is negative. This conclusion may be substantiated by the work of Phillips and Crowe,[10] Cohen, and others.[13]

If the theory gave correctly the absolute value for the cross section one could easily distinguish between a positive and a negative a_{nn} since the absolute cross section is very sensitive to the sign of a_{nn}. In our approximation the cross section at the high-energy peak is by a factor of ~2.5 higher for a negative a_{nn} of around −22 f than for a positive one of the same absolute magnitude. There are no reasons to believe that this ratio would change considerably if one used an exact theory.

It can be said in conclusion that the shape of the proton energy spectrum from the reaction D$(n,p)2n$ is definitely sensitive on the $n-n$ scattering length. In the experiment described in reference 14, this sensitivity has not been exploited to the full, due to the rather poor energy resolution as can be seen by comparing the smeared curves of Fig. 3 with the unsmeared ones of Fig. 2. If the present calculation could be put on a more rigorous basis, or if a more exact theory could be developed, a measurement of the shape of the proton spectrum could be used for a very accurate determination of a_{nn}.

ACKNOWLEDGMENTS

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Spectra of (p,α) and (p,p') Reactions and the Evaporation Model*†

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The energy spectra of α particles from V, Fe, Co, Ni, and Rb bombarded with 17.5-Mev protons were obtained at a number of angles. Beyond 30°, the spectra are essentially isotropic and have been analyzed in terms of the evaporation model. The level density deduced is of the form $\exp(E_x/T)$ for $E_x=3$ to 12 Mev. The values of T are in the range 1.2–1.5 Mev if the cross sections of the continuum model with $R=(1.4A^{\frac{1}{3}}+2.2)$ fermis are used, and in the range 1.3–1.6 Mev if optical model cross sections are used. This "constant temperature" level density is also consistent with alpha spectra of Ni and Co bombarded 15- and 19-Mev protons. Analysis of published inelastic proton scattering data for elements in the Ti-Zn region, with bombarding energies of 11.3 to 23 Mev, indicates that the (p,p') spectra (in the above excitation interval) are only partially attributable to evaporation. At 11.3 Mev the upper limit of the contribution of evaporation to the proton spectra is estimated to be ≤96%, while at 23 Mev it is probably much smaller than 22%. The value of T for Fe^{56} deduced from (p,α) and (α,α') reactions is ~1.5 Mev, while inelastic neutron scattering data yield a value of 0.95 Mev. The resolution of this disagreement will require modification of the usual calculations of inverse cross sections.

I. INTRODUCTION

In recent years there have been many investigations of the energy spectra of nuclear reaction products at intermediate bombarding energies. Where the final states are well-separated, the angular distributions can, in general, be described in terms of the optical and direct interaction models. Where the final states are not resolved, corresponding to high excitations of the residual nucleus, analysis of the experimental results has been carried out in terms of the evaporation model of Weisskopf and Ewing.[1,2] The present paper discusses experiments falling into this second category.

The evaporation model describes the spectra of particles emitted from a highly excited compound nucleus in which the available energy is shared by all the nucleons. The compound nucleus has a long lifetime and its decay is assumed to be independent of its mode of formation. In order to describe the spectra of the evaporated particles, it is assumed[1] that the intrinsic probabilities for the formation of all final states are the same. The differential energy spectra are then determined by phase space factors, transmission coefficients for the particles, and the level densities of the residual nuclei.

Theoretical expressions for the spectra are given by Eq. (1) and Eq. (2) in Sec. IV; in all subsequent references to the evaporation model we shall be referring

* This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.

† Some of the results of the present investigation have appeared in the following: P. C. Gugelot, Physica **22**, 1019 (1956); R. Sherr, *Proceedings of the University of Pittsburgh Conference on Nuclear Structure, 1957* (University of Pittsburgh and Office of Ordnance Research, U. S. Army, 1957), p. 376; F. P. Brady and R. Sherr, Bull. Am. Phys. Soc. **5**, 249 (1960); F. P. Brady, D. G. Cassel, and R. Sherr, *ibid.* **6**, 48 (1961). Part of this work was submitted by F. P. Brady in partial fulfillment of the requirements for the Ph.D. degree from the Department of Physics, Princeton University.

[1] V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).

[2] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 342.

to this specific formulation. According to this model, the particles are evaporated isotropically, so that it is generally assumed that particles with isotropic distributions are the products of an evaporation process.

Many spectra have been experimentally investigated and analyzed in terms of this model for (n,n'), (n,p), (p,p'), (p,n), (α,α'), and (α,p) reactions. From the first it was evident that this model by itself was inadequate to explain the observations.[3] The angular distributions were frequently peaked in the forward direction and the shape did not have the predicted dependence on incident energy. Subsequent work on the evaporation model and its application has been reviewed by Peaslee[4] and Le Couteur.[5] The theoretical aspects of this problem have recently been discussed by Ericson.[6]

Despite the limitations of the evaporation model, it appears to be capable of describing at least semiquantitatively many experimental observations. It is therefore of interest to explore various reactions at different energies to see whether there are regions in which this strong interaction model provides a consistent quantitative description. With this latter question in mind, Colli and her collaborators[7] have recently reviewed most of the available experimental results. If one considers the part of the spectra corresponding to the emission of only one particle, one is limited to a range of excitation from 0 to 7 or 10 Mev. As a statistical model should not be applied below about 3 Mev of excitation for medium weight nuclei, the range of excitation which is usable for analysis is only 3 to 7 Mev. (In the case of α emission a larger range can be investigated.) Colli *et al.*[7] analyzed the spectra of protons and neutrons emitted in the backward direction by medium-weight nuclei (Al-Mo) following bombardment with neutrons and protons of 14 Mev or less. Although there is some forward peaking, generally ascribed to direct interactions, the assumption is made that these interactions will not contribute significantly to the yield in the backward direction.

The level densities obtained[7] from the neutron induced spectra using Eq. (1) are of the form $\exp(E_x/T)$, with $T\approx 1$ Mev. Level densities of the same form were found by Lassen *et al.*[8] for the reaction (α,α') on medium-weight nuclei using 12–20-Mev α particles. The applicability of the statistical model is strongly supported by the approximate isotropy of the continuous part of the α spectra, a condition not satisfied in the (n,n'), (n,p), (p,n), and (p,p') reactions. Recently, Facchini *et al.*[9] used the evaporation model to calculate the relative total cross sections for proton and neutron emission for 14-Mev incident neutrons. The results were in good agreement with experimental observations. Similar analysis and conclusions have been reported by Allan.[10] On evaluating the results obtained at higher bombarding energies (>15 Mev for protons and 40 Mev for α particles), Colli *et al.*[7] conclude that instantaneous or nonequilibrium emission leads to an intensification of the high-energy part of the spectra at all angles; consequently, at these bombarding energies it is not possible to extract the evaporation component.

