Energy Dependence and Absorption in the Use of Recoil Polarization from $\pi^- - p$ Scattering*†

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The use of recoil polarization to determine angular momentum assignments for the second and third resonances in $\pi^- - p$ scattering is found to be qualitatively unaffected by the presence of absorption in the resonant states. The energy dependence of the polarized cross section in the vicinity of a resonance is shown to give some information about both the background and the resonant state. The polarized cross section is expected to increase across a resonance if $L=J+\frac{1}{2}$ and decrease if $L=J-\frac{1}{2}$.

1. INTRODUCTION

MORAVCSIK has suggested[1] that the polarization of the recoil proton in $\pi^- - p$ scattering might be used to determine angular momentum assignments for the second and third resonances observed in that scattering. We would like to develop two points with regard to the use of recoil polarization. The first point is concerned with the effect of inelastic processes upon the results of Moravcsik. For the sake of simplicity, the method discussed in reference 1 assumes that $\pi^- - p$ scattering is purely elastic, despite the fact that in the region of the second and third resonances the cross section for production of an additional pion is considerable.[2] Furthermore it has been shown[3,4] that the second and third resonances might be due to certain initial and final state interactions in the production channel, which would indicate that the partial waves for the resonant states were largely absorbed.

The second point is concerned with the energy dependence of the recoil polarization in the neighborhood of a resonance. Investigation of this energy dependence is suggested by the fact that the amplitude for a resonant state is strongly energy dependent in the resonance region while amplitudes for nonresonant states are essentially energy independent in this region—especially if the resonance is quite narrow. Hence, while effects that depend on the large size of the resonant amplitude might be masked by a large number of "background" amplitudes, effects that depend on the large energy variation of the resonant amplitude should show through.

2. SUMMARY OF USEFUL FORMULAS

We choose the coordinate system in which the z axis is parallel to the initial pion momentum in the c.m. system and the y axis is parallel to the normal to the scattering plane. The $\pi^- - p$ scattering is determined by two amplitudes

$$f(\theta)=\frac{1}{k}\sum_{ljt}(j+\tfrac{1}{2})\eta_t P_l(x)a_{2t\,2j}(l),$$

$$g(\theta)=\frac{1}{k}\sum_{ljt}(-1)^{j+l+\frac{1}{2}}\eta_t\frac{dP_l(x)}{dx}a_{2t\,2j}(l),$$

where k is the wave number in the c.m. system, x is the cosine of the c.m. scattering angle θ, $P_l(x)$ is the lth Legendre polynomial, t is the total isotopic spin, j is the total angular momentum, and η_t is a weighting factor which has the values $\frac{2}{3}$ for $t=\frac{1}{2}$ and $\frac{1}{3}$ for $t=\frac{3}{2}$. The quantities $a_{2t\,2j}(l)$ are the various partial wave amplitudes for a given t, j, and l. In the case of elastic scattering it is customary to impose the unitarity condition by the use of phase shifts:

$$a_{2t\,2j}(l)=e^{i\delta_{2t\,2j}(l)}\sin\delta_{2t\,2j}(l).$$

In the presence of absorption, the unitarity condition implies that

$$[\mathrm{Im}a_{2t\,2j}(l)-\tfrac{1}{2}]^2+[\mathrm{Re}a_{2t\,2j}(l)]^2=\rho^2/4\leq\tfrac{1}{4}, \quad (1)$$

where ρ is a measure of the extent to which the partial wave has been absorbed. The two extreme cases are given by

$\rho=1$ (elastic scattering),
$\rho=0$ (maximum absorption).

