-\bar\psi\sigma_{\mu\nu}\psi^c\bar\psi^c\sigma_{\mu\nu}\psi+\bar\psi\gamma_\mu\gamma_5\tau_i\psi^c\bar\psi^c\gamma_\mu\gamma_5\tau_i\psi].$$

This is seen to lead to two bound states: a pseudovector, isoscalar ($J=1^+$, $T=0$) coming from the first two interaction terms, and a scalar, isovector ($J+0^+$, $T=1$), coming from the last term. For the $J=1^+$, $T=0$ state (deuteron) the main contribution comes from the attractive tensor interaction, and we get

$$\Gamma_\mu=\sigma_{\mu\nu}q_\nu\Gamma^{A'},$$

$$1=\frac{g_0'}{4\pi^2}\int_{4m^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2-\mu^2}\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}}\left[\mu^2+\frac{2}{15}(\kappa^2-4m^2)\right],$$
$$\mu^2\geqslant 17m^2/5.$$

For the $J=0^+$, $T=1$ case we have

$$\Gamma=\gamma_5\gamma\cdot q\Gamma^{S'},$$
$$1=\tfrac{4}{5}m^2I^P(-\mu^2),$$
$$\mu^2\geqslant 16m^2/5.$$

IV. VIOLATION OF γ_5 INVARIANCE

Let us now discuss the violation of the γ_5 invariance as indicated by the finite mass of the real pion. It would be senseless, of course, to talk about the invariance if the observed pion mass implied a large departure from our original Lagrangian, for example, due to a bare nucleon mass as large as the observed mass. So we need to estimate the amount of violation in the Lagrangian.

In general, the bare mass operator, which does not

[7] Note that this is half the value of I because a pion (e.g., π_0) consists of two substates $\bar pp$ and $\bar nn$, which changes the normalization of the pion wave function.

[8] The ambiguity about the subtraction of the most divergent part was discussed in I, section 4. The gross qualitative feature is not altered even if we do not make a subtraction.

violate nucleon number conservation,[9] can have the following form

$$M^0 = m_1^0 + m_2^0 \boldsymbol{\tau}\cdot\mathbf{n} + m_3^0 i\gamma_5 + m_4^0 i\gamma_5 \boldsymbol{\tau}\cdot\mathbf{n}', \qquad (4.1)$$

where $\mathbf{n}$ and $\mathbf{n}'$ are arbitrary unit vectors in the isospin space. The observed mass generated by Eq. (4.1) will also have a similar form. Because of the invariance of the rest of the Lagrangian under the transformations (2.1), we can choose it to be

$$M = m_1 + m_2\tau_3 + m_3 i\gamma_5, \qquad (4.2)$$

which gives two eigenmasses

$$\begin{aligned} m_p &\equiv [(m_1+m_2)^2 + m_3^2]^{\frac{1}{2}}, \\ m_n &\equiv [(m_1-m_2)^2 + m_3^2]^{\frac{1}{2}}. \end{aligned} \qquad (4.3)$$

The self-consistent self-energy equation to be solved is now

$$\begin{aligned} M = M^0 + g_0\{&\mathrm{Tr}S_F^{(M)}(0) - \gamma_5\tau_i \,\mathrm{Tr}[\gamma_5\tau_i S_F^{(M)}(0)] \\ &+ \tfrac{1}{5}\gamma_5 \,\mathrm{Tr}[\gamma_5 S_F^{(M)}(0)] - \tfrac{1}{5}\tau_i \,\mathrm{Tr}[\tau_i S_F^{(M)}(0)]\}. \end{aligned} \qquad (4.4)$$

Equating the respective coefficients of both sides, we get

$$\begin{aligned} m_1 &= m_1^0 + \bar{I}m_1 + \tilde{I}m_2, \\ m_2 &= m_2^0 - \tfrac{1}{5}\bar{I}m_2 - \tfrac{1}{5}\tilde{I}m_1, \\ m_3 &= m_3^0 - \tfrac{1}{5}\bar{I}m_3, \\ 0 &= m_4^0 + \bar{I}m_3, \end{aligned} \qquad (4.5)$$

where

$$\begin{aligned} \bar{I} &= \tfrac{1}{2}[I(m_p) + I(m_n)], \\ \tilde{I} &= \tfrac{1}{2}[I(m_p) - I(m_n)], \\ I(m) &= -\frac{8ig_0'}{(2\pi)^4}\int \frac{d^4p}{p^2+m^2}F(p,\Lambda). \end{aligned} \qquad (4.5')$$