Despite the impressive amount of data on the 14-Mev (n,p) spectra which is consistent with the evaporation process, it is not certain that these neutron-induced reactions always involve the formation of a compound nucleus. The optical model predicts mean free paths of several fermis for nucleons at 14 Mev[4]; furthermore, the mean free path increases as the energy decreases. It would thus appear that the successive collisions required to attain thermal equilibrium would be improbable. It is by no means clear that one can distinguish evaporation and direct interaction processes on the basis of angular distribution above. While direct interactions to discrete states generally lead to strong emission in the forward direction, distortion and exchange effects can produce emission in the backward direction. It should also be noted that for highly excited final states the variation of momentum transfer with angle becomes small, thereby reducing the variation of direct process yield with angle. Thus, further experimental or theoretical information is needed in order to find the relative contributions of the two mechanisms to the (n,p) spectra. An investigation of the spectra obtained at different incident energies would provide valuable information. Such investigations have been carried out by Lassen *et al.*[8] for the (α,α') reaction, by Cohen and Rubin[11] for the (p,p') reaction, and by Fulmer and Goodman[12] for the (p,α) reaction.

The measurements of Lassen and his collaborators[8] on the (α,α') reaction using 12–20-Mev α particles show that the continuous parts of the α spectra agree very well with the predictions of the evaporation model. The direct interaction and evaporation portions of the spectra are apparently readily distinguishable. This separability of the two mechanisms is consistent with the optical model parameters[4,13] for α particles which indicate that nuclei are essentially black for α particles.

[3] P. C. Gugelot, Brookhaven Conference on Statistical Aspects of the Nucleus [Brookhaven National Laboratory Report BNL-331 (C-21), 1955 (unpublished), p. 89].

[4] D. C. Peaslee, Ann. Rev. Nuclear Sci. **5**, 99 (1955).

[5] K. J. Le Couteur, *Nuclear Reactions* (North Holland Publishing Company, Amsterdam, 1959), Vol. 1, p. 318.

[6] T. Ericson, *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1960), Vol. 9, p. 425.

[7] L. Colli, U. Facchini, I. Iori, M. G. Marcazzan, and A. M. Sona, Nuovo cimento **13**, 730 (1959). (This article contains 82 references; we shall make specific references only to work which is immediately relevant to the present paper.)

[8] N. O. Lassen and N. O. Roy Poulsen, *International Conference on Low-Energy Nuclear Physics, Paris, July, 1958* (Dunod, Paris, 1959); H. W. Fulbright, N. O. Lassen, N. O. Roy Poulsen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd **31**, No. 10 (1959).

[9] U. Facchini, I. Iori, and E. Menichella, Nuovo cimento **16**, 1109 (1960).

[10] D. L. Allen, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960* (University of Toronto Press, Toronto, 1960), p. 838.

[11] B. L. Cohen and A. G. Rubin, Phys. Rev. **113**, 579 (1959).

[12] C. B. Fulmer and C. D. Goodman, Phys. Rev. **117**, 1339 (1960).

[13] G. Igo, Phys. Rev. Letters **1**, 167 (1958).

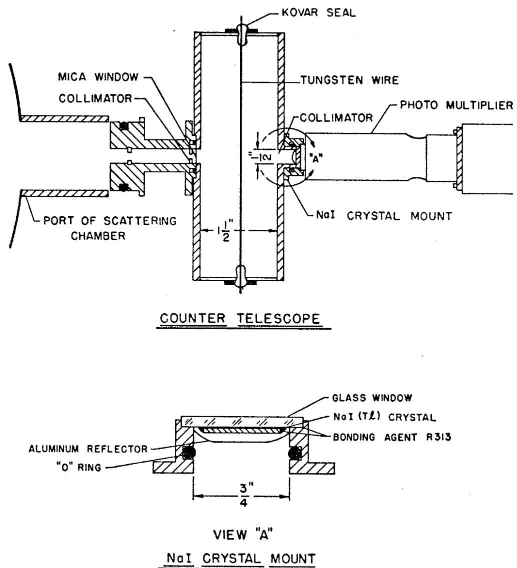

FIG. 1. Schematic diagram of experimental arrangement and details of the NaI-proportional counter telescope.

Thus, α particles passing through the nuclear surface dissolve so that direct interactions can occur only at the surface. Emission of α particles from the interior would then take place after an appreciable time as evaporation particles. On the other hand, the protons emitted in the (α,p) reaction at 20 Mev do not seem to be entirely due to evaporation[14]; they could come from knock-on particles from the surface or as fragments of the dissolved α particle.[5]

In the present paper we shall discuss several reactions with a view to determining the range of validity of the evaporation model in describing the spectra of α particles, protons, and neutrons. The experimental results of the investigation of the (p,α) reaction using 15–19-Mev protons are presented in Secs. II, III, and IV(c). The analysis of these results in terms of the evaporation model is described in Sec. IV. Comparisons with previous experiments involving α emission are given in Sec. V. In Sec. VI, the experimental results of Cohen and Rubin[11] on inelastic proton scattering are analyzed and discussed. In Sec. VII, various experimental results on the level density of Fe^{56}, and their implications with regard to the evaporation model, are discussed. A summary is given in Sec. VIII.

II. EXPERIMENTAL METHOD

The angular distribution and energy spectra of α particles from the (p,α) reaction were investigated using the 15–19-Mev proton beam of the Princeton FM cyclotron. The α-particle spectra were measured at different angles with a proportional counter–scintillation counter telescope. (Fig. 1.) The former discriminated well against protons, γ rays, deuterons, and tritons. A He^3 particle of a given energy has almost the same energy loss as an α particle and these would be difficult to separate. Fortunately, (p, He^3) reactions have large negative Q values and could only contribute through transitions to ground and near-ground states. Such transitions are strongly peaked at forward angles for (p,p'), (p,d), and (p,α) reactions for 18-Mev protons and one might expect the (p,He^3) reactions to be similar. No evidence for this reaction was found, but there remains the possibility of a slight He^3 contamination at the lowest He^4 energies.

The NaI(Tl) crystal was cleaved just thick enough to stop 30-Mev α particles. 18-Mev protons lose 3 Mev in such a crystal but because of their better response in NaI give a pulse equivalent to an α particle of about 6 Mev. The latter is considerably below the Coulomb barrier for α particles for the targets used and it would not be important if some proton pulses were gated in by chance coincidences. The crystal itself was exposed directly to the counter gas except for a 0.15-mg/cm^2 aluminum reflector. With this arrangement, 2.9-Mev α particles just reached the crystal and 5.3-Mev polonium α particles gave a peak whose resolution was 350 kev.

Energy loss curves for protons, deuterons, tritons, He^3, and alpha particles passing through the counter gas were computed. The energy losses were calibrated with Po alphas and known reactions. An integral discriminator was used to select those pulses from the proportional counter which corresponded to the desired range of α-particle energies. The scintillation spectrum was recorded on a 20-channel analyzer which was gated by the pulses from the integral discriminator.

The energy response of the system was obtained with a Po^{210} source and with α particles from the $C^{12}(p,\alpha)B^9$, $Al^{27}(p,\alpha)Mg^{24}$, and $F^{19}(p,\alpha)O^{16}$ reactions. As can be seen from Fig. 2, the calibration curve is nonlinear. The energy of the proton beam was measured several times with an energy measuring and regulating device.[15] At

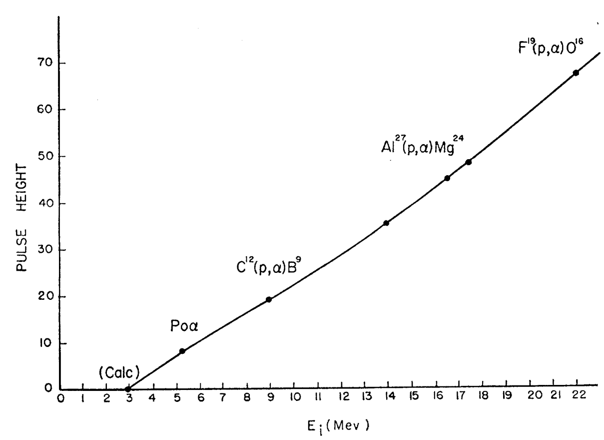

FIG. 2. Energy calibration curve of counter telescope. E_i is the laboratory energy of the α particle on entry into the telescope.