The total cross section for $\pi^- + p$ can be obtained by use of the optical theorem,

$$\sigma^{\mathrm{tot}}=\frac{4\pi}{k}\,\mathrm{Im}f(0)=\frac{4\pi}{k^2}\sum_{ljt}(j+\tfrac{1}{2})\eta_t\,\mathrm{Im}a_{2t\,2j}(l),$$

while the polarized cross section[5] is given by

$$I_0P=2\sin\theta\,\mathrm{Im}[f^*(\theta)g(\theta)]$$
$$=2\sin\theta\sum_{tt'}\sum_{l\leq l'}\sum_{j\leq j'}\eta_t\eta_{t'}A_{lj,l'j'}(x)\times\mathrm{Im}[a_{2t\,2j}^*(l)a_{2t'\,2j'}(l')],$$

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1 M. J. Moravcsik, Phys. Rev. 118, 1615 (1960).
2 J. C. Brisson, J. F. Detouf, P. Falk-Vairant, L. Van Rossum, and G. Valladas, Nuovo cimento 19, 210 (1961).
3 R. F. Peierls, Phys. Rev. 118, 325 (1960).
4 P. Carruthers, Ann. Phys. 14, 229 (1961).
5 L. Wolfenstein, Ann. Rev. Nuclear Sci. 6, 43 (1956).

TABLE I. Polarization between second and third resonances for various amounts of absorption in the resonant states.

Assignment of second and third resonances		Polarization $P(x)$ $\mu_{(2)}=1.0$, $\mu_{(3)}=1.0$ $x=0.3$	$x=-0.3$	$\mu_{(2)}=1.0$, $\mu_{(3)}=0.5$ $x=0.3$	$x=-0.3$	$\mu_{(2)}=0.5$, $\mu_{(3)}=1.0$ $x=0.3$	$x=-0.3$
P_{13}	D_{15}	+0.97	+0.97	+0.90	+0.90	+0.70	+0.70
D_{13}	D_{15}	−0.97	+0.97	−0.90	+0.90	−0.70	+0.70
P_{13}	F_{15}	+0.97	−0.97	+0.90	−0.90	+0.70	−0.70
D_{13}	F_{15}	−0.97	−0.97	−0.90	−0.90	−0.70	−0.70

where

$$A_{lj,l'j'}(x)=(j+\tfrac{1}{2})(-1)^{l'+j'+\frac{1}{2}}P_l(x)dP_{l'}(x)/dx - (j'+\tfrac{1}{2})(-1)^{l+j+\frac{1}{2}}P_{l'}(x)dP_l(x)/dx,$$

and I_0 is the unpolarized cross section, which is given by

$$I_0=|f(\theta)|^2+\sin^2\theta|g(\theta)|^2.$$

Particular states are referred to as S_{11}, P_{33}, etc., where the first subscript is twice the isotopic spin and the second subscript is twice the total angular momentum.

3. ABSORPTION AND THE MORAVCSIK METHOD

Before looking at the effects of absorption we shall comment briefly on the Moravcsik method in general. The essential feature of the method is that the interference between two large amplitudes that are 90° out of phase will dominate over the interference of much smaller amplitudes and tend to give a rather large polarization. The absence of such a dominant interference will in general lead to a smaller polarization, if any. Thus, for example, between the first (P_{33}) and second resonances, at $\theta=90°$, we will have a large interference between resonant amplitudes if the second is D_{13}, while we will have no interference if the second resonance is P_{13}. Actually, there are two difficulties involved in this example. In the first place, phase shift analysis[6,7] shows that $\delta_{33}(P)$ is already 135° at 310 Mev while neither $\delta_{13}(P)$ nor $\delta_{13}(D)$ is large, so that it seems unlikely that "maximum" interference is ever attained between the first and second resonances. Secondly, in the event that the second resonance is P_{13} we should expect to find a "small" polarization; but in fact this "small" polarization (i.e., polarization arising from the interference of small amplitudes with one another and with the resonant amplitudes) can be quite sizeable. The result of the above is that most likely no clear-cut choice can be made between P_{13} or D_{13} by measurement of the recoil polarization between the first and second resonances.

