Calculable Model for Compound Nucleus-Direct Interaction Interference*

LEONARD S. RODBERG†
University of Maryland, College Park, Maryland
(Received May 31, 1961)

The formation of the compound nucleus is described. The importance of two-particle excited states is stressed and the possibility of experimentally observing the interference between the associated "two-particle resonances" and direct-interaction processes is discussed. Formulas are presented which permit the calculation of both the direct-interaction term and the amplitude associated with the resonance.

THE concept of the compound nucleus has been used for several decades to explain the existence of resonances in the scattering of nucleons by nuclei, and a mathematical formalism which permits phenomenological descriptions of the experiments has been developed.[1] More recently the direct-interaction theory has been used to explain some prominent features of medium-energy reactions.[2] While formal theories have been developed which contain both possibilities and which show some features of their interrelation,[3] no one has discussed in detail the compound nucleus–direct interaction interference, nor has any calculation of this interference been performed. In this note we wish to discuss a simple model which demonstrates the connection between these two concepts and which permits detailed calculations of the cross section for inelastic scattering in the interference region.

FIG. 1

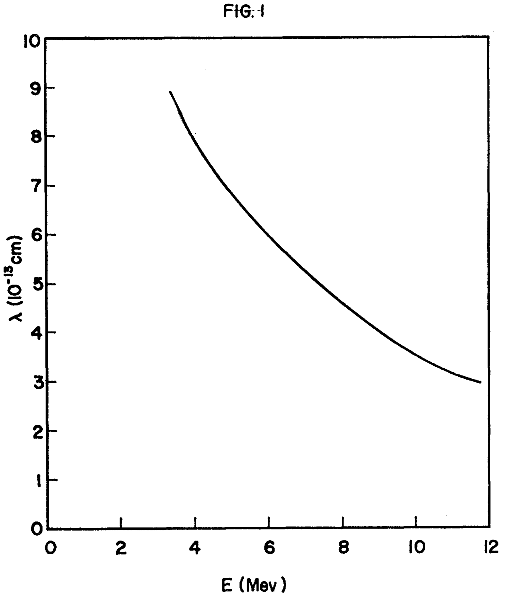

FIG. 1. Mean-free path of a nucleon in nuclear matter, as determined from the measured optical potentials (see *Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies, No. 32,* edited by A. E. S. Green, C. E. Porter, and D. S. Saxon (The Florida State University, Tallahassee, Florida, 1959).

With the successful application of the optical model and the shell model to nuclear physics, the picture of the formation of a compound nucleus has undergone serious revisions. The long mean-free path of a nucleon in nuclear matter implied by the observed depth of the optical potential (Fig. 1) contradicts the older strong-interaction picture. In the newer picture a nucleon entering a nucleus may completely traverse the nucleus without suffering a "hard" collision (one in which kinetic energy is transferred). Even if the nucleon loses no energy during this traversal, it may not escape. There is a finite probability that it will be reflected at the boundary of the nucleus. This probability increases rapidly as the incident kinetic energy is reduced to zero. The resulting multiple reflections at the surface require the nucleon to traverse a long path length within the nucleus so that it may have sufficient opportunity to transfer energy to the nucleus. After one or two hard collisions the nucleon is likely to have too little energy to escape and the system will proceed into the "chaotic" state envisioned by the older model.

Let us consider, for illustrative purposes, a process in which a neutron is inelastically scattered by a target consisting of a closed-shell core plus one nucleon.[4] The incident neutron may collide with this outer nucleon and immediately emerge from the target; this is a direct-interaction process. At the other extreme, it may be captured into a long-lived state which subsequently decays yielding a nucleon; this is a compound-nucleus process. According to the picture described above, the simplest compound state is a "two-particle excited state"[5] formed after just one hard collision (for instance, a collision with the outer nucleon in which this nucleon is excited while the incident neutron loses energy and is captured). The formation of a two-particle

* This research was supported in part by the U. S. Atomic Energy Commission under contract with the University of Maryland.

† Present address: U. S. Disarmament Administration, Department of State, Washington 25, D. C.

[1] A. M. Lane and R. G. Thomas, Revs. Modern Phys. **30**, 257 (1958) and references therein.

[2] L. S. Rodberg, Ann. Phys. **9**, 373 (1960) and references therein.

[3] C. Bloch, Nuclear Phys. **4**, 503 (1957), G. E. Brown, Revs. Modern Phys. **31**, 893 (1959), and references therein.

[4] In actually considering experimental observations of the processes described, all charge combinations should be considered. Since the Coulomb barrier inhibits proton emission, it acts as a "filter" which reduces the relative magnitude of the compound-nucleus contribution. Hence combinations of reactions such as (n,n'), (n,p), etc., will allow various processes to be distinguished.