We are interested in a small change of the non-trivial solution due to M^0. From Eq. (4.5) it is clear that $m_3 = 0$ unless

$$m_3^0 = -(\tfrac{1}{5} + \bar{I}^{-1})m_4^0 \neq 0.$$

The term m_3 implies a violation of time and space reflections. Since we are not interested in such a violation, we will assume $m_3 = m_3^0 = m_4^0 = 0$ from now on. We further note that

$$\bar{I} = I(m_1) + O[(m_2/m_1)^2], \quad \tilde{I} = O[m_2/m_1].$$

In fact, up to the first order in m_2/m_1, we may put

$$m_1 = m_1^0 + I(m_1)m_1, \qquad (4.6\text{a})$$

$$m_2 = m_2^0 - \tfrac{1}{5}[I(m_1) + 2I'(m_1)m_1^2]m_2, \qquad (4.6\text{b})$$

where

$$I'(m) = dI(m)/d(m^2) < 0.$$

Equation (4.6a) determines m_1 in terms of m_1^0.

The self-consistency condition required, for $m_1^0 = 0$, is that $I(m) = 1$. We may thus expand $I(m)$:

$$I(m_1) = 1 + I'(m)(m_1^2 - m^2),$$

and obtain

$$\Delta m^2 = m_1^2 - m^2 = -m_1^0/[mI'(m)]. \qquad (4.7)$$

Since $I'(m)$ is of the order of $-I(m)/m^2$ (see below), this means

$$\Delta m = m_1 - m \approx m_1^0. \qquad (4.8)$$

From (4.6b), then

$$\begin{aligned} m_2 &\approx m_2^0\{1 - \tfrac{1}{5}[1 + I'(m)m^2]\}^{-1} \\ &\approx m_2^0. \end{aligned} \qquad (4.8')$$

We note that originally there were two solutions $\pm|m|$, which now split into opposite directions according to Eq. (4.7) or (4.8). The meaning of this is as follows. Under the strict γ_5 invariance, there is a complete degeneracy with respect to the transformation (2.1c). The perturbation m_1^0 removes this degeneracy, so that the energy of the vacuum will depend on the orientation of the "γ_5 spin" of the negative energy fermions present in the "vacuum" with respect to this preferred direction. Obviously, the self-consistent procedure, which is similar to the variational method, gives the two extremum configurations corresponding to parallel ($m_0/m > 0$) or antiparallel ($m_0/m < 0$) γ_5-spin lineup. The parallel case has the larger "gap parameter" $|m|$ than the antiparallel case, so that the former will correspond to the stable ground state. The latter, on the other hand, should correspond to a metastable world.

It is perhaps interesting to see the general behavior of the self-consistency equation for arbitrary magnitude of m_1^0, assuming $m_2^0 = 0$ for simplicity. The relevant equation,

$$m[1 - I(m)] = m^0,$$

is plotted schematically in Fig. 1.

Note that the trivial branch of the solution, which goes through the origin, has $m_0/m < 0$. In other words, even in this case the self-consistent solution is qualitatively different from the simple perturbation result. As m^0 increases, it approaches the metastable nontrivial solution, and finally both go into the complex plane.

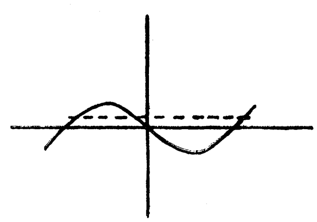

FIG. 1. The three self-consistent mass solutions m (ordinate) as a function of the bare mass m^0 (abscissa).

[9] The most general form of the self-energy Lagrangian (neglecting isospin dependence) is

$$\bar{\psi}[i\gamma\cdot p\,\Sigma_1(p^2) + \Sigma_2(p^2) + i\gamma\cdot p\gamma_5\Sigma_3(p^2) + i\gamma_5\Sigma_4(p^2)]\psi + \bar{\psi}^c[i\gamma\cdot p\,\Sigma_5(p^2) + \Sigma_6(p^2) + i\gamma\cdot p\gamma_5\Sigma_7(p^2) + i\gamma_5\Sigma_8(p^2)]\psi + \mathrm{H.c.}$$

We do not attempt to study such a problem at this place.

We now come to the meson problem. The pion mass will be determined from

$$\Gamma_j = ig_0'\tau_i \int \mathrm{Tr}[\tau_i S_F(p'+\tfrac{1}{2}q)\Gamma_j S_F(p'-\tfrac{1}{2}q)]d^4p', \quad (4.9)$$

but

$$\underset{\text{isospin}}{\mathrm{Tr}}[\tau_i S_F{}^{(M)}\tau_j S_F{}^{(M)}]$$

$$= \underset{\text{isospin}}{\mathrm{Tr}}\left[\tau_i\left(S_F{}^{(m_p)}\frac{1+\tau_3}{2}+S_F{}^{(m_n)}\frac{1-\tau_3}{2}\right)\times\tau_j\left(S_F{}^{(m_p)}\frac{1+\tau_3}{2}+S_F{}^{(m_n)}\frac{1-\tau_3}{2}\right)\right]$$

$$= 2\delta_{ij}\frac{S_F{}^{(m_p)}+S_F{}^{(m_n)}}{2}\frac{S_F{}^{(m_p)}+S_F{}^{(m_n)}}{2} + 2(2\delta_{i3}\delta_{j3}-\delta_{ij})\frac{S_F{}^{(m_p)}-S_F{}^{(m_n)}}{2}\frac{S_F{}^{(m_p)}-S_F{}^{(m_n)}}{2}.$$

