Scattering of μ Mesons from the Nuclei of Bromine and Silver*

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The μ-meson elastic scattering cross section and polarization asymmetry factor are calculated for bromine for positive and negative muons, and for silver for negative muons, for the values of $v/c=0.4$, 0.6, and 0.7. These results, when combined with the ones available for cadmium, permit the comparison of theory with measurement of the scattering of muons in emulsions. The nuclei are taken to be extended, with a charge distribution as derived from electron scattering, the Dirac equation is assumed valid, and the calculation is done with the aid of an IBM-650 computer utilizing the same program as used previously for the calculations on cadmium and mercury.

I. INTRODUCTION

THE recent accurate determination[1] of the magnetic moment of the μ meson provides strong evidence that the muons[2] have the same properties as electrons, with the exception of a heavier mass and the possibility of beta decay through the coupling to the weak interaction. In some experiments cosmic-ray μ mesons appeared to have an abnormally large scattering cross section,[3] but recent laboratory experiments[4] on high-energy muons gave results consistent with theory, the calculations being based on the assumption that muons are heavy electrons. In order to increase the accuracy in the comparison of muons to electrons, elastic scattering experiments of nearly relativistic μ mesons on several nuclei have been conducted by various groups.[5,6] Measurements of the elastic scattering of muons on the nuclei of nuclear emulsions are presently being undertaken by a group at Cornell University,[6] and it is the purpose of the present note to provide the corresponding theoretical values of the cross sections in the range[7] of v/c between 0.4 and 0.7. The present results include values for the polarization asymmetry factor[8] S, and extend results previously available,[9,10] for $Z=48$ and 80. In Sec. II the values for the differential cross section σ and of S are listed for the following cases: for the nucleus of bromine ($Z=35$), v/c has the values 0.2, 0.4, 0.6, and 0.7, and both μ^+ and μ^- are considered; for silver ($Z=47$) only μ^- are considered for $v/c=0.4$, 0.6, and 0.7. In Sec. III the dependence of the cross section on Z and on v/c is examined with the purpose of exploring the possibility of extrapolation in these parameters.

The comparison of the scattering cross section of muons to that of electrons incident on the same target nucleus will be of value, not only in seeing the relation of the intrinsic properties of muons to those of electrons, but also in the determination of the charge distribution of the target nucleus. In the calculation of electron and muon scattering cross sections several effects have up to now been neglected, such as[11] the effect of nuclear excitation on the elastic scattering, the deformation from sphericity occurring during the scattering, the effect of neutrons in the nucleus, etc. These effects will produce corrections on the scattering cross sections which are expected to be of different magnitude for μ^+, μ^-, e^+, and e^-. Consequently, comparison of the measurements of all four cross sections may show the presence of the effects mentioned above.

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[1] G. Charpak, F. J. M. Farley, R. L. Garwin, T. Mueller, J. C. Sens, V. L. Telegdi, and A. Zichichi, Phys. Rev. Letters **6**, 128 (1961). This paper reports on a $g-2$ experiment and contains references to earlier determinations of g.

[2] For a summary of μ-meson properties see G. N. Fowler and A. W. Wolfendale, *Progress in Elementary Particle and Cosmic-Ray Physics* (North Holland Publishing Company, Amsterdam, 1958), Vol. 4, p. 123.

[3] R. L. Sen Gupta, S. Gosh, A. Acharya, M. M. Biswas, and K. K. Roy, Nuovo cimento **19**, 245 (1961); R. Burnstein, T. Kitamura, D. D. Millar, Nuclear Phys. **19**, 665 (1960); A. I. Alikhanyan, *Proceedings of the Moscow Cosmic-Ray Conference, 1959* (International Union of Pure and Applied Physics, Moscow, 1960), English ed., Vol. 1, p. 327; J. L. Lloyd and A. W. Wolfendale, Phys. Rev. **117**, 247 (1960). No anomaly is reported by S. Fukui, T. Kitamura, and Y. Watase, Phys. Rev. **113**, 315 (1959); E. Amaldi and G. Fidecaro, Nuovo cimento **7**, 535 (1950). An experiment on $\mu-e$ collisions by R. F. Deery and S. H. Neddermeyer, Phys. Rev. **121**, 1803 (1961) also is consistent with the picture of the muon being a heavy electron.

[4] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams, Phys. Rev. **122**, 937 (1961).

[5] B. G. Chidley *et al.*, Can. J. Phys. **36**, 801 (1958).

[6] P. L. Connolly, J. G. McEwan, and J. Orear, Phys. Rev. Letters **6**, 554 (1961).