[14] N. O. Lassen and V. A. Sidorov, Nuclear Phys. **19**, 579 (1960).

[15] G. Schrank. Rev. Sci. Instr. **26**, 677 (1955).

these times, the calibration spectra were taken. During each run on the (p,α) spectra of other targets, the over-all gain of the system was checked with the Po^{210} source and the beam energy was monitored by observing alpha particles from the $C^{12}(p,\alpha)B^9$ reaction.

Although the proportional counter distinguishes easily between alphas and protons, or deuterons, spurious counts can arise from pileup or from chance gating of protons by neutron recoils of the counter gas. Chance coincidences due to neutron recoils were observed at high beam currents and forward angles, and were avoided during the final measurements.

III. EXPERIMENTAL RESULTS

The α-particle spectra at various angles were obtained for natural targets of V, Fe, Co, Ni, and Rh. Each pulse-height spectrum $N(h)$ was converted to an energy spectrum $N(E_i)$, where E_i is the energy of the α particle incident on the telescope. $N(E_i)$ included the correction for the variation of slope of the (E_i vs h) calibration curve (Fig. 2). Next, correction was made for the foil thickness to obtain $N(E_l)$, where E_l is the laboratory energy of the α particles on leaving the nucleus. As the foils were usually 2 to 3 mg/cm² thick, it was assumed that all the α particles passed through half the foil and the energy scale was corrected for the corresponding energy loss; the correction to $N(E_l)$ due to the variation of energy loss with energy in the target was ignored. (Trial calculations were made for an 8-mg/cm² target, integrating over the foil thickness with appropriate correction for variation of energy loss. It was found that this correct spectrum would be about 200 kv broader and the peak position lowered by a similar amount relative to the spectrum obtained with

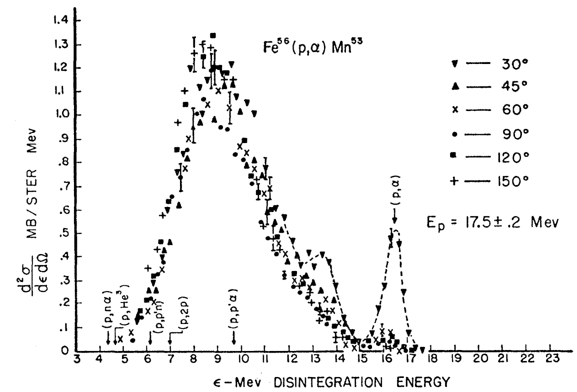

FIG. 4. Spectrum of α particles from $Fe^{56}(p,\alpha)Mn^{53}$.

the less complete correction. It was estimated that the value of T for V in Table II should be increased by a few percent and the total cross section of V in Table III should be increased by $\sim 10\%$; corrections for the other targets would be smaller). Having obtained $N(E_l)$ vs E_l, conversion to channel energy was then carried out.

The final results of these transformations of the experimental data for V, Fe, Co, Ni, and Rh are shown in Figs. 3, 4, 5, 6, and 7, where the absolute center-of-mass differential cross section is plotted against the channel energy. The angles refer to laboratory angles; these differ only slightly from the center-of-mass angles. The target thicknesses were 8.1, 2.3, 2.5, 2.8, and 2.5 mg/cm², respectively. The spectra for a target at different reaction angles were obtained at different times. The quality of the statistics is evident from the few error flags shown. Within the uncertainties in statistics and absolute calibration, the spectra are fairly isotropic for V, Fe, and Co for angles beyond 30°, except at the high-energy end of the spectra. The same conclusion is suggested by the Ni data. In the case of Rh, the 90° spectrum appears to be lower than the 60° and 120° spectra near the peak of the spectra, but the experi-

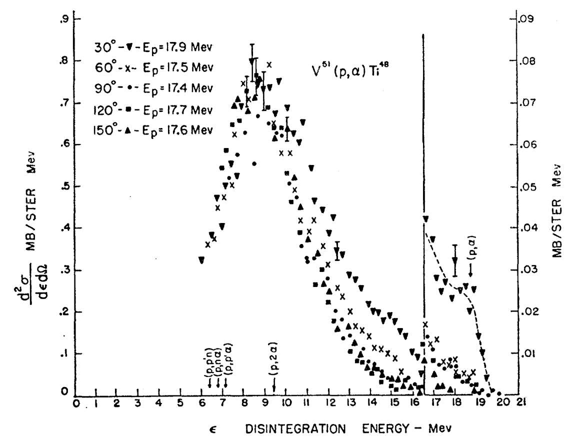

FIG. 3. Spectrum of α particles from $V^{51}(p,\alpha)Ti^{48}$ reaction. The vertical scale at the high energy end of the spectrum has been enlarged. In this and succeeding figures of spectra, the maximum energies corresponding to various reactions are indicated by arrows. The laboratory bombarding energies and laboratory angles are given in the graphs. The abscissa is the total kinetic energy of disintegration (α particle plus recoil) in the center of mass system.

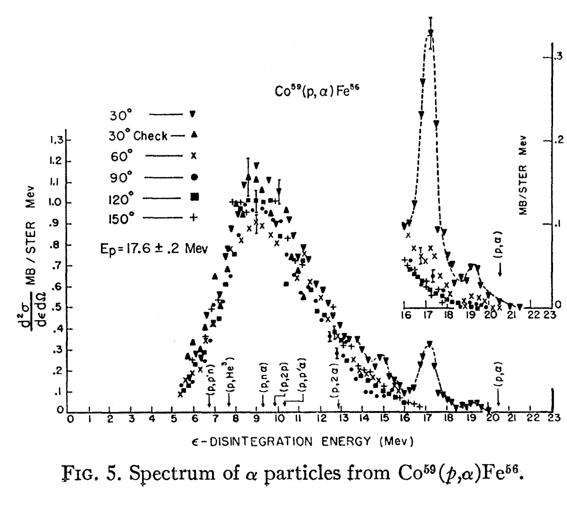

FIG. 5. Spectrum of α particles from $Co^{59}(p,\alpha)Fe^{56}$.

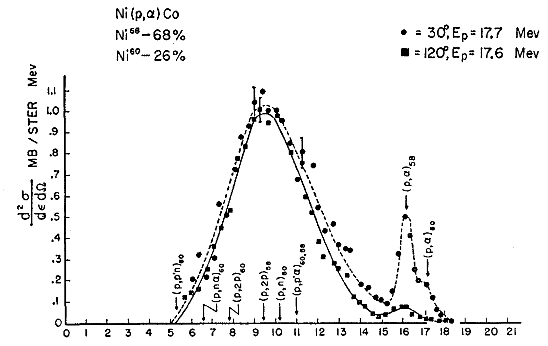

FIG. 6. Spectrum of α particles from Ni(p,α)Co for a natural target (Ni^{58} 68%, Ni^{60} 26%). The maximum energies for various reactions are indicated with subscripts 58 and 60 corresponding to these two most abundant isotopes.

mental inaccuracy precludes a firm conclusion on this point. Forward peaking of high-energy α particles is evident in all reactions at least up to 4-Mev excitation and is particularly marked for the $Rh^{103}(p,\alpha)Ru^{100}$ reaction. Silver as a target element gave similar results. Rhodium was chosen as a representative nucleus for this region of A.