The second term yields convergent results, and is $O[(\Delta m/m)^2]$. To the order $\Delta m/m$, therefore, only the first term is important; moreover,

$$(S_F{}^{(m_p)}+S_F{}^{(m_n)})/2 \approx S_F{}^{(m_1)}.$$

In other words, there will be no first-order mass splitting of the pion. The mass is then determined from

$$1 = J_{p1}(-\mu^2) = \frac{g_0'}{4\pi^2}\int_{4m_1^2}^{\Lambda^2}\frac{\kappa^2 d\kappa^2}{\kappa^2-\mu^2}\left(1-\frac{4m_1^2}{\kappa^2}\right)^{\frac{1}{2}}. \quad (4.10)$$

For $m_1^0=0$, we had originally

$$1 = J_p(0) = \frac{g_0'}{4\pi^2}\int_{4m^2}^{\Lambda^2}d\kappa^2\left(1-\frac{4m^2}{\kappa^2}\right)^{\frac{1}{2}},$$

which should now be replaced by

$$1 = \frac{m_1^0}{m_1}+\frac{g_0'}{4\pi^2}\int_{4m_1^2}^{\Lambda^2}d\kappa^2\left(1-\frac{4m_1^2}{\kappa^2}\right)^{\frac{1}{2}}, \quad (4.11)$$

according to Eq. (4.6a).

From Eqs. (4.10) and (4.11) follows

$$\frac{m_1^0}{m_1} = \mu^2\frac{g_0'}{4\pi^2}\int_{4m_1^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2-\mu^2}\left(1-\frac{4m_1^2}{\kappa^2}\right)^{\frac{1}{2}} \approx \mu^2\frac{g_0'}{4\pi^2}\int_{4m_1^2}^{\Lambda^2}\frac{d\kappa^2}{\kappa^2}\left(1-\frac{4m_1^2}{\kappa^2}\right)^{\frac{1}{2}} \lesssim \frac{\mu^2}{4m_1^2}\left(1-\frac{m_1^0}{m_1}\right). \quad (4.12)$$

For the observed value of $\mu^2/4m_1^2 \approx 1/200$ we then have, for the stable solution

$$m_1^0 \lesssim m_1/200 \approx 5\text{ Mev}. \quad (4.13)$$

The amount of bare mass needed to produce the pion mass is thus surprisingly small.

On the other hand, the metastable solution ($m_1^0/m<0$) produces an imaginary pion mass, indicating the unphysical nature of the solution.

The pion-nucleon coupling constant at the pion pole becomes [see Eq. (2.3)]

$$G_P^2/4\pi \approx g_0'[4\pi I_P(-\mu^2)]^{-1},$$

which is changed from the old one only by an order $\mu^2/m_1^2 \sim m_1^0/m_1$.

The other heavy meson states can be treated similarly. We see easily that the changes induced by m_1^0 are quite small: In general $\Delta\mu^2/m_1^2 = O(m_1^0/m_1)$ and $\Delta G^2/G^2 = O(m_1^0/m_1)$. Thus the effect of M^0 shows up dramatically only in the pion mass because it was originally zero.

Finally we remark that instead of a bare mass, we could assume slightly different coupling constants g_s and g_p ($<g_s$) for the scalar and pseudoscalar interaction terms in the Lagrangian (2.3). The nature of the solution is somewhat different from the previous case because the Lagrangian still retains the mass reversal invariance $\psi \rightarrow \gamma_5\psi$, and the solution is twofold degenerate ($\pm m$). The fractional change of the coupling necessary to produce the pion mass is again small: $|\Delta g/g| \approx \mu^2/4m_1^2$.

V. IMPLICATIONS OF THE MODEL

Let us now discuss the relevance of our present model to the physical realities of the nucleons and mesons.

1. We have seen that our Lagrangian (2.3) leads to the nucleon of isospin $\frac{1}{2}$ and the pion of isospin 1. The pion-nucleon coupling constant (pseudoscalar) depends on the cutoff parameter. For the observed large value (≈ 15) of $G_P^2/4\pi$, we see from Eqs. (1.5) and (2.3) that Λ must be of the same order of magnitude as the nucleon mass itself. This is not unreasonable, since the effective nucleon-nucleon interaction in higher approximations would proceed with the exchange of nucleon pairs.