The situation between the second and third resonances appears to be more favorable since there is greater "overlap" here and by looking at a suitable angle we are assured of obtaining a large interference for any of the four possible angular momentum assignments (assuming $j=\frac{3}{2}$ for the second resonance and $j=\frac{5}{2}$ for the third). This assurance is subject somewhat to limitations on the relative amounts of absorption in the two resonant states.

We shall write the amplitudes for the second and third resonances as $a_{(2)}$ and $a_{(3)}$, respectively. Then we consider the situation corresponding to "maximum overlap":

$$a_{(2)}=\mu_{(2)}(i-1)/2,\quad a_{(3)}=\mu_{(3)}(i+1)/2.$$

From Eq. (1) we see that $\mu_{(2)}=\mu_{(3)}=1$ indicates that both resonances are elastic while $\mu_{(2)}=\mu_{(3)}=\frac{1}{2}$ indicates the maximum possible absorption in each resonant state. Using only resonant amplitudes, the polarization has been calculated for the various possible angular momentum states and for different degrees of absorption in the two resonances. Since the two amplitudes are taken to be 90° out of phase, the angular distribution will be symmetric about $\theta=90°$ and hence the polarization will be symmetric (or antisymmetric) about $\theta=90°$. Furthermore, the value of the resonance polarization is quite insensitive to small background amplitudes provided it is close to unity; for this reason it is desirable to make observations near an angle at which the resonance polarization peaks. It was found that $x=\pm0.3$ is quite close to the peak position in all situations. The polarization at these angles is given in Table I. The peak value is smallest for the case in which the second resonance is completely absorptive ($\mu_{(2)}=\frac{1}{2}$) and the third resonance is completely elastic ($\mu_{(3)}=1$). This situation will tend to be more sensitive to the presence of small amplitudes than will the other situations. The results of Table I are in qualitative agreement with the results obtained by Moravcsik.

4. VARIATION OF THE POLARIZED CROSS SECTION WITH ENERGY

In order to discuss the energy dependence of the polarized cross section in the vicinity of a resonance, we need to specify the energy dependence of the amplitudes involved. The simplest procedure is to describe the resonant amplitude by a Breit-Wigner formula and to assume that all other amplitudes are independent of the energy. If we specify the resonant state quantum numbers by capital letters, then the energy-dependent

[6] O. Chamberlain, J. Foote, E. Rogers, and H. Steiner, Phys. Rev. **122**, 959 (1961).

[7] V. G. Zinov, S. M. Korenchenko, N. I. Polumordvinova, and G. N. Tentyukova, J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 1407 (1960). [Translation: Soviet Phys.—JETP **11**, 1016 (1960)].

part of the polarized cross section is given by

$$I_0P_E=(2/k^2)\sin\theta\eta_T\,\mathrm{Im}[Q^*(\theta)a_{2T\;2J}(L)],$$

where

$$Q(\theta)=\sum_{ljt}A_{lj,LJ}(x)\eta_t a_{2t\;2j}(l),$$

and

$$a_{2T\;2J}(L)=-\tfrac{1}{2}\Gamma_0/(E-E_R+\tfrac{1}{2}\Gamma).$$

Γ_0 and Γ are the "particle and "total" widths, respectively, E is the total energy in the c.m. system, and E_R is the resonance energy. The resonance is elastic when $\Gamma_0=\Gamma$ and is most absorptive when $\Gamma_0=\frac{1}{2}\Gamma$. It is easy to see that $k^2I_0P_E$ peaks at $E=E_R+\frac{1}{2}\Gamma\tan[\frac{1}{2}\phi(\theta)]$ and at $E=E_R-\frac{1}{2}\Gamma\cot[\frac{1}{2}\phi(\theta)]$ where $\tan\phi(\theta)=\mathrm{Im}Q(\theta)/\mathrm{Re}Q(\theta)$. If we restrict the energy range so that $E_R-\frac{1}{2}\Gamma\leq E\leq E_R+\frac{1}{2}\Gamma$ then only one peak will be observed except when $Q(\theta)$ is purely imaginary. In this case $k^2I_0P_E$ peaks at $E=E_R-\frac{1}{2}\Gamma$ passes through zero at resonance, and then peaks with the opposite sign at $E=E_R+\frac{1}{2}\Gamma$; it is antisymmetric about the resonance. Another special case occurs when $Q(\theta)$ is real. Then $k^2I_0P_E$ peaks *at* and is symmetric about the resonance. We can obtain some information about the relative importance of various background amplitudes if we should observe that the polarized cross-section peaks at the resonance for some angle θ. As an example, suppose we observe a peaking at the resonance when looking at $\theta=90°$. Then we have the condition:

