assumption (b) to indicate that $\Delta[(1/k^2)\,\mathrm{Re}a_{2T\ 2J}(L)]<0$, we arrive at the result

$$\Delta(I_0P)=\gamma\sum_{tjl}A_{lj,LJ}(x)\,\mathrm{Im}a_{2t\ 2j}(l),$$

where γ is some positive quantity. By unitarity we also know that $\mathrm{Im}a_{2t\ 2j}(l)\geqslant 0$. Reference to the inequalities (2) now indicates that if $L=J+\frac{1}{2}$ then $A_{lj,LJ}(x)>0$ except when $l=j+\frac{1}{2}$ and $l>L$ (provided θ is sufficiently small). But, if $L=J-\frac{1}{2}$, then $A_{lj,LJ}(x)<0$ except when $l=j-\frac{1}{2}$ and $l>L$. If partial waves with $l>L$ are assumed to be unimportant, then we have the result

$$\Delta(I_0P)>0 \quad \text{if} \quad L=J+\tfrac{1}{2},$$

and

$$\Delta(I_0P)<0 \quad \text{if} \quad L=J-\tfrac{1}{2}.$$

As an example, let us take the second and third $\pi-p$ resonances and, contrary to the assumption in Sec. 3, assume that they are sufficiently separated so that the amplitude of one can be regarded as constant over the other's width. Then calculations indicate that $\theta\approx 40°$ should be small enough for the second resonance, while $\theta\approx 30°$ is required for the third resonance.

Note added in proof. Ball and Frazer have suggested recently [Phys. Rev. Letters **7**, 204 (1961)] that a rapid *rise* in the π-p absorption cross section will cause a peak in the elastic scattering. The use of a Breit-Wigner formula in Sec. 3 implies, of course, that there is a peak in *both* the absorption and elastic cross sections, and that the ratio of the two cross sections is constant for constant Γ_0 and Γ.

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Effect of the ($\Lambda-\pi$) Resonance in Inelastic Nucleon-Hyperon Collisions

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Using an extension of the Chew-Low extrapolation procedure, the $Y+N\rightarrow Y'+N+\pi$ differential cross sections have been calculated. The calculation requires a knowledge of the energy dependence of the total pion-hyperon elastic scattering cross sections, for which we have made use of the results of Dalitz and Tuan based on the analysis of $\bar{K}-N$ data. The effect of Y_1^* in the present processes appears in the strong peaking at low energy and a "knee" at a higher energy in the energy spectrum of the recoil nucleon. In view of the rare strong decay of $Y_1^*\rightarrow\Sigma+\pi$, it is suggested that the reaction $\Lambda+N\rightarrow\Lambda+N+\pi$ would be best suited for experimental study.

INTRODUCTION

THE existence of a $\Lambda-\pi$ resonance (denoted by Y_1^*) in $I=1$ state at 1385 Mev in $K^-+p\rightarrow\Lambda+\pi+\pi$ is now firmly established by recent experiments.[1] Dalitz[2] has interpreted Y_1^* as a bound state of the K^--p system with angular momentum $J=\frac{1}{2}$ and a strong decay via $S_{\frac{1}{2}}$ if the $K-\Lambda$ parity is odd. Block *et al.*[3] have analyzed the data on the production of Y_1^* in K^-+He^4 reactions, assuming an initial S wave and neglecting final-state interactions. Their analysis favors $J=\frac{1}{2}$ but this conclusion is severely limited by their assumptions. Recent data of Berge *et al.*[4] on Y_1^* points to a odd $K-\Lambda$ parity and $J=\frac{1}{2}$, though $J=\frac{3}{2}$ is not excluded.

We have recently[5] pointed out that Y_1^* should also be observable in the reactions

$$Y+N\rightarrow Y'+N+\pi, \tag{1}$$

where Y or Y' stand for either the Λ or Σ hyperon and N represents a nucleon. In reference 5, on the basis of charge independence, gross tests (like inequalities and equalities) were pointed out to test the existence of a Y_1^* as a dominant $I=1$ isotopic spin state of the pion-hyperon system. In this paper we present the calculation of the energy spectrum of the recoil nucleon in reactions (1) as a specific test of the existence of Y_1^*. The method of calculation is analogous to that used for one-pion production in nucleon-nucleon collisions[6] and is based on a generalization of the "extrapolation method" of Chew and Low.[7] The details are given in Sec. II.