Indicated in Figs. 3, 4, 5, and 6 are the energies available for various reactions. In the case of V, secondary α particles (following a primary neutron or proton) could not compete in an evaporation process with a secondary nucleon. For Fe, secondary α particles would not compete with secondary protons following neutron emission. If a proton were emitted first with an energy between 7.0 and 9.6 Mev, only a secondary α particle or γ ray could be emitted; however, the maximum energy of the α particle would be 2.6 Mev and would not pass through the counter window. Lower energy primary protons would (with rapidly increasing probability) be followed by a proton or neutron rather than an α particle. Similar considerations hold for Co and Ni. It is therefore unlikely that secondary (evaporated) α particles with energies greater than 5 Mev would be observed, nor is a significant contamination of

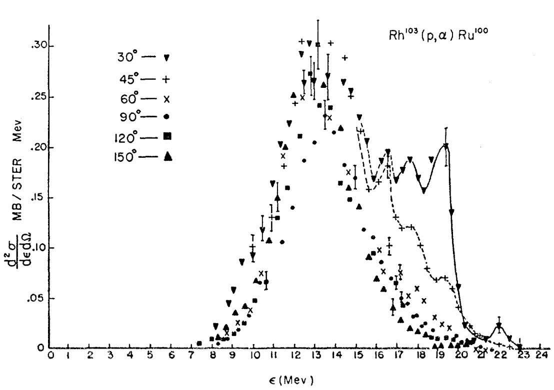

FIG. 7. Spectrum of α particles from $Rh^{103}(p,\alpha)Ru^{100}$.

TABLE I. Comparison of present Q values with previously determined Q values. Column 2 gives the differential cross section per Mev of the maxima of the ground-state experimental peaks.

Reaction	$d^2\sigma(30°)/d\Omega d\epsilon$ (mb/sr Mev)	Experimental Q (Mev)	Mass value Q (Mev)
$V^{51}(p,\alpha)Ti^{48}$	$\lesssim 0.025$	1.1 ± 0.4	1.16 ± 0.005
$Fe^{56}(p,\alpha)Mn^{53}$	0.50	-1.0 ± 0.3	-1.05 ± 0.01
$Co^{59}(p,\alpha)Fe^{56}$	0.05	1.95 ± 0.25	3.23 ± 0.01
$Ni^{58}(p,\alpha)C^{55}$	0.50	1.30 ± 0.25	-1.34 ± 0.02
$Ni^{60}(p,\alpha)Co^{57}$	0.20	-0.25 ± 0.25	-0.025 ± 0.03
$Rh^{103}(p,\alpha)Ru^{100}$	0.02	4.5 ± 0.3	5.6 ± 1.0

He^3 likely. The experimental spectra are consistent with these arguments in that there is nothing seen which points to secondary α or He^3 particles; nevertheless, one cannot rule out such events with complete certainty.

The highest energy groups at 30° can be used to measure Q values, assuming that these groups correspond to ground-state transitions. Table I compares the experimental Q values with those obtained from mass measurements and other reaction data,[16] and includes the cross sections at the maxima of the corresponding groups.

The agreement in Q values is good for V, Fe, Rh, and the Ni isotopes, but poor for Co. It would appear that the highest energy peak (at ~19.3 Mev) for the Co^{59} is not due to the ground-state transition; it would correspond to an excited state at $\sim1.2\pm0.3$ Mev in Fe^{56}. The known[17] states in Fe^{56} are at 0.845, 2.085, 2.661 Mev, and higher. The group at 1.2 Mev is apparently due to the first two excited states (unresolved). It is evident that the cross section for the ground state of Fe^{56} is appreciably less than 0.05 mb/sr Mev at 30°.

The ground-state cross sections for the V, Fe, Co, and Ni targets show an interesting variation. For Fe and Ni, the peak cross sections are of the order of 0.5 to 0.8 mb/sr Mev, while for V and Co they are <0.025 and 0.01, respectively. For the former pair, the initial and final states are 0^+ and $\frac{7}{2}^-$, respectively, while for the latter the states are $\frac{7}{2}^-$ and 0^+. Thus, the angular momentum changes involved are the same and the linear momentum transfers are approximately the same in all four cases. It is probable that this large variation in cross section must be attributed to the shell structure of the nuclei involved. Preliminary calculations by Tobocman and by Bayman[18] indicate that such large variations can occur in both pickup and knockout descriptions of the (p,α) reaction.

IV. ANALYSIS OF THE (p,α) SPECTRA

A. Theory

According to the evaporation model, the differential cross section at a fixed bombarding energy for (p,α)

[16] V. J. Ashby and H. C. Catron, University of California Radiation Laboratory Report UCRL-5419, 1959 (unpublished).
[17] M. Mazari, A. Sperduto, and W. W. Buechner, Phys. Rev. **107**, 365 (1957).
[18] W. Tobocman, B. F. Bayman (private communications).

reaction (assumed to be isotropic) is given by[2]

$$\frac{d^2\sigma(\epsilon,\epsilon_p)}{d\Omega d\epsilon}=\sigma_c(\epsilon_p)\frac{(2M_\alpha/\hbar^2)\epsilon\sigma_c(\epsilon)\omega(E_x)}{4\pi\sum_i F_i}, \qquad (1)$$

where ϵ_p and ϵ are the incident and exit channel energies (the sum of the kinetic energies of the particle and target (or residual) nucleus in the center-of-mass system). $\sigma_c(\epsilon_p)$ and $\sigma_c(\epsilon)$ are the cross sections for protons and alpha particles for forming the compound nucleus with the target and (excited) residual nucleus, respectively. The level density of the residual nucleus at excitation $E_x=\epsilon_0-\epsilon$ is represented by $\omega(E_x)$. ϵ_0 is the maximum channel energy available for decay and is given by $\epsilon_0=\epsilon_p+Q_{p\alpha}$. M_α is the reduced mass of the α particle. F_i is the energy integral of the numerator of (1) for each of the possible reactions (p,n) (p,p') (p,α), etc. These integrals over the various spectra are constants at a given bombarding energy. Equation (1) is useful in a limited range of ϵ; the lower limit is set by the possible presence of low energy secondary α particles, and the upper limit should correspond to $E_x\approx 3$ Mev for medium heavy nuclei.

The energy dependence of the level density is obtained from the differential spectrum (1):

$$\omega(E_x)=\omega(\epsilon_0-\epsilon)\propto\frac{1}{\epsilon\sigma_c(\epsilon)}\frac{d^2\sigma}{d\Omega d\epsilon}. \qquad (2)$$

Total cross sections can be calculated integrating (1) over energy and solid angle. In the following sections the experimental results are analyzed in terms of these theoretical expressions.