[5] These states have been discussed previously by K. A. Brueckner, R. J. Eden, and N. C. Francis, Phys. Rev. **100**, 891 (1955) and G. L. Shaw, Ann. Phys. **8**, 509 (1959).

excited state leads to a broad resonance which we may term a "two-particle resonance." If this state decays back into the entrance channel, with the target left in its ground state or in an excited state, this resonance process can interfere with the direct interaction process. Alternatively one of the particles may collide with a third nucleon within the nucleus. These further collisions will lead to more complicated excitations having long lifetimes and contributing a large part of the total reaction cross section. On the other hand, only the initial two-particle excitation will have a sufficiently short lifetime to interfere strongly with the direct interaction process. Our central thesis is that reactions proceeding through such two-particle excited states will interfere strongly with direct interaction processes and further, that their contribution can be explicitly calculated.

The interference will be observed in those inelastic reactions which require just one hard collision within the target. In such reactions the target is left in a state whose predominant configuration is a single-particle excitation. The experimental distinction between the two reaction modes appears in the angular distribution, since the direct-interaction contribution will usually have the characteristic forward peak, while the compound-nucleus contribution from a single level will be symmetric about 90°. Both the angular distribution and the polarization of the outgoing nucleon will be sensitive to the interference between these two reaction modes.

From the picture of compound-nucleus formation discussed above, it follows that two-particle excited states will make a significant contribution to inelastic scattering when the diameter of the target is comparable with the mean-free path, and the energy is high enough to reduce reflection at the surface. On the other hand, the energy should not be so high that the nearby compound states are too broad and short-lived. These requirements are of course closely related, since it is the surface reflection which gives the intermediate states their finite lifetime. We have estimated that for $A<100$ and $E\lesssim 6$ Mev, the widths of two-particle resonances will be ~ 250 kev, while their mean separation will be ~ 1 Mev. Each of these parameters will show strong variations with mass number due to shell model effects.

The ground-state configuration of the target nucleus should be in the neighborhood of a closed shell so that few-particle excitations will predominate at low energies. In general the wave function of each intermediate state may be expanded in terms of a complete set of shell-model wave functions. The interference between the direct-interaction mode and the contribution of a given intermediate state will be proportional to the probability of finding two-particle excited state configurations in the intermediate state. Thus interesting information about shell-model configurations can be obtained if the magnitude of the interference term can be determined.

We now want to exhibit formulas which will permit the calculation of the scattering amplitude in the presence of both reaction modes. This is most easily done in the direct-interaction formalism. We introduce the initial and final wave functions of the target, denoted by ϕ_a and ϕ_b, and the distorted-wave functions, $\eta_i^{(+)}$ and $\eta_f^{(-)}$, representing motion of the incident and outgoing nucleons in the average field of the target. These are usually taken to be solutions of the Schrödinger equation in a single-particle optical potential. In an approximation which neglects multiple inelastic collisions, the amplitude for inelastic scattering is

$$M_{fb,ia}=-(2M/4\pi\hbar^2)T_{fb,ia},$$

where[2]

$$T_{fb,ia}=(\eta_f^{(-)}\phi_b|\sum_{\alpha=1}^{A}t_\alpha|\eta_i^{(+)}\phi_a). \qquad (1)$$

The transition operator t_α represents the effective interaction between the incident particle (denoted by 0) and target nucleon α in the presence of the remainder of the target. It satisfies the integral equation

$$t_\alpha=v_\alpha+v_\alpha(1/E-H+i\epsilon)t_\alpha, \qquad (2)$$

where $H=H_T+T_0+U_0$, H_T is the Hamiltonian of the target, T_0 is the kinetic energy of 0, U_0 is the optical potential for 0, and v_α is the potential acting between 0 and α. In the single-particle or shell model, H_T is replaced by $T_\alpha+U_\alpha$, the Hamiltonian for particle α, while the total energy E is replaced by the sum of the energies of 0 and α in the initial state. To facilitate the discussion we shall assume that this model is valid.

The second term of Eq. (2) contains a sum over the eigenstates of H. These represent intermediate states which are reached after a single collision between the incident neutron and target nucleon α. (Note that energy need not be conserved in this collision.) The low-energy eigenstates will be characteristic of the target nucleus; for instance, they will depend on the radius, strength, and spin-dependence of the single-particle potential, as well as on the multiplet structure of the levels of the target. Low-energy eigenstates of H which are bound or which represent low-lying virtual levels will give discrete or nearly discrete contributions to the sum over intermediate states. They can therefore lead to resonant enhancement of the cross section at energies corresponding to the positions of these levels. The high-energy intermediate states will be nearly independent of the target nucleus and may be approximated by free-particle eigenstates. We observed earlier that in a time-dependent description the direct-interaction and compound-nucleus contributions were distinguished by the time delay inherent in the latter mode. In the present time-independent description the analogous distinction is that high-energy intermediate states in Eq. (2) contribute to the direct interaction mode while low-energy states contribute to compound nucleus processes.

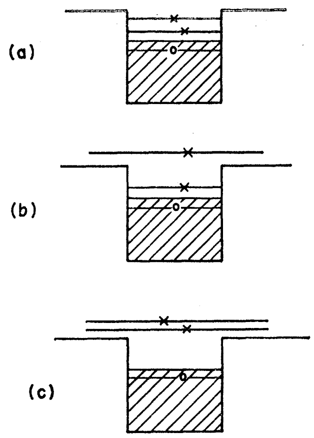

FIG. 2. Possible configurations of two-particle excited states in the shell model. × denotes a filled state, o denotes an empty state, and the cross-hatching denotes filled states occupied by all other particles. (a) is a discrete eigenstate of H, while (b) and (c) are continuum eigenstates.