A third parameter, the bare mass, enters our picture in order to make the meson mass finite. It would seem rather unsatisfactory and embarrassing that after all one has to break the postulated symmetry in an *ad hoc* manner. In order to clear up this point, the origin of the effective bare mass then becomes an interesting and important question. Since the required bare mass [Eq. (4.13)] seems to be quite small, a tempting possibility suggests itself that the bare nucleon field is the same as the electron-neutrino field. The electron mass itself could be either intrinsic or of electromagnetic origin.[10] Under this assumption, the bare mass operator would have the form $M^0=m_e^0(1+\tau_3)/2$, where the word "bare" is used relative to the interaction under consideration. According to the results of the previous section, it is only the isoscalar part of M^0 that produces the large shift of the pion mass, and the amount of violation of the isospin invariance will remain small.

2. Besides the pion, we have also derived vector mesons of both isoscalar ($T=0$) and isovector ($T=1$) types, and a scalar isoscalar meson which is actually unbound. No state corresponding to the isoscalar pion ($\pi_0{}^0$) is found. Of course, these results should depend sensitively on the choice of the interaction in the first place, and to a lesser extent also on the degree of approximation. At any rate, it seems to be a rather interesting and satisfactory feature of the model that these same vector mesons have been anticipated theoretically from various grounds,[11] even though there do not seem to be convincing experimental indications of their existence as yet.[12]

The mass values obtained here are rather high, and these mesons should actually decay into pions very quickly. The coupling constants are generally of the same order as the pion coupling constant, which means a very strong interaction for the vector and scalar mesons. These results, however, may be considerably altered in a better approximation. For one thing, the heavy mesons are coupled strongly to many-pion states which would make the former mere resonances of the latter. Moreover, the nucleon-nucleon and meson-meson interactions can go through long-range forces due to the exchange of these same mesons, which would in turn change the meson states themselves. These processes (the so-called left-hand cuts in the language of the dispersion theory) have not been taken into account in our ladder approximation.

This is a highly cooperative mechanism, and if one wants to handle it in a systematic way, one may be led to the same dispersion theoretical approach that is now widely pursued in pion physics. As a result of such effects, it is then conceivable that the masses of the vector mesons, for example, may come down.[13] Alternatively, it is also conceivable that we have more than one resonance having the same quantum numbers, of which we have obtained the higher ones. These high-energy poles may in turn determine the low-energy resonances.

In addition to the vector mesons, we expect a $T=0$, $J=0^+$ resonance, which has also been postulated by some people.[14] We should try to check these predictions against experimental evidence, such as the characteristic Q-value distributions and angular correlations in meson production processes.

Turning to the nucleon number 2 states, we expect two bound states ($T=0$, $J=1^+$ and $T=1$, $J=0^+$) with comparable masses to those for the vector mesons. This is a qualitatively satisfactory feature in view of the observed deuteron and the singlet virtual states, even though the actual binding is considerably weaker.[15]

3. As was already mentioned in I, our particular model was motivated by the approximate axial vector conservation observed in the nuclear β decay and the role of the pion in it.[5,16] The only difference from I is that (a) we now have the conservation of the isovector axial vector current $i\bar{\psi}\gamma_\mu\gamma_5\tau_i\psi$ instead of the simple axial vector current $i\bar{\psi}\gamma_\mu\gamma_5\psi$, and (b) a small violation of conservation is explicitly introduced. The general treatment of the problem will be completely analogous to the previous case.

Assuming that the β decay occurs through an additional term in the Lagrangian

$$L_\beta=g_\beta\bar{\psi}\gamma_\mu(1+\gamma_5)\tau_+\psi l_{\mu-}+\text{H.c.}\quad[\tau_+=\tfrac{1}{2}(\tau_1+i\tau_2)],$$

where l_μ refers to the lepton current, the nuclear β-

[10] The electromagnetic interaction is invariant under the simple γ_5 transformation, but not under the Gürsey transformation since it fundamentally distinguishes between the charged and neutral components. Thus there is a built-in violation which can eventually produce the pion mass.

[11] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1609 (1960); Y. Nambu, *ibid.* **106**, 1366 (1957); G. Chew, Phys. Rev. Letters **4**, 142 (1960); J. J. Sakurai, Ann. Phys. **11**, 1 (1960).

[12] J. A. Anderson, Vo X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Phys. Rev. Letters **6**, 365 (1961); A. Abashian, N. Booth, and K. M. Crowe, *ibid.* **5**, 258 (1960).