B. Energy Distribution

The isotropy of the observed spectra beyond 30° is consistent with an evaporation process. Using an average of the 90°, 120°, and 150° data for each target, the present results were analyzed in terms of Eq. (2). Semilogarithmic plots were made of differential cross section divided by $\epsilon\sigma_c(\epsilon)$ versus ϵ, or E_x, the excitation energy. These plots should give the logarithm of the relative density of the residual nucleus as a function of excitation energy. (We shall refer to these graphs as level density plots.)

In order to carry out this analysis, one needs values of the inverse cross sections $\sigma_c(\epsilon)$. Tables of $\sigma_c(\epsilon)$ for α particles have been published by Shapiro,[19] Blatt and Weisskopf,[20] and Feshbach, Shapiro, and Weisskopf.[21] These tables are based on the boundary condition model[20] with the assumption of a Coulomb potential which is sharply cut off at an interaction radius R. $\sigma_c(\epsilon)$ is very sensitive to the choice of this interaction radius, particularly for α-particle energies which are below the Coulomb barrier (approximately $3Z/R$ Mev, with R in fermis). Thus the form of the energy dependence of relative level density one obtained depends strongly on the choice of R. This point is illustrated in Fig. 8 for the Fe data. The various choices of R are indicated together with the values of r_0 which correspond to

$$R=r_0A^{\frac{1}{3}}+2.2. \qquad (3)$$

The use of this formula is suggested by the analysis of the elastic scattering of α particles in terms of the sharp-cutoff model of Blair.[22,23] The values of R used in Fig. 8 were chosen for convenience in the use of the tables for $\sigma_c(\epsilon)$.

The various models which are used to predict the level density lead to $\omega(E_x)\propto\exp(bE_x^\beta)$ with $\frac{1}{2}\leq\beta\leq 1$. It is evident from Fig. 8 that a value of R can be chosen to correspond to any choice of β. We will restrict the discussion to $\beta=\frac{1}{2}$, corresponding to the Fermi gas model, and to $\beta=1$. Thus for $\beta=\frac{1}{2}$, the curve for the level density in Fig. 8 should be concave downward, as for $R=8.45$ fermis, while for $\beta=1$, one expects a straight line, as for $R=7.22$ f. We shall refer to the case $\beta=\frac{1}{2}$ as a "Fermi-gas level density"; this is usually expressed as $\omega(E_x)\propto\exp[2(aE_x)^{\frac{1}{2}}]$. We shall refer to the case $\beta=1$ as the "constant-temperature level density", and use the usual expression $\omega(E_x)\propto\exp(E_x/T)$.

Similar plots were made for the other targets using an

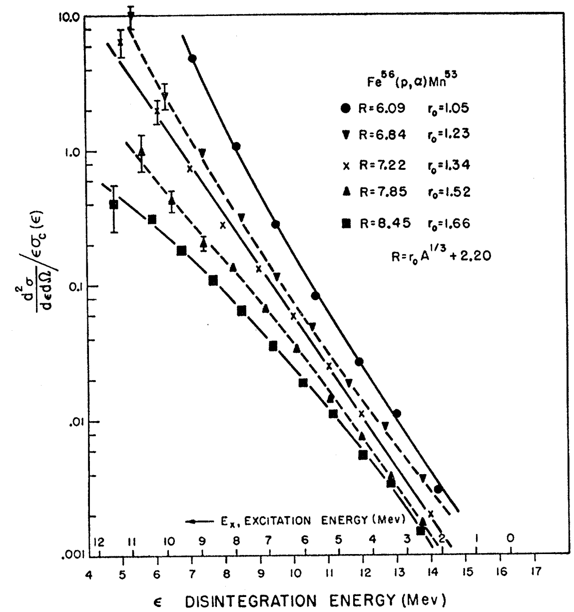

FIG. 8. Relative level density of Mn^{53} from the $Fe^{56}(p,\alpha)Mn^{53}$ reaction. The various values of R and r_0 (in fermis) refer to the radius of Mn^{53} used in calculating the inverse cross sections. Two scales are shown for the abscissa, the disintegration energy and the excitation energy.

[19] M. M. Shapiro, Phys. Rev. **90**, 171 (1953).
[20] Reference 2, Table 4.1, p. 352.
[21] H. Feshbach, M. M. Shapiro, and V. F. Weisskopf, Atomic Energy Commission Report NOA 25-B-3, (NYO-3077) (unpublished).
[22] J. S. Blair, Phys. Rev. **95**, 1218 (1954).
[23] D. D. Kerlee, J. S. Blair, and G. W. Farwell, Phys. Rev. **107**, 1343 (1957).

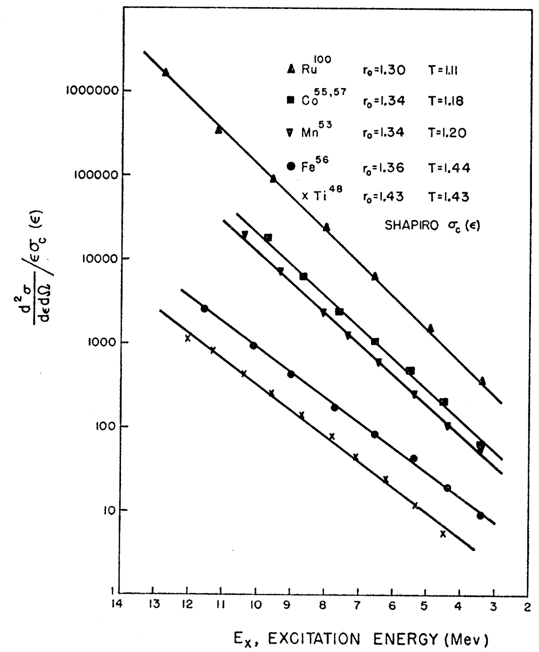

FIG. 9. Relative level density vs E_x for radii appropriate to a constant temperature level density. The corresponding values of r_0 (in fermis) and T (in Mev) are given for the various residual nuclei. The inverse cross sections were calculated using the tables of Shapiro.[19]

average of the 90°, 120°, and 150° data for each target. For values of $r_0 \approx 1.35$ fermis, the level densities have the constant-temperature form, while for $r_0 \geq 1.53$ f, the Fermi-gas form is obtained. Semilog plots of relative level density which are linear when plotted vs E_x and $E_x^{\frac{1}{2}}$ are shown in Figs. 9 and 10; the corresponding values of r_0 and T for the former, and of r_0 and a for the latter, are given in the figures.

It is evident that the experimental data can be represented equally well by either level density form. The analysis of elastic scattering of α particles by Kerlee, *et al.*[23] using the sharp-cutoff model,[22] yielded a value of of 1.4 f for r_0. With this radius our results are in much better agreement with the constant temperature level density than with the Fermi-gas level density.

Another method of obtaining the inverse cross section $\sigma_c(\epsilon)$ is based on the analysis of α scattering using the optical model. Igo[24] recently carried out this analysis and obtained the total reaction cross section $\sigma_c(\epsilon)$ for the ground state using a Woods-Saxon potential based on the Hill-Ford charge distribution. These two potentials produce a barrier which is smaller in height but thicker radially than the Coulomb potential with a sharp cutoff radius of $1.4A^{\frac{1}{3}}+2.2$ f. Igo compared his calculated cross section with α-induced excitation functions and concluded that the agreement is good.