$$\mathrm{Im}Q(\pi/2)=0.$$

If we suppose that we are at the second $\pi^{-}-p$ resonance and that it is D_{13}, then the above condition can be expressed as

$$2\,\mathrm{Im}a_{11}(P)+\mathrm{Im}a_{31}(P)-2\,\mathrm{Im}a_{13}(P)-\mathrm{Im}a_{33}(P)-3\,\mathrm{Im}a_{15}(F)-\tfrac{3}{2}\,\mathrm{Im}a_{35}(F)=0,$$

where we have kept terms up through $j=\frac{5}{2}$. This condition could be made more restrictive by using some information from phase shift analysis at a lower energy.

With some algebraic manipulation it can also be seen that if θ_R is an angle for which $P_L(x)$ vanishes, then $\phi(\theta_R)$ is simply the phase of the nonresonant part of the non-spin-flip amplitude (at $\theta=\theta_R$). This phase could be obtained by looking at $\theta=\theta_R$ and observing the energy at which k^2I_0P peaks. Similarly if θ_R' is an angle for which $dP_L(x)/dx=0$ then $\phi(\theta_R')$ is the phase of the nonresonant part of the spin-flip amplitude.

It should be noted that the amount of absorption in the resonant state, as indicated by the ratio Γ_0/Γ, does not influence the shape of the polarized cross section as long as we use a Breit-Wigner formula for the resonant amplitude.

As a second application of the energy dependence of the polarized cross section, we consider the situation in which a single resonance (or two resonances with little overlap) occurs in the presence of numerous background states which may contribute considerably to the polarization. The distinguishing feature of the resonant amplitude is its rapid variation in the resonance region compared to the slowly varying background. It is just this feature which can be utilized to give some indication of the parity of the resonance (we assume knowledge of the total angular momentum).

First, we note some properties of the quantities $A_{lj,LJ}(x)$ (l, j refer to nonresonant states and, as before, L, J refer to the resonant state). It is possible to show that

$$A_{j+\frac{1}{2}\;j,J+\frac{1}{2}\;J}(x)=-A_{j-\frac{1}{2}\;j,J-\frac{1}{2}\;J}(x),$$

and

$$A_{j+\frac{1}{2}\;j,J-\frac{1}{2}\;J}(x)=-A_{j-\frac{1}{2}\;j,J+\frac{1}{2}\;J}(x).$$

These relations are related to the Minami ambiguity.[8] Since

$$P_l(1)=1\quad\text{and}\quad dP_l(1)/dx=l(l+1)/2,$$

we can write

$$A_{lj,LJ}(1)=(j+\tfrac{1}{2})(-1)^{L+J+\frac{1}{2}}L(L+1)/2-(J+\tfrac{1}{2})(-1)^{l+j+\frac{1}{2}}l(l+1)/2.$$

We now consider the four possible cases:

$$\begin{aligned}
&\text{If } L=J+\tfrac{1}{2},\; l=j+\tfrac{1}{2},\; \text{then } A_{lj,LJ}(1)>0 \text{ for } L>l;\\
&\text{if } L=J+\tfrac{1}{2},\; l=j-\tfrac{1}{2},\; \text{then } A_{lj,LJ}(1)>0 \text{ for all } l;\quad (2)\\
&\text{if } L=J-\tfrac{1}{2},\; l=j+\tfrac{1}{2},\; \text{then } A_{lj,LJ}(1)<0 \text{ for all } l;\\
&\text{if } L=J-\tfrac{1}{2},\; l=j-\tfrac{1}{2},\; \text{then } A_{lj,LJ}(1)<0 \text{ for } L>l.
\end{aligned}$$

By continuity there is some angle $\theta_0>0$ for which the inequalities (2) are still valid. The value of θ_0 taken depends on how large a value of l is thought to be important.