[7] The velocity of the muon is v and the ratio v/c is denoted by β, where c is the velocity of light. For a muon rest mass of 105.7 Mev, the values of β corresponding to kinetic energies of 2.179, 9.628, 26.425, and 42.309 Mev are 0.2, 0.4, 0.6, and 0.7, respectively.

[8] For an incident beam transversely polarized in the "up" direction, the polarization asymmetry factor S is given in terms of the scattering cross section to the "left" (L) and that to the right (R) by the relation $S(\theta)=(L-R)/(L+R)$. A more complete definition in terms of scattering amplitudes can be found for instance after Eq. (17A) of the paper listed in reference 10.

[9] J. Franklin, B. Margolis, and H. Oberthal, Phys. Rev. **111**, 296 (1958) contains values for the cross section σ and for S for the nuclei of cadmium and mercury, for μ^- at $\beta=0.2$ and 0.4. The charge distributions of the nuclei are assumed uniform in this calculation.

[10] G. H. Rawitscher, Phys. Rev. **112**, 1274 (1958), lists values of σ and S for the following cases: μ^-, $\beta=0.6$, $Z=48$, 80, and 92; μ^+, $\beta=0.4$ and 0.6, $Z=48$, 80, and 92; μ^+, $\beta=0.8$, $Z=48$ and 80. The nuclear charge distribution assumed for these nuclei is described in reference 14.

[11] The author is indebted to Professor G. Breit for an interesting conversation concerning these corrections. Compare also with G. Breit, G. B. Arfken, and W. W. Clendenin, Phys. Rev. **78**, 391 (1950) concerning effect of nuclear excitation on polarization.

TABLE I. σ, $Z=35$. Differential scattering cross section σ, in 10^{-26} cm^2 per sr calculated for the extended[14] nucleus of bromine; β denotes v/c[7]. The quantity in parentheses denotes the power of 10 by which each entry has to be multiplied.

θ (deg)	$\beta=0.2$ μ^-	$\beta=0.2$ μ^+	$\beta=0.4$ μ^-	$\beta=0.4$ μ^+	$\beta=0.6$ μ^-	$\beta=0.6$ μ^+	$\beta=0.7$ μ^-	$\beta=0.7$ μ^+
30	8.16(3)	7.54(3)	4.23(2)	4.01(2)	5.16(1)	5.31(1)	1.80(1)	1.93(1)
45	1.76(3)	1.57(3)	7.55(1)	8.12(1)	7.03	9.22	1.87	2.75
60	5.65(2)	5.37(2)	1.98(1)	2.67(1)	1.28	2.49	2.27(−1)	5.74
75	2.21(2)	2.44(2)	6.27	1.16(1)	2.55(−1)	8.68(−1)	2.62(−2)	1.46(−1)
90	9.97(1)	1.33(2)	2.24	6.10	5.18(−2)	3.61(−1)	3.44(−3)	4.30(−2)
105	5.08(1)	8.40(1)	8.77(−1)	3.69	1.11(−2)	1.74(−1)	1.28(−3)	1.43(−2)
120	2.94(1)	5.90(1)	3.82(−1)	2.50	3.34(−3)	9.55(−2)	8.44(−4)	5.44(−3)
135	1.96(1)	4.55(1)	1.92(−1)	1.87	1.91(−3)	5.98(−2)	5.29(−4)	2.41(−3)
150	1.49(1)	3.80(1)	1.16(−1)	1.53	1.58(−3)	4.27(−2)	3.26(−4)	1.28(−3)
165	1.28(1)	3.43(1)	8.70(−2)	1.36	1.45(−3)	3.48(−2)	2.24(−4)	8.57(−4)

TABLE II. $100S$, $Z=35$. Polarization asymmetry factor[8] $100S$ calculated for the extended[14] nucleus of bromine; β denotes v/c.[7]

θ (deg)	$\beta=0.2$ μ^-	$\beta=0.2$ μ^+	$\beta=0.4$ μ^-	$\beta=0.4$ μ^+	$\beta=0.6$ μ^-	$\beta=0.6$ μ^+	$\beta=0.7$ μ^-	$\beta=0.7$ μ^+
30	0.176	0.040	−0.198	0.202	−0.307	0.187	−0.240	0.146
45	−0.098	0.072	−0.738	0.364	−0.667	0.316	−0.494	−0.010
60	−0.871	0.097	−1.47	0.506	−0.986	0.269	−0.618	−0.491
75	−2.08	0.115	−2.35	0.601	−1.18	0.032	−0.989	−1.46
90	−3.63	0.122	−3.43	0.636	−1.60	−0.360	−5.01	−2.91
105	−5.29	0.119	−4.85	0.613	−4.20	−0.819	−10.28	−4·66
120	−6.58	0.107	−6.71	0.540	−12.2	−1.19	−9.25	−6.24
135	−6.82	0.087	−8.47	0.431	−17.8	−1.32	−7.34	−6.98
150	−5.61	0.058	−8.54	0.301	−15.2	−1.15	−5.29	−6.31
165	−3.15	0.032	−5.49	0.152	−8.42	−0.668	−2.80	−3.79