[13] A crude way to see the general tendency will be to argue as follows: The $T=1$ vector meson is coupled to the nucleon mainly through tensor coupling, so that it will cause a nucleon-antinucleon interaction of the type $-g^2(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)e^{-\mu r}/r$. This tends to raise $T=1$, $J=0^+$ and $T=0$, $J=1^-$ meson states, and lower $T=0$, $J=0^+$ and $T=1$, $J=1^-$ states. Any change in the binding force, however, will be offset by the corresponding change in the nucleon mass, which automatically adjusts the pion mass to lie where it should be. The exchange of the $T=0$, $J=1$, and $J=0$ mesons, therefore, would not be so important in determining the relative shift of the meson levels.

[14] J. Schwinger, Ann. Phys. **2**, 407 (1957); M. Gell-Mann and M. Lévy, Nuovo cimento **16**, 705 (1960); S. Gupta, Phys. Rev. **111**, 1436 (1958), Phys. Rev. Letters **2**, 124 (1959); M. H. Johnson and E. Teller, Phys. Rev. **98**, 783 (1955); H. P. Duerr and E. Teller, *ibid.* **103**, 469 (1956). The σ meson mass obtained here is independent of the cutoff Λ, so that there may be some point in arguing that it is more reliable than for the vector mesons. If so, we may expect a nucleon-antinucleon resonance near zero kinetic energy (taking account of the mass shift due to M^0). The width may be quite broad.

[15] In fact, both $T=0$ and $T=1$ vector meson exchanges work in the direction to reduce the binding relative to the nucleon-antinucleon case.

[16] S. Bludman, Nuovo cimento **9**, 433 (1958); F. Gürsey, Ann. Phys. **12**, 91 (1961); Y. Nambu, reference 1; M. Gell-Mann and M. Levy, reference 14; J. Bernstein, N. Gell-Mann, and L. Michel, Nuovo cimento **16**, 560 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, *ibid.* **17**, 757 (1960); Chou Kuang-Chao, J. Exptl. Theoret. Phys. (U.S.S.R.) **39**, 703 (1960) [Soviet Phys.—JETP **12**, 492 (1961)].

decay vertex becomes

$$\Gamma_\mu = g_\beta[i\gamma_\mu\tau_+ F_{V1}(q^2) - i\sigma_{\mu\nu}q_\nu\tau_+ F_{V2}(q^2) + \{i\gamma_\mu\gamma_5\tau_+ + [2m_1\gamma_5 q_\mu\tau_+/(q^2+\mu_\pi^2)]f(q^2)\}F_A(q^2)],$$

where q is the momentum change. In the ladder approximation, $F_{V1}(q^2)$ arises from the vector-type nucleon pairs, and $F_{V1}(0)=1$ (in accordance with the Ward identity, applicable to the isospin current, which shows that $F_{V1}(0)=1$ in general.[17])

In the axial vector part, $F_A(q^2)=1$ in our approximation. $f(q^2)$ arises because of the violation of the γ_5 invariance, but it deviates from 1 only to the order $m_1^0/m_1 \sim \mu^2/m_1^2$, as was already seen in the previous section. For practical purposes, therefore, the axial vector current has the desired form which would lead to the Goldberger-Treiman relation[18]

$$2m_1 g_A \approx \sqrt{2}G_\pi g_\pi,$$

where $g_A = g_\beta F_A(0)$ and G_π, g_π are, respectively, pion-nucleon and pion-lepton couplings.

In higher orders, however, $F_A(q^2)$ will be present, and in general $F_A(0)\neq 1$ even under the strict γ_5 invariance. People have conjectured in the past that $F_A(0)=g_A/g_V=1$ as $\mu_\pi\to 0$, but this does not seem to be easily guaranteed. The generalized Ward identity for the axial vector current[19] suffices to prove the Goldberger-Treiman relation, but is not enough to make $F_A(0)=1$. In order that the latter should come out rigorously, we would need a more subtle mechanism. Nevertheless, we can try a working hypothesis that $g_A/g_V=1$ under the strict invariance, and then estimate the deviation due to the violation. This scheme is carried out in the Appendix.

VI. FURTHER PROBLEM

We will consider here some of the general problems which have not been explored, but which seem to be important in a more comprehensive understanding of the elementary particles.

1. The hyperons. In order to incorporate the strange particles into our picture we would have to increase the dimensions of the fundamental field unless we do further unconventional things (see below). The simplest possibility from the point of view of quantum numbers would be to add a bare Λ-particle field as was originally proposed by Sakata.[20] We would then postulate, in addition, the generalized γ_5 symmetry, which would mean the invariance of the left-handed and right-handed components separately under the unitary transformation among the three fields or some subgroups of it. The mass splitting of the three baryons will be obtained from bare masses of similar magnitude, which destroys the otherwise rigorous symmetry.