We now make two assumptions: (a) All the energy dependence is in the amplitude $a_{2T\;2J}(L)$; (b) $\mathrm{Re}a_{2T\;2J}(L)$ is a decreasing function of the energy across the resonance. We do not assume that $a_{2T\;2J}(L)$ is described by a Breit-Wigner formula. If we let E_1 be some energy below the resonance and E_2 be some energy above the resonance, then

$$\begin{aligned}
\Delta(I_0P)&=I_0P(E_2)-I_0P(E_1)\\
&=2\sin\theta\sum_{tlj}A_{lj,LJ}(x)\eta_t\eta_T\\
&\quad\times\{\mathrm{Re}a_{2t\;2j}(l)\Delta[(1/k^2)\,\mathrm{Im}a_{2T\;2J}(L)]\\
&\quad-\mathrm{Im}a_{2t\;2j}(l)\Delta[(1/k^2)\,\mathrm{Re}a_{2T\;2J}(L)]\}.
\end{aligned}$$

Now $(1/k^2)\,\mathrm{Im}a_{2T\;2J}(L)$ is proportional to the total cross section in the resonant state and, by assumption (a) above, changes in the total cross section as we pass through the resonance are due to changes if the contribution from the resonant state. Hence

$$\Delta[(1/k^2)\,\mathrm{Im}a_{2T\;2J}(L)]\propto\Delta\sigma^{\mathrm{tot}}.$$

If we pick E_1 and E_2 such that $\sigma^{\mathrm{tot}}(E_1)=\sigma^{\mathrm{tot}}(E_2)$ and use

[8] S. Hayakawa, M. Kawaguchi, and S. Minami, Progr. Theoret. Phys. **11**, 332 (1954).

assumption (b) to indicate that $\Delta[(1/k^2)\,\mathrm{Re}a_{2T\,2J}(L)]<0$, we arrive at the result

$$\Delta(I_0P)=\gamma\sum_{tjl}A_{lj,LJ}(x)\,\mathrm{Im}a_{2t\,2j}(l),$$

where γ is some positive quantity. By unitarity we also know that $\mathrm{Im}a_{2t\,2j}(l)\geqslant 0$. Reference to the inequalities (2) now indicates that if $L=J+\frac{1}{2}$ then $A_{lj,LJ}(x)>0$ except when $l=j+\frac{1}{2}$ and $l>L$ (provided θ is sufficiently small). But, if $L=J-\frac{1}{2}$, then $A_{lj,LJ}(x)<0$ except when $l=j-\frac{1}{2}$ and $l>L$. If partial waves with $l>L$ are assumed to be unimportant, then we have the result

$$\Delta(I_0P)>0 \quad \text{if} \quad L=J+\tfrac{1}{2},$$

and

$$\Delta(I_0P)<0 \quad \text{if} \quad L=J-\tfrac{1}{2}.$$

As an example, let us take the second and third $\pi-p$ resonances and, contrary to the assumption in Sec. 3, assume that they are sufficiently separated so that the amplitude of one can be regarded as constant over the other's width. Then calculations indicate that $\theta\approx 40°$ should be small enough for the second resonance, while $\theta\approx 30°$ is required for the third resonance.

Note added in proof. Ball and Frazer have suggested recently [Phys. Rev. Letters **7**, 204 (1961)] that a rapid *rise* in the π-p absorption cross section will cause a peak in the elastic scattering. The use of a Breit-Wigner formula in Sec. 3 implies, of course, that there is a peak in *both* the absorption and elastic cross sections, and that the ratio of the two cross sections is constant for constant Γ_0 and Γ.