II. RESULTS

The assumptions and methods of calculation are the same as those used in our previous work.[10,12] The muon obeys the Dirac equation, the nucleus has a static spherical charge distribution of a Woods-Saxon form, also called "Fermi" in the literature, with parameters taken from the electron scattering work.[13] Nuclear recoil is neglected, as well as the effects mentioned in the introduction. The calculation is numerical, the IBM-650 program is the same as used for an earlier calculation.[10] The accuracy of the polarization asymmetry[8] factor S is estimated to be better than 10% for the $\beta=0.7$ cases, and improves rapidly with decreasing energy. The cross sections are accurate to about 0.1% for all energies, but for simplicity are listed only to 3 significant figures. The accuracy estimates are based on considerations similar to the ones done in connection to a previous electron-positron comparison.[12] The results are given in Tables I to IV. (See reference 14.)

In order to determine the sensitivity with which the cross section depends on the diffuseness of the nuclear surface, μ^+ and μ^- cross sections were calculated at $v/c=0.6$ for the nucleus of $Z=48$ for which the surface thickness parameter[13] t was taken equal to 2.64 fermis, which is 10% larger than the value assumed in a previous calculation.[10] The resulting μ^+ cross section is smaller by about 1% at the scattering angle θ of 45°,

TABLE III. $100S_0$ and σ_0, $Z=35$, for point nucleus. Polarization asymmetry factor[8] $100S_0$ and differential scattering cross section σ_0 (in 10^{-26} cm^2 per sr) calculated for the nucleus of bromine for which the charge is concentrated at one point; β denotes v/c.[7] The quantity in parentheses behind an entry denotes the power of ten by which that entry has to be multiplied.

θ (deg)		$\beta=0.2$ μ^-	$\beta=0.4$ μ^+	$\beta=0.6$ μ^-	$\beta=0.6$ μ^+
30	$100S_0$	0.327	0.276	−0.495	0.353
	σ_0	7.65(3)	4.01(2)	6.85(1)	5.79(1)
45	$100S_0$	0.028	0.410	−1.84	0.860
	σ_0	1.63(3)	8.26(1)	1.46(1)	1.16(1)
60	$100S_0$	−1.36	0.624	−3.90	1.41
	σ_0	5.70(2)	2.79(1)	5.01	3.83
75	$100S_0$	−3.46	0.806	−6.40	1.94
	σ_0	2.64(2)	1.25(1)	2.22	1.67
90	$100S_0$	−5.53	0.929	−8.94	2.38
	σ_0	1.47(2)	6.77	1.16	8.79(−1)
105	$100S_0$	−6.96	0.970	−11.0	2.63
	σ_0	9.46(1)	4.22	6.90(−1)	5.34(−1)
120	$100S_0$	−7.35	0.922	−12.1	2.61
	σ_0	6.77(1)	2.94	4.53(−1)	3.63(−1)
135	$100S_0$	−6.65	0.784	−11.7	2.31
	σ_0	5.30(1)	2.26	3.26(−1)	2.72(−1)
150	$100S_0$	−5.00	0.571	−9.34	1.73
	σ_0	4.49(1)	1.88	2.57(−1)	2.23(−1)
165	$100S_0$	−2.67	0.300	−5.22	0.927
	σ_0	4.08(1)	1.69	2.22(−1)	1.98(−1)

[12] G. H. Rawitscher and C. R. Fischer, Phys. Rev. **122**, 1330 (1961).

[13] R. Herman and R. Hofstadter, *High-Energy Electron Scattering Tables* (Stanford University Press, Stanford, California, 1960).

[14] The nuclear charge distribution is assumed to be of the Woods-Saxon form (also called "Fermi" in the literature). The two parameters which specify this distribution are the surface thickness t and r_1, the half-density radius divided by $A^{\frac{1}{3}}$. Reference 13 contains a complete discussion of the parameters. For the present calculations, as well as the ones listed in reference 10, t and r_1 are taken equal to be 2.40 and 1.08 fermis, respectively.