The $\pi-Y$ scattering cross sections used in the calculation are those predicted by Dalitz and Tuan,[8] from low energy $\bar{K}-N$ scattering data. The advantage of using the above approach is that it enables one to correlate the cross sections, etc., for reactions (1) with the parameters for $\bar{K}-N$ scattering and absorption.

[1] M. M. Alston *et al.*, Phys. Rev. Letters **5**, 520 (1960); O. Dahl, *et al.*, *ibid.* **6**, 142 (1961).
[2] R. H. Dalitz, Phys. Rev. Letters **6**, 239 (1961).
[3] M. M. Block *et al.*, Nuovo cimento **20**, 715, 724 (1961).
[4] J. P. Berge *et al.*, Phys. Rev. Letters **6**, 557 (1961).
[5] S. N. Biswas and V. Gupta, Nuclear Phys. **24**, 620 (1961).
[6] F. Selleri, Phys. Rev. Letters **6**, 64 (1961); V. N. Gribov, Zhur. Eksp. i Teoret. Fiz. (to be published).
[7] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1652 (1959).
[8] R. H. Dalitz and S. F. Tuan, Ann. Phys. **10**, 307 (1960).

In Sec. III we give the numerical results and their discussion.

II. CALCULATION TECHNIQUE

The calculation of the reactions (1) is based on the diagram shown in Fig. 1. The Chew-Low approach[7] essentially consists in calculating the Feynman amplitude for Fig. 1 and replacing the square of the $YY'\pi\pi$ vertex by $\sigma_{YY'}$, the physical cross section for $\pi+Y\to\pi+Y'$. A knowledge of $\sigma_{YY'}$ would then enable one to calculate the relevant cross sections for reactions (1). Of course, one may determine $\sigma_{YY'}$, and thus $\pi-Y$ resonance, by extrapolating the experimental data on reactions (1) to the pole at μ^2 (μ=pion mass) introduced through the pion propagator. Though a rigorous justification is lacking, the above procedure (or conjecture) has been fairly successful in its applications.[6,7]

In the above approximation one can write down the differential cross section,

$$\frac{\partial^2\sigma(Y+N\to Y'+N+\pi)}{\partial\omega^2\partial\Delta^2}=\frac{1}{2\pi}\frac{f^2}{\mu^2}\frac{1}{p_I^2}\frac{\Delta^2}{(\Delta^2+\mu^2)^2}\omega q_Y\sigma_{YY'}(\omega),\quad(2)$$

where ω and q_Y are the energy and momentum in the center of mass system of π and Y. The magnitude of the momentum in the laboratory system of the incident hyperon Y is denoted by p_I. Also, Δ^2 is the square of the four-momentum transfer (at the $NN\pi$ vertex) and is directly related to the laboratory kinetic energy, T, of the recoil nucleon via[7] $\Delta^2=2mT$, where m is the nucleon rest mass. Further, f is the renormalized pion-nucleon coupling constant. Relation (2) is valid when a π^0 is exchanged; for the exchange of $\pi^\pm$, f^2 is to be replaced by $2f^2$.

In writing down Eq. (2), we have not considered the possibility that the pion can be produced through the $NN\pi\pi$ vertex. Note that this is not possible if both Y and Y' are Λ's, because the $\Lambda\Lambda\pi$ vertex is forbidden by charge independence. By not considering the possible $NN\pi\pi$ vertex we are neglecting the effect of a possible pion-nucleon resonance. This may be a serious drawback for reactions in which π^+p or π^-n appear in the final state. Consequently, we will confine our discussion to reactions in which $\pi-N$ are not in a pure isotopic spin state (cf. Sec. III also).

As mentioned earlier, Dalitz and Tuan[8] predicted an $I=1$ resonance in pion-hyperon scattering from an analysis of $\bar{K}-N$ data. The $I=1$ resonance occurred only for the a_- solution for the scattering parameters. In this note we assume Dalitz and Tuan's interpretation[2,8] of the Y_1^* with $J=\frac{1}{2}$ and odd parity. Further, for the mass and half-width of Y_1^*, we take[2] 1382±20 Mev and 18 Mev, respectively, which are consistent with the scattering length, $a+ib=-1.08(\pm0.2)+i0.20(\pm0.06)$ fermi, for the $I=1$ state. The expression for $\sigma_{YY'}(\omega)$ predicted near resonance below $\bar{K}-N$ threshold is