This approach will produce easily the pions and K mesons and probably more, and their masses can again be related to the baryon bare masses. But we do not yet have a comparable dynamical method to predict Σ and Ξ particles. Consequently, we shall not be able to say whether or not the present model is dynamically satisfactory in this respect.

2. The leptons. In connection with the above model we are naturally led to the lepton problem. Gamba, Marshak, and Okubo[21] have pointed out an interesting parallelism between the $pn\Lambda$ and $\nu e\mu$ triplets. As was remarked in the beginning, our theory gives a special incentive for speculation about this relation because we have obtained two solutions: one ordinary and one extraordinary, differing in masses. Could they both be realized in nature simultaneously? According to our results in I, the answer is no because they belong to different Hilbert spaces. Moreover, the trivial solution gives rise to unphysical mesons at least under the assumption of fixed cutoff, with a large mass ($-\mu^2 \gtrsim \Lambda^2$) but not necessarily a weak coupling ($G^2 \lesssim \Lambda^4/\mu^4$). Nevertheless, it would seem too bad if Nature did not take advantage of the two solutions. A straightforward way to make the two solutions co-exist in the same world is obviously to postulate that the world is represented by the direct product of two Hilbert spaces[22]:

$$\mathcal{H} = \mathcal{H}^{(0)} \otimes \mathcal{H}^{(m)}, \tag{6.1}$$

built upon the vacuum state

$$\Omega = \Omega^{(0)} \otimes \Omega^{(m)}. \tag{6.1$'$}$$

It is true that this is effectively the same as doubling the fields, but here the choice of the two solutions (particles) is dictated by the dynamics of the original nonlinear theory. In order to describe this situation, we may adopt an *effective* Lagrangian

$$L = L^{(1)} + L^{(2)}, \tag{6.2}$$

where each of the $L^{(i)}$ has the same form, only differing in the charge assignments of the respective triplet fields. The Lagrangian obviously yields four subspaces

$$\mathcal{H}_1^{(0)}\otimes\mathcal{H}_2^{(m)},\quad \mathcal{H}_1^{(m)}\otimes\mathcal{H}_2^{(0)},\quad \mathcal{H}_1^{(0)}\otimes\mathcal{H}_2^{(0)}$$

and

$$\mathcal{H}_1^{(m)}\otimes\mathcal{H}_2^{(m)}.$$

According to our plan, we must say that we happen to live in the first subspace. [In the second space, the masses of $\nu e\mu$ and $pn\Lambda$ are interchanged, whereas in the third (fourth) case we have two kinds of leptons (baryons).]

[17] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[18] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 356 (1958).
[19] J. Bernstein *et al.*, reference 16.
[20] S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956).
[21] A. Gamba, R. E. Marshak, and S. Okubo, Proc. Natl. Acad. Sci. U. S. **45**, 881 (1959); Z. Maki, M. Nakagawa, Y. Ohnuki, and S. Sakata, Progr. Theoret. Phys. **23**, 1174 (1960).
[22] S. Okubo and R. E. Marshak, Nuovo cimento **19**, 1226 (1961), have independently proposed a similar idea. We thank the authors for valuable communications.

So far there is no interaction between leptons and baryons (except the electromagnetic, which is trivial). To introduce the weak interactions, we may, for example, add to Eq. (5.2) a third nonlinear term involving all the (left-handed) fields. This would complete our program of dealing with the strong and weak interactions.

But, of course, it is not yet a truly unified theory; the weak interaction is introduced only as an *ad hoc* additional process. Moreover, we do not know the mathematical consistency of such a procedure, because the additional interaction, if taken seriously, may qualitatively affect the baryon and lepton solutions we already have.

There is an alternative, but less drastic scheme; namely, to assume six different fields from the beginning, of which three (becoming eventually the baryon fields) have additional strong interactions in the Lagrangian. This may not be devoid of elegance if the interaction is mediated by a vector Bose field coupled to the baryon charge. The intermediate bosons, including the photons and possibly also the weak bosons, could then be interpreted as the agents that distinguish between different components of the bare fermions, which otherwise would enjoy a high degree of symmetry.

We would like to throw in another remark here that there may be also a possibility of utilizing the ordinary and extraordinary solutions in distinguishing between electron and muon, or baryons of different strangenesses.

3. The γ_5 invariance for general systems. In our theory the γ_5 invariance is a very essential element. It is a particular symmetry which exists in the Lagrangian, but is masked in reality because of the (approximate) degeneracy of the vacuum with respect to that symmetry. We have used the pion and the β decay in support of the assumption. In order to firmly establish its validity, however, we must try to find more evidences. For one thing, the induced pseudoscalar terms in nucleon β decay and μ capture should be examined more closely.