The cross sections published by Igo[24] were inadequate for analysis of the present data. Recently appropriate cross sections were calculated by Igo and Huizenga[25] and we have used these cross sections to analyze the present data. The results are seen in Fig. 11. Constant-temperature level densities are again found, with temperatures somewhat higher than those obtained with continuum model cross sections. By interpolation of the curves of Fig. 8 for the various targets we obtained the temperatures corresponding to $r_0=1.4$ f for the Shapiro cross sections. These temperatures and the corresponding ones, for the optical potential cross sections are compared in Table II.

It should be noted that the optical model calculations yield reaction cross sections which include direct interaction as well as compound nucleus reactions. It is therefore probable that the optical model cross sections are larger than the inverse cross sections required here and furthermore that this difference increases with increasing energy. From our data the total direct-interaction cross section is small ($\leq 10\%$ for V$-$Ni), so that this effect is unimportant here. However, at higher bombarding energy this effect could become significant.

We conclude on the basis of the above discussion that analysis of the observed spectra in terms of the evaporation model leads to a constant-temperature level density for the range of excitation from 3 to 12 Mev. The value of T depends only slightly (10%) on the choice of model for calculating $\sigma_c(\epsilon)$. (The error in the temperature assigned to each curve, based on the

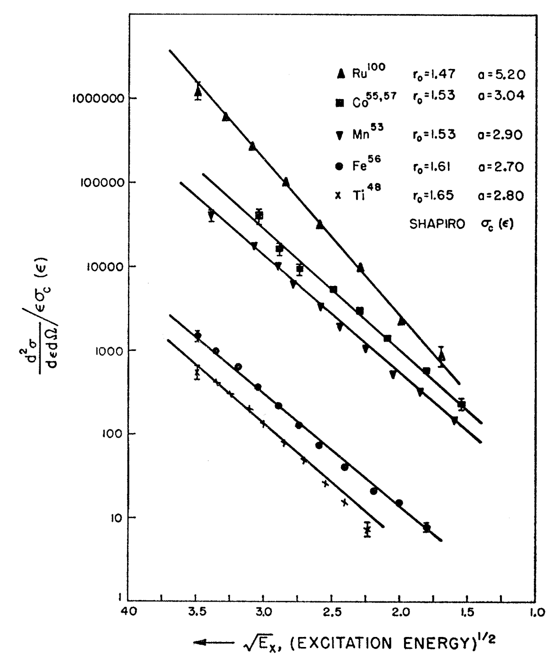

FIG. 10. Relative level density vs $E_x^{\frac{1}{2}}$ for radii leading to a shape consistent with the Fermi level density $\exp[2(aE_x)^{\frac{1}{2}}]$. The values of r_0 (in fermis) and a (in Mev^{-1}) for the various residual nuclei are shown. The inverse cross sections used were obtained from the tables of Shapiro.[19]

[24] G. Igo, Phys. Rev. **115**, 1665 (1959).

[25] G. Igo and J. R. Huizenga (private communication).

TABLE II. Comparison of temperatures obtained using the $\sigma_c(\epsilon)$ values of Shapiro with the Blair-Kerlee-Farwell radius, $1.4A^{1/3}+2.2$ fermis, and Igo-Huizenga optical model values of $\sigma_c(\epsilon)$.

Target	Residual nucleus	T (Shapiro) (Mev)	T (Igo) (Mev)
V^{51}	Ti^{48}	1.42	1.59
Fe^{56}	Mn^{53}	1.21	1.33
Co^{59}	Fe^{56}	1.46	1.54
$Ni^{58,60}$	$Co^{55,57}$	1.20	1.32
Rh^{103}	Ru^{100}	1.17	1.31

scatter of points, is estimated to be ~5%.) We note for the first four nuclei, closely spaced in A, that the even-even residual nuclei appear to have higher temperatures than the odd-even nuclei, so that the level density increases more rapidly with excitation in the latter case.

C. Dependence of α Spectra on Bombarding Energy

Fulmer and Goodman[12] have investigated the Ni(p,α) reaction using bombarding energies of 11.3, 13.1, 14.9, 17.2, 20.02, and 22.8 Mev. They found that the energy of the peak of the spectra remains essentially constant independent of bombarding energy. They calculated that for a Fermi-gas level density, the energy of the peak should drop by 15% on reducing the bombarding energy from 22.8 to 11.3 Mev. To account for the absence of this expected shift, they attributed this effect to a decrease of Coulomb barrier with excitation. They note, however, that a constant-temperature level density would also explain the constancy of the peak. The latter follows immediately from Eq. (1), for with $E_x=\epsilon_0-\epsilon$ and $\epsilon_0=\epsilon_p+Q$, the spectrum would factor into a product of a function of bombarding energy and Q, and a function of disintegration energy:

$$N(\epsilon)\propto f(\epsilon_p,Q)[\epsilon\sigma_c(\epsilon)e^{-\epsilon/T}]. \tag{4}$$

$f(\epsilon_p,Q)$ would determine the relative cross section for α emission, but the spectral shape would be given by the bracketed factors. The latter has its maximum at some particular value of ϵ, independent of bombarding energy if T is a constant.

There is, however, a way of testing the hypothesis of constant temperature in a more detailed way than is provided by the constancy of the energy of the peak. According to Eq. (4), the spectra at different bombarding energies should be similar except for multiplicative factors. Intercomparing the experimental spectra of Fulmer and Goodman (excluding energies corresponding to excitation energies less than 2 Mev for Co^{55}), we found that the similarity was reasonably good.

It should be emphasized that the comparison of spectra at different bombarding energies is a more direct way of testing for a constant-temperature level density than attempting to fit straight lines to the level density plots. The latter depend critically on the choice of parameters for $\sigma_c(\epsilon)$. For a comparison of shapes it is sufficient to use the experimental pulse height spectra without the various corrections and transformations since these depend only on ϵ for a given target. (There is a negligible correction for bombarding energy in the transformation from laboratory to c.m. coordinates.)

With this in mind, the (p,α) spectra of Ni and Co at different proton energies were determined. In Fig. 12, the experimental yields vs channel number for Ni (at 135°) at 19.4- and 15.6-Mev bombarding energy are shown. The two spectra were normalized at the peaks of the distributions. The agreement is very good from channel 11 to channel 35 ($\epsilon=7$ to 12 Mev). These channels correspond to $E_x=2$ to 7 Mev for the 15.6-Mev spectrum and $E_x=6$ to 11 Mev for the 19.4-Mev spectrum. Beyond channel 35 one would not expect agreement since this region would involve $E_x<2$ Mev. In Fig. 13 a similar comparison is made for Co^{59} (at 135°) for bombarding energies of 19.0 and 15.4 Mev. In this case the curves agree from channel 15 to channel 55. (There is some indication of a weak group at channel 48 at the higher bombarding energy.) Because the Q is higher for the Co reaction, the range of ϵ (7 to 15 Mev) corresponds to $E_x=3.5$ to 11.5 Mev and $E_x\cong 7$ to 15 Mev at the lower and higher bombarding energies. Finally, as an indication of the sensitivity of this test for a constant temperature level density, Fig. 14 displays spectra for Co calculated on the basis of a Fermi