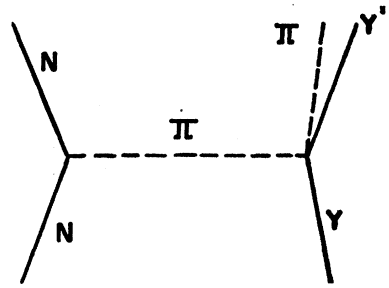

FIG. 1. Feynman diagram for reaction (1).

$$\sigma_{YY'}(\omega)=\frac{4\pi}{q_Y^2}\frac{\kappa^2\beta_Y^2q_Y\beta_{Y'}^2q_{Y'}}{[(1+\kappa a)^2+(\kappa b)^2]},\quad(3)$$

where $\kappa=[2\mu_K(m+m_K-\omega)]^{\frac{1}{2}}$, m_K being the K-meson mass and $\mu_K=mm_K/(m+m_K)$. Further, β_Y is the off-diagonal matrix element, $\langle\bar{K}N|K|\pi Y\rangle$, of the Hermitian K matrix.[8]

In general, β_Σ and β_Λ should be energy dependent, but there is no clear guidance as to the form of this dependence. Consequently, we assume for simplicity that they are constant with respect to energy; i.e., S-wave, $\pi-Y$ scattering.[8] For a discussion of the effect of their energy dependence on the recoil-nucleon energy spectrum see Sec. III.

The expression for $\sigma_{YY'}(\omega)$ above the $\bar{K}-N$ production threshold ($\kappa=0$) is to be obtained by changing $\kappa\to i\kappa$ in the expression for the phase shift, that is, by changing the denominator $[(1+\kappa a)^2+(\kappa b)^2]$ in (3) to $[(1-\kappa b)^2+(\kappa a)^2]$. This should be remembered when using (4) below.

Using (3) in (2), we have for the recoil-nucleon energy spectrum (for π^0 exchange)

$$\frac{d\sigma(Y+N\to Y'+N+\pi)}{d\Delta^2}=\sigma_0(Y)\phi(\Delta^2,Y)\int_{m_Y+\mu}\frac{q_{Y'}\kappa^2\beta_Y^2\beta_{Y'}^2\omega^2d\omega}{[(1+\kappa a)^2+(\kappa b)^2]},\quad(4)$$

where

$$\sigma_0(Y)=4f^2/\mu^2p_I^2$$

and

$$\phi(\Delta^2,Y)=\Delta^2/(\Delta^2+\mu^2)^2.$$

The upper limit for ω is a function of Δ^2 and initial center-of-mass energy W. For a given ω and W, the corresponding Δ^2 is calculated from the relation[7]

$$\Delta^2=+2(E_TE_S-m^2)+2\cos\theta[E_T^2-m^2]^{\frac{1}{2}}[E_S^2-m^2]^{\frac{1}{2}},\ (5)$$

where θ is the c.m. recoil angle;