To formulate this mathematically, we define a projection operator P_d which selects the discrete eigenstates of H [Fig. 2(a)]. The operator $P_c=1-P_d$ selects the continuum eigenstates [Fig. 2(b), (c)]. We may define a transition operator which contains only the continuum intermediate states by

$$t_\alpha^{(c)}=v_\alpha+v_\alpha\left(\frac{P_c}{E-H+i\epsilon}\right)t_\alpha^{(c)}. \tag{3}$$

Since only high-energy intermediate states are included in $t_\alpha^{(c)}$, it may usually be represented by a short-range "pseudo-potential" with parameters chosen to reproduce scattering data at an appropriate energy (this is the "impulse approximation"). Some algebra now shows that v_α may be eliminated from Eq. (2) to give

$$t_\alpha=t_\alpha^{(c)}+t_\alpha^{(c)}\frac{P_d}{E-H-P_d t_\alpha^{(c)}P_d+i\epsilon}t_\alpha^{(c)}. \tag{4}$$

This result is independent of the definition of P_d, but our choice permits us to give a simple interpretation to each part of Eq. (4). The first term is just the direct interaction contribution since it contains the high-energy, short-lived, intermediate states. The second term, containing only discrete intermediate states, is the compound nucleus contribution.[6] Since this contribution is obtained by the action of the two-body[7] operator $t_\alpha^{(c)}$ upon the initial state, it is proportional to the admixture of two-particle excited state in each intermediate state. More complicated excitations are included in the higher-order corrections to Eq. (1), but are not expected to interfere with $t_\alpha^{(c)}$.

[6] Of course the distinction between compound nucleus and direct interaction contributions is not this sharp. A practical requirement is that the direct interaction contribution be only weakly dependent on energy. Thus it may be desirable to include the low-lying continuum states in P_d in order to insure that $t_\alpha^{(c)}$ contains no resonant effects.

[7] Strictly speaking, $t_\alpha^{(c)}$ is a many-body operator since H acts on the coordinates of all particles. Only in the independent particle model does $t_\alpha^{(c)}$ become a true two-body operator.

If the eigenstates of H are denoted by χ_n (these are simply linear combinations of products $\phi_k\eta_j^{(+)}$ having well-defined angular momenta), the transition amplitude may be written[8]

$$T_{fb,ia}=\sum_{\alpha=1}^{A}\Bigg[(\eta_f^{(-)}\phi_b|t_\alpha^{(c)}|\eta_i^{(+)}\phi_a) + \sum_{n\,(\text{discrete})}\frac{(\eta_f^{(-)}\phi_b|t_\alpha^{(c)}|\chi_n)(\chi_n|t_\alpha^{(c)}|\eta_i^{(+)}\phi_a)}{E-E_n+\frac{1}{2}i\Gamma_n+i\epsilon}\Bigg]. \tag{5}$$

The complex "energy" is given by

$$E_n-\tfrac{1}{2}i\Gamma_n=(\chi_n|H+t_\alpha^{(c)}|\chi_n). \tag{6}$$

This is the Brueckner approximation to the energy.[9] Although the Hamiltonian H contains no interaction between 0 and α, the expectation value of $t_\alpha^{(c)}$ gives the level shift due to that interaction. Hence the real part E_n, which gives the position of the resonance, is the energy eigenvalue of the state χ_n. The imaginary part $\Gamma_n/2$, which gives the width of the resonance, can be related to the decay modes of the state χ_n. The imaginary parts of U_0 and U_α correspond to the finite lifetimes of excited single-particle states, whether bound or unbound, as a result of collisions with the remainder of the target. (In our case these collisions will lead to three-particle excited states.) The imaginary part of $t_\alpha^{(c)}$ corresponds to the rescattering of 0 and α leading, for instance, to one of the continuum levels [Fig. 2(b), (c)].

Virtual levels very near zero energy may lead to striking resonance effects, and should be included in the states selected by P_d if they are present.[6] The contributions of such levels may be calculated if one approximates χ_n by a bound state wave function and includes in the total width the partial width due to leakage across the potential barrier.

Valuable information on reaction mechanisms and level structure will be obtained from inelastic scattering experiments in the low-energy low-mass region. The angular distributions for reactions leading to distinct final states should be studied both experimentally and theoretically as a function of the incident energy. In addition, there are many related questions (such as the relation of collective effects to the model described here) which require further examination.

ACKNOWLEDGMENTS

It is a pleasure to acknowledge useful discussions with Dr. Wm. M. MacDonald and Dr. James E. Young, and to thank Mr. Leonard Garside for performing some preliminary calculations.

[8] This assumes that $t_\alpha^{(c)}$ does not mix eigenstates of H; this approximation may be easily removed if one desires to include several coupled levels.

[9] K. A. Brueckner, C. A. Levinson, and H. Mahmoud, Phys. Rev. **95**, 217 (1954); L. S. Rodberg, Ann. Phys. **2**, 199 (1957).