TABLE IV. 100*S* and σ for μ^- scattering on the extended[14] nucleus of silver ($Z=47$). The notation is the same as in Tables I and II.

θ (deg)		β=0.4	β=0.6	β=0.7
30	100*S*	−0.206	−0.405	−0.385
	σ	8.15(2)	9.56(1)	3.20(1)
45	100*S*	−0.859	−0.929	−0.782
	σ	1.45(2)	1.20(1)	2.89
60	100*S*	−1.78	−1.53	−1.39
	σ	3.59(1)	1.92	2.91(−1)
75	100*S*	−2.90	−2.39	−3.79
	σ	1.05(1)	3.30(−1)	3.28(−2)
90	100*S*	−4.44	−4.84	−7.21
	σ	3.37	6.35(−2)	8.41(−3)
105	100*S*	−7.06	−9.95	−6.46
	σ	1.19	1.86(−2)	3.94(−3)
120	100*S*	−11.3	−13.4	−4.38
	σ	4.84(−1)	9.39(−3)	1.74(−3)
135	100*S*	−15.6	−13.9	−2.07
	σ	2.50(−1)	6.04(−3)	6.83(−4)
150	100*S*	−15.2	−12.1	−0.477
	σ	1.72(−1)	4.24(−3)	2.64(−4)
165	100*S*	−9.06	−7.53	−0.526
	σ	1.47(−1)	3.32(−3)	1.23(−4)

and smaller by about 7% at $\theta=150°$. For μ^- scattering, the 10% increase in t reduces the cross section by about 3% at $\theta=45°$ and by about 15% for $\theta=150°$. At this value of v/c the sensitivity of the cross section to surface thickness is therefore larger for μ^- than for μ^+, but the reverse would be expected for some larger values of v/c, judging from indications obtained in the comparison of positron to electron scattering.[12]

III. DEPENDENCE ON *Z* AND v/c

The present results for $Z=35$, when compared to those previously available for $Z=48$ and 80,[9,10] show that for μ^+ the dependence of the cross section on Z is more regular than is the case for μ^-. Figure 1 serves to illustrate this comparison, in which the ratio of the extended nucleus to the point nucleus cross section is plotted versus the momentum transfer q. For the case of μ^+, the spacing between the three curves for $Z=35$, 48, and 80 is not quite linear in Z and varies monotonically with q. A spacing nearly linear in Z is achieved by plotting the ratio of extended to point nucleus cross sections on a logarithmic scale versus $q\times A^{\frac{1}{3}}$, where A is the mass number corresponding to the nucleus of atomic number Z. However an accuracy of about 5% can be obtained in the extrapolation to values of Z contained between 35, 48, and 80 by plotting the cross sections on a logarithmic scale versus transfer momentum and assuming a spacing linear in Z.

The μ^- cross sections have a less regular trend in Z as can be seen from the three lower curves in Fig. 1. Since a reliable extrapolation in Z is not feasible, the cross sections for $Z=47$ have been calculated explicitly. The dependence of the cross sections on v/c is illustrated in Fig. 2. The points representing μ^- cross sections are connected by lines so as to distinguish them from the μ^+ results.

As pointed out by Connolly *et al.*,[6] the μ^+ cross sections, plotted versus transfer momentum, are nearly independent of v/c. From first Born approximation considerations one would expect instead that the ratio of the extended nucleus to point nucleus cross section should be independent of v/c, and hence that the extended nucleus cross section should depend on v/c in the same way as the point nucleus cross section. From the first Born approximation one also would expect that any dependence of $\sigma_{\text{extended}}/\sigma_{\text{point}}$ on v/c should be more pronounced for μ^- than for μ^+ mesons, since the distortion of the wave functions from plane waves should be larger for μ^- than for μ^+. The reverse however is the case, as can be seen from the comparison of the $v/c=0.6$ and 0.8 curves of $\sigma_{\text{extended}}/\sigma_{\text{point}}$ for the case[10] of $Z=80$. Apparently, the repulsion of the μ^+ wave function from the center of the nucleus lessens the effect of the extension of the nuclear charge distribution on the cross section, while the opposite is the case for μ^- mesons. This effect is apparent in Fig. 1, where the $Z=80$ curve lies below the others in the μ^- case, while the order is inverted for the μ^+ curves. As v/c is increased, the repulsion of the μ^+ wave function from the nuclear