$$E_T=(W^2+m^2-m_Y^2)/2W$$

and

$$E_S=(W^2+m^2-\omega^2)/2W$$

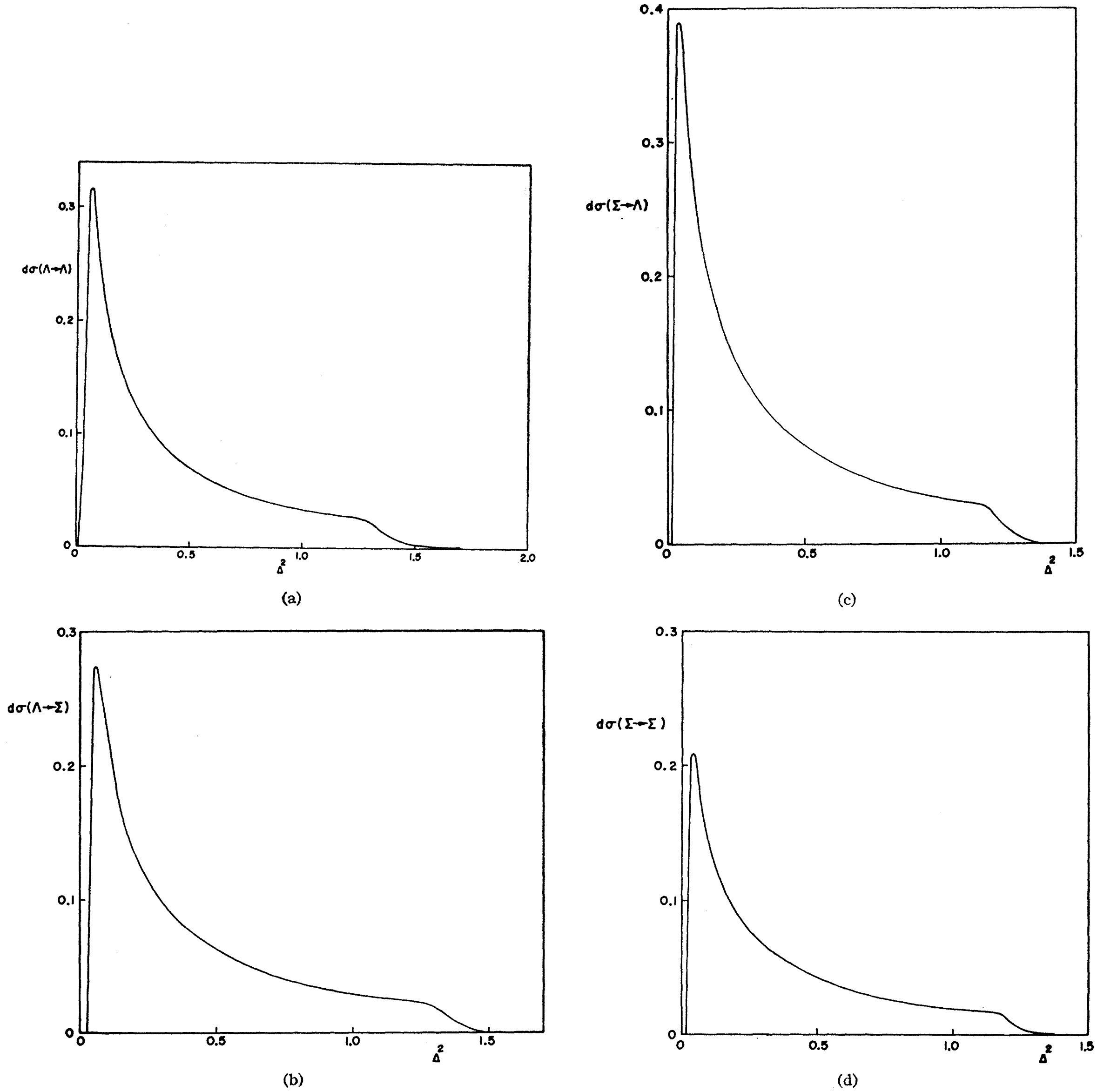

FIG. 2. Differential cross section, $d\sigma(Y \to Y')$, in dimensionless units, for the reaction $Y+N \to Y'+N+\pi$. (a) $\Lambda \to \Lambda$; (b) $\Lambda \to \Sigma$; (c) $\Sigma \to \Lambda$; (d) $\Sigma \to \Sigma$. Δ^2 in $(\text{Bev})^2$ is directly related to recoil nucleon kinetic energy T, via $\Delta^2 = 2mT$, where m is the nucleon mass. The ordinate $d\sigma(Y \to Y')$ is essentially the differential cross section $d\sigma(Y+N \to Y'+N+\pi)/d\Delta^2$. For an exact definition see text.

are, respectively, the over-all laboratory system energies of the target and the recoil nucleons; m_Y denotes the mass of the incident hyperon.

III. RESULTS

The nucleon recoil energy spectrum has been calculated for $W=2.5$ Bev. The value of W is so chosen that the range for ω includes the resonance energy ω_r for the pion-hyperon system.

For numerical computation the following mass values (in Bev): $\mu=0.139$, $m_K=0.494$, $m=0.939$, $m_\Lambda=1.115$, and $m_\Sigma=1.190$, have been used. The value for the square of the renormalized pion-nucleon coupling, $f^2=0.08$, is taken.

For convenience of discussion, since β_Λ and β_Σ have been taken independent of energy ω, the dimensionless quantity

$$(\beta_Y \beta_{Y'})^{-2} \frac{d\sigma}{d\Delta^2}(Y+N \to Y'+N+\pi),$$

denoted by $d\sigma(Y \to Y')$, has been plotted against Δ^2, in Figs. 2(a)–(d). All the quantities have been expressed in Bev using $\hbar = c = 1$.