Furthermore, if such a symmetry is to have a general meaning, we must be able to consider partially conserved currents for processes such as

$$\begin{aligned} H^3 &\rightarrow He^3, \\ He^6 &\rightarrow Li^6, \\ C^{14} &\rightarrow N^{14}, \\ \Sigma^- &\rightarrow \Sigma^0, \\ \Sigma^- &\rightarrow \Lambda. \end{aligned} \tag{6.3}$$

An elementary definition of the γ_5 transformation for the general system is obvious: When the wave function of a system is expressed in terms of the fundamental (bare) spinors obeying the rules Eq. (2.1), the transformation is unambiguously defined for each component, and thereby the total axial vector current is determined.

For superallowed transitions with spin $\frac{1}{2}$, the problem is particularly simple, since it is the same as for the neutron case. Thus for $H^3 \rightarrow He^3$ we have the same Goldberger-Treiman relation

$$(M_H + M_{He}) g_A(\mathrm{H,He})/\sqrt{2} G_\pi(\mathrm{H,He}) \approx g_\pi, \tag{6.4}$$

where g_A, G_π now characterize the β decay and the (unknown) pion coupling for the transition under consideration.

Similar relations hold for the Σ decays.[23] For the Σ-Σ case, we have

$$2m_\Sigma g_A(\Sigma\Sigma)/G_\pi(\Sigma\Sigma) \approx g_\pi. \tag{6.5}$$

For the Σ-Λ case, the axial vector vertex becomes[24]

$$\begin{aligned} \Gamma_A \approx [i\gamma_\mu\gamma_5 + (m_\Sigma + m_\Lambda)\gamma_5 q_\mu/(q^2+\mu_\pi^2)] F_{1A}(q^2) \\ + i\gamma_5 \sigma_{\mu\nu} q_\nu F_{2A}(q^2) \\ (m_\Sigma + m_\Lambda) F_{1A}(0)/G_\pi(\Sigma\Lambda) \approx g_\pi, \end{aligned} \tag{6.6}$$

if the relative Σ-Λ parity is even. The vector current conservation is also violated because of the Σ-Λ mass difference, and it looks as though this would predict a corresponding scalar meson term. However, the analogy is rather superficial. Firstly, the violation disappears if $m_\Sigma = m_\Lambda$, in which case there would be no need for a scalar meson. The Σ-Λ mass difference itself might be due to the breakdown of the γ_5 symmetry. Secondly, it is an "unfavored" transition ($\Delta T = 1$), so that the vector part, corresponding to the off-diagonal element of the isospin current, should vanish in the ideal limit of strict isospin invariance and $q \rightarrow 0$. In other words, we expect

$$\Gamma_V \approx [q^2 i\gamma_\mu - (m_\Sigma - m_\Lambda) q_\mu] F_{1V}(q^2) + \sigma_{\mu\nu} q_\nu F_{2V}(q^2). \tag{6.7}$$

In case the Σ-Λ parity is odd,[25] the vector and axial vector parts will interchange their roles. The vector part, which now looks like the axial vector current, would have the form

$$\begin{aligned} \Gamma_V \approx [q^2 i\gamma_\mu\gamma_5 + (m_\Sigma + m_\Lambda)\gamma_5 q_\mu] F_{1V}(q^2) \\ + i\gamma_5 \sigma_{\mu\nu} q_\nu F_{2V}(q^2). \end{aligned} \tag{6.8}$$

The axial vector part can similarly be put in the form

$$\begin{aligned} [i\gamma_\mu - q_\mu(m_\Sigma - m_\Lambda)/(q^2+\mu_\pi^2) f(q^2)] F_{1A}(q^2) \\ + \sigma_{\mu\nu} q_\nu F_{2A}(q^2). \end{aligned} \tag{6.9}$$

But $f(q^2)$ need not be ≈ 1 if the Σ-Λ mass difference is also due to the violation of the γ_5 symmetry.

There are other processes for which the chirality conservation can be tested in a direct way. Although

[23] L. B. Okun', *Ann. Rev. Nuclear Sci.* **9**, 61 (1959); M. Gell-Mann, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 522.

[24] We have in this case three independent terms.

[25] See S. Barshay, Phys. Rev. Letters **1**, 97 (1958); Y. Nambu and J. J. Sakurai, *ibid.* **6**, 377 (1961).

extraordinary solutions are in general not eigenstates of chirality (even under strict γ_5 invariance), the conservation law should still apply to the expectation values of chirality. In fact, we can express the chirality conservation law $\langle X_i \rangle = \langle X_f \rangle$ for any reaction $i \to f$; for example

$$p+\pi \to p+\pi, \quad p+\pi+\pi', \text{ etc.,}$$
$$p+p \to p+p, \quad p+p+\pi, \text{ etc.,}$$

as a relation between the change of nucleon chirality and the magnitude of the pion production amplitude.