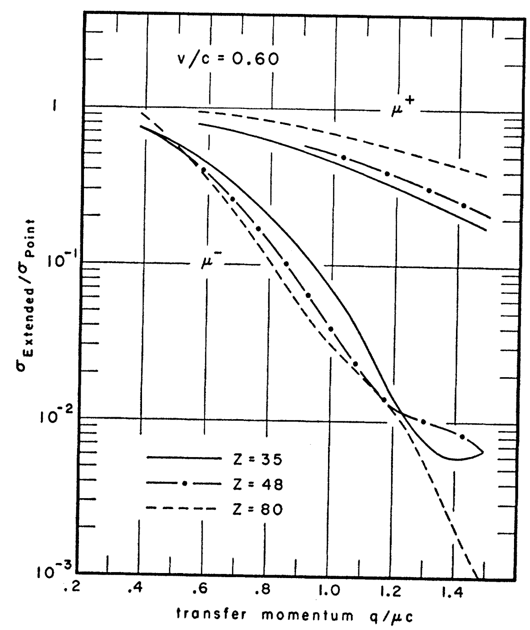

FIG. 1. Dependence of the cross section on atomic number Z. The cross section for which the nuclear charge distribution is spread out over a finite region is denoted by σ_{extended}, while for σ_{point} the nuclear charge is assumed to be concentrated at the center of the nucleus. The abscissa represents the momentum transfer q, in units of muon rest mass times c, and the ordinate represents the ratio of the extended to point cross section. All cross sections are calculated for $v/c=0.6$. Note that for the μ^+ results, the curve for $Z=80$ (dashed line) lies above the other two, while the reverse is the case for μ^- through most of the range of q. The spacing between the curves for the μ^+ cases is sufficiently regular so as to permit extrapolation in Z.

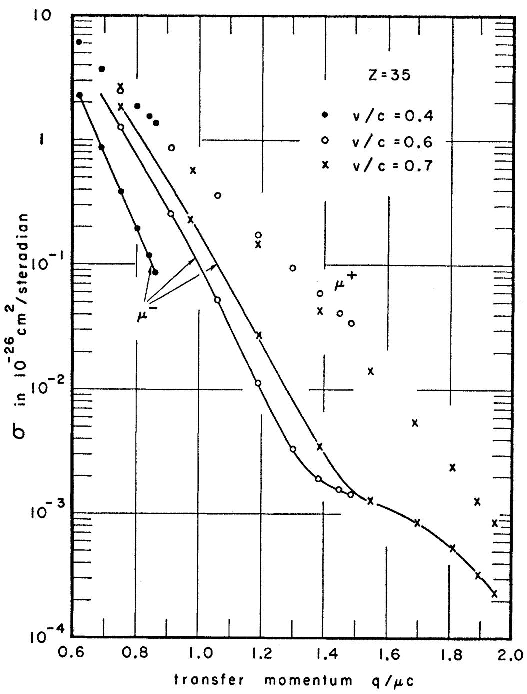

FIG. 2. Dependence of μ^+ and μ^- cross sections on v/c. The differential scattering cross section for the nucleus of bromine is plotted versus momentum transfer q measured in units of the muon rest mass times c, μc. The μ^- points are connected by lines so as to distinguish them from the μ^+ points. The μ^+ cross sections are seen to be nearly independent of v/c, when plotted versus the momentum transfer. This feature is not peculiar to the nucleus of bromine, but holds also, although not as well, for the nucleus of mercury.

interior is lessened and the effect of the finite extension of the nuclear charge distribution upon the cross section becomes more pronounced. For a given momentum transfer, the increase with v/c of the cross section for a point nucleus is offset in the μ^+ case by the reduction due to the nuclear size. The Coulomb distortion should also affect the various theoretical corrections mentioned in the introduction. Since, for the same momentum transfer, muons have less energy than electrons,[15] the Coulomb distortion effects should be larger for muons than for electrons.

IV. SUMMARY

Reasons are given for believing that the measurement of scattering of muons on nuclei might be useful in the determination of the nuclear charge distribution. Previous calculations[10] of the cross section and polarization asymmetry factor[8] are extended to the case of bromine and silver in the energy range from 9.6 to 42.3-Mev muon kinetic energy, which corresponds to v/c between 0.4 and 0.7. Comparison of results with measurements[6] of the scattering of μ^+ in nuclear emulsions shows good agreement. Therefore, up to momentum transfer of 160 Mev/c there is no indication of a scattering anomaly. The dependence of the cross section on Z and v/c is discussed.

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[15] A theoretical comparison of electron and positron scattering by extended nuclei has recently been carried out by S. D. Drell and R. H. Pratt, Phys. Rev. (to be published).