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Effect of the $(\Lambda-\pi)$ Resonance in Inelastic Nucleon-Hyperon Collisions

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Using an extension of the Chew-Low extrapolation procedure, the $Y+N\rightarrow Y'+N+\pi$ differential cross sections have been calculated. The calculation requires a knowledge of the energy dependence of the total pion-hyperon elastic scattering cross sections, for which we have made use of the results of Dalitz and Tuan based on the analysis of $\bar{K}-N$ data. The effect of Y_1^* in the present processes appears in the strong peaking at low energy and a "knee" at a higher energy in the energy spectrum of the recoil nucleon. In view of the rare strong decay of $Y_1^*\rightarrow\Sigma+\pi$, it is suggested that the reaction $\Lambda+N\rightarrow\Lambda+N+\pi$ would be best suited for experimental study.

INTRODUCTION

THE existence of a $\Lambda-\pi$ resonance (denoted by Y_1^*) in $I=1$ state at 1385 Mev in $K^-+p\rightarrow\Lambda+\pi+\pi$ is now firmly established by recent experiments.[1] Dalitz[2] has interpreted Y_1^* as a bound state of the K^--p system with angular momentum $J=\frac{1}{2}$ and a strong decay via $S_{\frac{1}{2}}$ if the $K-\Lambda$ parity is odd. Block *et al.*[3] have analyzed the data on the production of Y_1^* in $K^-+\mathrm{He}^4$ reactions, assuming an initial S wave and neglecting final-state interactions. Their analysis favors $J=\frac{1}{2}$ but this conclusion is severely limited by their assumptions. Recent data of Berge *et al.*[4] on Y_1^* points to a odd $K-\Lambda$ parity and $J=\frac{1}{2}$, though $J=\frac{3}{2}$ is not excluded.

We have recently[5] pointed out that Y_1^* should also be observable in the reactions

$$Y+N\rightarrow Y'+N+\pi, \qquad (1)$$

where Y or Y' stand for either the Λ or Σ hyperon and N represents a nucleon. In reference 5, on the basis of charge independence, gross tests (like inequalities and equalities) were pointed out to test the existence of a Y_1^* as a dominant $I=1$ isotopic spin state of the pion-hyperon system. In this paper we present the calculation of the energy spectrum of the recoil nucleon in reactions (1) as a specific test of the existence of Y_1^*. The method of calculation is analogous to that used for one-pion production in nucleon-nucleon collisions[6] and is based on a generalization of the "extrapolation method" of Chew and Low.[7] The details are given in Sec. II.

The $\pi-Y$ scattering cross sections used in the calculation are those predicted by Dalitz and Tuan,[8] from low energy $\bar{K}-N$ scattering data. The advantage of using the above approach is that it enables one to correlate the cross sections, etc., for reactions (1) with the parameters for $\bar{K}-N$ scattering and absorption.

[1] M. M. Alston *et al.*, Phys. Rev. Letters **5**, 520 (1960); O. Dahl, *et al.*, *ibid.* **6**, 142 (1961).
[2] R. H. Dalitz, Phys. Rev. Letters **6**, 239 (1961).
[3] M. M. Block *et al.*, Nuovo cimento **20**, 715, 724 (1961).
[4] J. P. Berge *et al.*, Phys. Rev. Letters **6**, 557 (1961).
[5] S. N. Biswas and V. Gupta, Nuclear Phys. **24**, 620 (1961).
[6] F. Selleri, Phys. Rev. Letters **6**, 64 (1961); V. N. Gribov, Zhur. Eksp. i Teoret. Fiz. (to be published).
[7] G. F. Chew and F. E. Low, Phys. Rev. **113**, 1652 (1959).
[8] R. H. Dalitz and S. F. Tuan, Ann. Phys. **10**, 307 (1960).