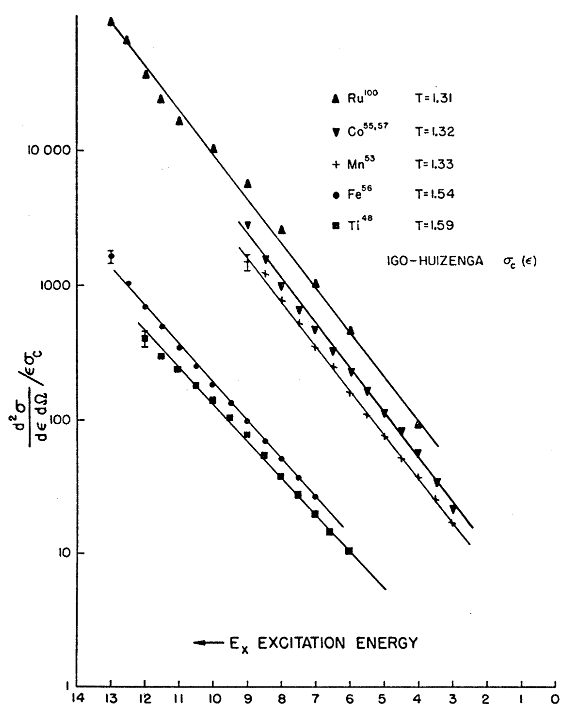

FIG. 11. Relative level density vs E_x. The inverse cross sections used are the optical model cross sections calculated by Igo and Huizenga.[24,25]

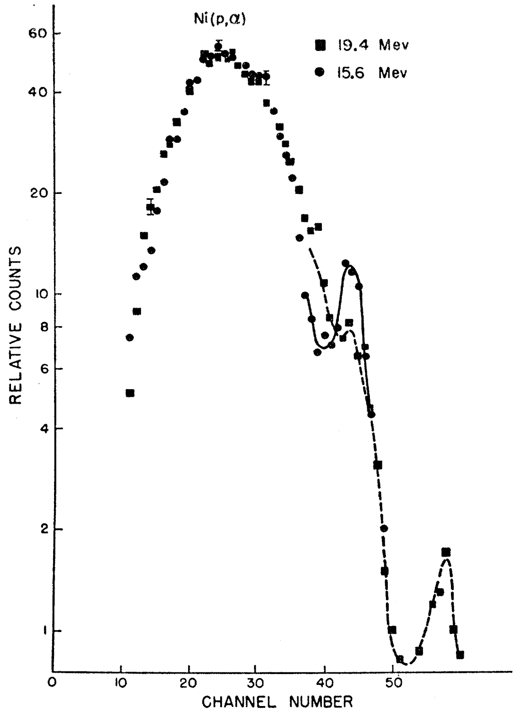

FIG. 12. Ni(p,α) spectra at 15.6- and 19.4-Mev bombarding energy.

level density using the optical-model cross sections for values of $a=3.0$ and $a=3.5$ Mev^{-1} which are in the appropriate range in the present case. Comparison of Figs. 12, 13, and 14 shows that a constant-temperature level density is consistent with the experimental results while the Fermi level density is not. We conclude that the (p,α) spectra are consistent with a constant-temperature level density and that T is independent of bombarding energy in the range 15–19 Mev. It should be emphasized that these conclusions are based on the

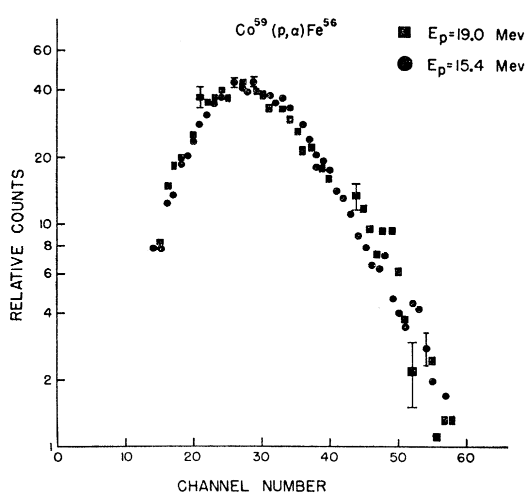

FIG. 13. Co(p,α) spectra at 15.4- and 19.0-Mev bombarding energy.

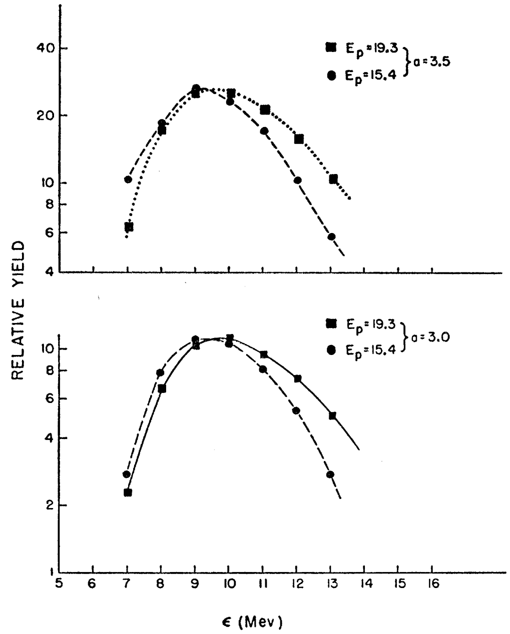

FIG. 14. Calculated spectra for Co(p,α) at 15.4- and 19.0-Mev bombarding energy for a Fermi-gas level density.

assumption that $\sigma_c(\epsilon)$ is independent of E_x; this point is discussed in Sec. VIII.

D. Total Cross Section

By integrating Eq. (1), one can, in principle, calculate the total (p,α) cross section. However, one has to assume appropriate level densities for the residual nuclei following proton and neutron emission. The (p,p')[26] and (p,n)[27] spectra for 18-Mev incident protons are forward peaked, and in general do not appear to be entirely due to evaporation. Ignoring this difficulty and assuming that the level densities are in all cases to be $\omega(E_x)=C\exp(E_x/T)$, total (p,α) cross sections were calculated for several sets of parameters. These calculations are summarized in Table III. C was assumed to be the same for all the residual nuclei.

The calculations in Table III were made several years ago and are based partially on incorrect Q values for the (p,α) reaction; the correct values for the latter would decrease the calculated total cross section for Fe^{56} and V^{51}. Further refinements could be made regarding the level density dependence for even-even, even-odd and odd-odd nuclei. However, these, the temperatures, and the relative importance of direct interaction in the (p,n) and (p,p') reactions are not sufficiently well known to warrant more detailed calculations. One can conclude that the observed (p,α) total cross sections are not inconsistent with the evaporation model.

[26] P. C. Gugelot, Phys. Rev. **93**, 425 (1954).
[27] D. M. Thomson, Proc. Phys. Soc. (London) **A69**, 447 (1956).

V. COMPARISON OF PRESENT RESULTS WITH PREVIOUS OBSERVATIONS

A. (p,α) Spectra

Fulmer and Cohen[28] investigated the (p,α) spectra for a variety of nuclei using 23-Mev protons. They found that for $Z<50$, the spectra are moderately isotropic beyond 60° if the highest energy particles are excluded. Fulmer and Goodman[12] studied the dependence of (p,α) spectra on bombarding energy [see Sec. IV (C)]. In general, our results are in qualitative agreement with the results reported in these two papers. However, the maxima of the spectra of Fulmer and his co-workers occur at a lower energy. They found the peak for Ni, at a bombarding energy of 17.2 Mev, at a total kinetic energy of 8.15 Mev,[12] whereas our spectrum peaks at 9.4 Mev. Similarly, they found the Rh peak at 17-Mev bombarding energy[12] about 1 Mev lower than the peak in our Rh spectra. The total cross section for Ni at 17.6 Mev was found to be 55 mb in the present experiment, in excellent agreement with 53 mb obtained by interpolation of their total cross-section data.