The numerical values of $d\sigma(Y\rightarrow Y')$ in these figures are for a neutral pion exchange as in Fig. 1. In Figs. 2(a) and 2(b) $d\sigma(\Lambda\rightarrow\Lambda)$ and $d\sigma(\Lambda\rightarrow\Sigma)$ have been plotted, and both show a sharp peak for $\Delta^2=0.057$, that is, at recoil nucleon kinetic energy $T=30.3$ Mev. The plots for $d\sigma(\Sigma\rightarrow\Lambda)$ and $d\sigma(\Sigma\rightarrow\Sigma)$ [see Figs. 2(c) and 2(d)] also show a sharp peak, but for $\Delta^2=0.037$, that is, $T=19.7$ Mev. The peak is highest in case of $d\sigma(\Sigma\rightarrow\Lambda)$.

For larger Δ^2 all the graphs show a "knee" shape at $\Delta^2=1.288$ in Figs. 2(a) and 2(b) and at $\Delta^2=1.166$ in Figs.[9] 2(c) and 2(d). In all the cases the peak and the "knee" are at values of Δ^2 corresponding to $\omega=1.40$ rather than $\omega=\omega_r=1.382$ for the $\pi-Y$ system.

Our results are to be compared with those of Selleri[6] for $N+N\rightarrow N+N+\pi$, where he obtains a peak at the small as well as at the larger value of Δ^2. The reason for this is that in Selleri's case both the vertices are identical and the pion can emerge at either, while in our case the vertices are different and moreover we have neglected the $NN\pi\pi$ vertex. The presence of the $NN\pi\pi$ vertex would show up as a peak in the energy spectrum of the outgoing hyperon (excepting $\Lambda+N\rightarrow\Lambda+N+\pi$). The inclusion of the pion-nucleon resonance (via $NN\pi\pi$ vertex) would also modify the recoil nucleon spectrum. However, kinematically one would expect the πN and πY resonances to be mutually exclusive,[10] so that the shape of the nucleon energy spectrum in Figs. 2(b), (c), and (d) should not be much altered.

The $\pi Y\rightarrow\pi Y'$ scattering (except $\pi\Lambda\rightarrow\pi\Lambda$) through Y_1^* is expected to be small because the experiment shows that the coupling of Y_1^* to $\pi\Sigma$ is weak. In fact, less than 10% decays of Y_1^* are into $\pi+\Sigma$ the rest being into $\pi+\Lambda$. This implies that the contribution of Fig. 1 to the $\Lambda+N\rightarrow\Sigma+N+\pi$ and $\Sigma+N\rightarrow Y'+N+\pi$ cross sections will be small. Consequently, the experimental observation of the nucleon recoil spectrum given in Figs. 2(b), (c), (d) would be rather difficult. Thus, the $\Lambda+N\rightarrow\Lambda+N+\pi$ reactions would be the most suitable for the observance of Y_1^* and testing our results; for example, $\Lambda+p\rightarrow\Lambda+p+\pi^0$.

In the other three cases the best reactions, from the experimental point of view, are

$$\Lambda+p\rightarrow\Sigma^-+p+\pi^+,$$
$$\Sigma^\pm+p\rightarrow\Lambda+p+\pi^\pm,$$
$$\Sigma^-+p\rightarrow\Sigma^-+p+\pi^0.$$

All these reactions involve a π^0 exchange and thus may be directly compared with the graphs given.

As remarked earlier, β_Λ and β_Σ have been taken to be energy independent. β_Y would have an energy dependence of the form $(q_Y)^l$ if the $\pi-Y$ system has an orbital angular momentum l. Such an energy dependence will increase the height of the peak in Figs. 2(a)–(d). However, for low l this increase in height would be small. Furthermore, such energy dependences of β_Λ and β_Σ would be different if the $\Lambda-\Sigma$ parity were odd. Constant β_Λ and β_Σ act as scaling factors and, since their absolute values are not known, a rough estimate of the total cross section for the reactions (1) on the present model would not be significant.

At present, the data on the reactions considered here are very scanty owing to experimental difficulties. However, data on the reactions (1) would be very desirable as they would enable one to correlate $\bar{K}-N$ scattering parameters by means of the approach adopted here.

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[9] Note that the two values 1.288 and 0.057 correspond to the same ω in the case of the $\Lambda\rightarrow Y$ reactions. This is also true for the $\Sigma\rightarrow Y$ reactions.

[10] For in the c.m. system, when π goes in the same direction as N, the hyperon Y must go in the opposite direction; and when π goes with Y, then N must travel in the opposite direction.