The ideas outlined in this section will be taken up in more detail elsewhere.

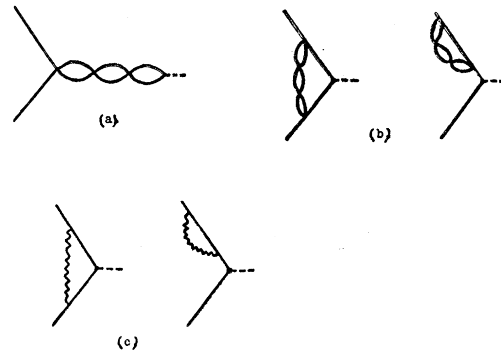

FIG. 2. Typical graphs considered in the evaluation of the axial vector vertex.

APPENDIX

We calculate here the renormalization of the axial vector (Gamow-Teller) coupling constant g_A for nuclear β decay under the following assumptions:

(1) Under strict γ_5 invariance (Gürsey type), there is no renormalization, namely $g_A = g_{A0}$ $(=g_{V0}=g_V)$, where g_{A0} is the bare coupling constant.

(2) The violation of the invariance gives rise to the finite pion mass as well as the deviation of the ratio $R = g_A/g_{A0} = g_A/g_V$ from unity, so that there is a functional relation between the two quantities.

Let us first consider the isovector axial vector vertex Γ_A in the usual perturbation theory. In our model, it consists of various graphs, some of which are shown in Fig. 2(a) and (b). The "ladder" graphs 2(a) have been considered in I as well as in the present paper, since they are intimately related to the γ_5 gauge transformation. In I (Appendix) we found that $R>1$ when both pseudoscalar and pseudovector type interactions are present.[26] The graphs 2(b) have not been considered yet. These will come into our consideration as soon as we take corresponding higher-order approximations for the self-energy, which was briefly discussed in I. The chain of bubbles in these graphs will act like a meson when there is such a dynamical pole [Fig. 2(c)].

The (divergent) renormalization effect due to intermediate mesons is always negative,[27] irrespective of the type of the meson, so that the effect of these meson-like bubble graphs is also expected to be similar. When the chain does not produce a pole, however, the effect can be opposite.

Combining all these effects, we have no way to predict the resultant magnitude and sign of the renormalization correction. So we simply assume these contributions to cancel out under strict γ_5 invariance.

Next let us suppose that the invariance is slightly violated. This will cause changes in the propagators in all these graphs. Most of these changes are, however, quite small, being of the order of $m^0/m \approx \mu^2/4m^2$, as will be clear from the results of Sec. IV. The largest effect is naturally expected to come from the "pion" contribution in Fig. 2(b), as this is a change from zero mass (infinite range) to a finite one.

Let us accordingly take the effective pion graph from Fig. 2(c) with an arbitrary pion mass μ. Call its contribution to the vertex renormalization (for zero momentum transfer) $\Lambda(\mu)$. Then according to the above assumption

$$\begin{aligned} R &= \Gamma_A(\mu_\pi)/g_A = \Gamma_A(\mu_\pi)/\Gamma_A(0) \\ &\approx 1 + \Lambda(\mu_\pi) - \Lambda(0). \end{aligned} \tag{A1}$$

The difference $\Lambda(\mu) - \Lambda(0)$ is convergent, which turns out to be

$$\Lambda(\mu)-\Lambda(0)=\frac{G^2}{16\pi^2}\frac{\mu^2}{m^2}\left\{\left(3-\frac{5\mu^2}{2m^2}\right)\ln\frac{m^2}{\mu^2}-5 + \frac{16\mu}{\sqrt{3}m}\left[\tan^{-1}\left(\frac{4m^2}{3\mu^2}-\frac{2}{3}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]\right\}, \tag{A2}$$

where G is the phenomenological pion coupling constant.

As was expected, this goes like $(\mu^2/m^2)\ln(m^2/\mu^2)$ for small μ, which is more important than the contributions from the neglected processes behaving like μ^2/m^2.

With $(G^2/4\pi)(\mu^2/4m^2) = f^2/4\pi = 0.08$, Eq. (A2) gives

$$R-1=\begin{cases} 0.18 \\ 0.24. \end{cases} \tag{A3}$$

The first figure is the entire contribution from Eq. (A2), while the second is the contribution from the leading logarithmic term alone. Experimentally, R is estimated to be ≈ 1.25.[28]

[26] See also Z. Maki, Progr. Theoret. Phys. (Kyoto) **22**, 62 (1959).

[27] We owe Dr. J. de Swart the mathematical check on this point.

[28] M. T. Burgy, V. E. Krohn, T. B. Novey, G. R. Ringo, and V. L. Telegdi, Phys. Rev. **120**, 1829 (1961).