The disagreement in energy scale is somewhat larger than the estimated uncertainties (~0.2 Mev in our scale, and ~0.5 in theirs[12,28]). To resolve this discrepancy, Hill and Sherr[29] measured the Ni(p,α) spectrum at 28° and 17.6 Mev with a magnetic analyzer. A comparison between their results and the 30° curve of Fig. 6 is shown in Fig. 15. The agreement is good both in energy scale and in shape, and also in absolute cross section. We have recently determined the (p,α) spectra of Co and Ni using a telescope with a p-n junction detector, with the same calibration procedures used in obtaining Fig. 2. The results substantiate our earlier measurements.

Fulmer and his coauthors[12,28] ascribed their α spectra to the compound nucleus reaction. However, because the Maxwellian peaks of their spectra occurred at unexpectedly low energy they concluded that the Coulomb barrier for excited states was lower than for the ground state. The preceding discussion on energy calibrations eliminates the necessity for this conclusion for the above reason. As was mentioned in Sec. IV (C), these authors also ascribed the lack of dependence of the energy of the Maxwellian peak on bombarding energy, if the Fermi gas level density is assumed to be correct, to a decrease of barrier with excitation. Their analysis suggests that the radius of $Co^{55,57}$ increases by a factor of 1.28 at 12.5 Mev of excitation. Lane and Parker[30] have calculated that the increase in nuclear radii should be negligible even at 30 Mev of excitation. Therefore, the Fermi-gas level density can be maintained only by modifying the inverse cross section for reasons other than variation of radius.

[28] C. B. Fulmer and B. L. Cohen, Phys. Rev. **112**, 1672 (1958).
[29] A. Hill and R. Sherr, Bull. Am. Phys. Soc. **5**, abstract III, 249 (1960).
[30] A. M. Lane and K. Parker, Nuclear Phys. **16**, 690 (1960).

TABLE III. Comparison of calculated and observed total cross sections for the (p,α) reaction. The T's refer to the temperatures assumed for the neutron, proton, and α channels, and the Q's are the respective Q values for the reactions. The experimental cross sections are accurate to about 10%.

Target	T_n (Mev)	$Q(p,n)$ (Mev)	T_p (Mev)	T_α (Mev)	$Q(p,\alpha)$ (Mev)	$\sigma_t(p,\alpha)$ (mb) (Calc.)	(Exp.)
Fe^{56}	1.15	−5.3	1.15	1.15	−0.47	34	64
Co^{59}	1.44	−1.86	1.40	1.44	+1.95	95	65
	1.15	−1.86	1.15	1.44	+1.95	12	
	1.15	−1.86	1.15	1.44	+3.23	30	
V^{51}	1.34	−1.53	1.34	1.34	+1.53	76	50
	1.16	−1.53	1.15	1.34	+1.53	22	

B. Comparison of (p,α) and (α,α') Reactions

Lassen and his collaborators[8] have investigated (α,α') reaction using 20-Mev α particles. For targets in the Fe-Zn region, the α-particle spectra show sharp peaks superimposed on a broad continuum. The continuum was found to be almost the same at 45°, 90°, and 135°. Analysis in terms of the evaporation model with inverse cross sections corresponding to the sharp-cutoff radius ($R=1.4A^{\frac{1}{3}}+2.2$) yielded a constant-temperature level density with T in the range 1.04–1.25 Mev.

In trying to compare the temperatures in Table II with those obtained in other experiments,[8,28] we have found disagreement which is in part attributable to an ambiguity in the instructions for the use of the published cross-section tables.[19,20] The energy parameter is stated to be the kinetic energy of the particle in the c.m. system, but the cross sections are tabulated as a function of the total kinetic energy of particle and recoil nucleus. The total energy is $(A+4)/A$ greater than the particle energy. For low-energy α particles this small difference in energy can make a large difference in $\sigma_c(\epsilon)$. For example, for $\epsilon=8$ Mev, incorrect use of the tables will yield a cross section which is 0.57 of the correct value for $Z=20$, while for $Z=30$ the factor is 0.43. At $\epsilon=7$ Mev, the corresponding factors are 0.45 and 0.34. The effect of this error in the level density plots is to raise the low-energy points relative to the high-energy points and therefore decrease the value of T. For this reason, the temperatures of reference 8 are lower than they should be.

The spectra of Lassen *et al.*,[8] are very similar to the (p,α) data presented above. Dr. Lassen sent us a comparison between an unpublished 90° spectrum for Fe(α,α')Fe at 20 Mev and the 90° spectrum for $Co^{59}(p,\alpha)Fe^{56}$ of Fig. 5. This comparison is shown in Fig. 16. The agreement is striking. The excitation energies of the compound nucleus Ni^{60} involved in both reactions are nearly the same (26.8 for the protons and 25.0 for the α particles) and the emission spectra are very similar. Thus the assumption of the independence of production and decay in the compound nucleus model is strongly supported by the comparison of these two experiments. Both spectra are quantitatively consistent with the evaporation model if a constant-temperature level density is assumed. With the latter assumption, the evaporation spectrum was shown Sec. IV(C) to be independent of the energy of the compound nucleus. Hence the difference in excitation of Ni^{60} in the present comparison will affect only the absolute cross sections, but not the spectral shapes.

There are a few differences between the results of the present (p,α) investigation and the (α,α') experiments. Lassen *et al.*[8] found that T for Cu decreased from 1.12 to 0.94 Mev when the incident energy was reduced from 20 to 16 Mev, while in the present investigations of the Ni and Co (p,α) reactions, T was found to be constant for an equivalent change in bombarding energy. The (α,α') angular distributions are slightly more anisotropic than the (p,α) distributions. The values of T given in Table I indicate that even-odd nuclei have a lower T than even-even nuclei, while the (α,α') experiments lead to the opposite conclusion. These minor differences, if real, between the two reactions are not considered important enough to mask the impressive similarity of the two reactions exhibited in Fig. 16.

C. (n,α) and (d,α) Reactions

The α spectra from Al, S, V, Mn, and Co bombarded with 14.8-Mev neutrons have been observed by Kumabe and collaborators.[31] These authors find spectra and angular distributions which are completely inconsistent with the (p,α) and (α,α') results discussed above. Their angular distributions are markedly anisotropic though symmetrical about 90° and the spectra are fitted with temperatures which are about 35% smaller than those reported above.

Mead and Cohen[32] have observed the α spectra resulting from the (d,α) reaction on various targets using 15-Mev deuterons. They find for $Z<50$, a Maxwellian low-energy component which they attribute to evaporation. In the region of Cu they find a temperature of about 1.2 Mev, in fair agreement with the (p,α) and (α,α') results. They show angular distributions for Ni which are slightly peaked in the forward direction. Insufficient detail is given to make an accurate comparison with our results. However, the agreement is sufficiently good to support the view that in the 10–20 Mev range of bombarding energies for protons, α particles, and deuterons, most of the α particles emitted from medium-weight targets are the result of evaporation from a compound nucleus.

VI. ANALYSIS AND DISCUSSION OF (p,p') EXPERIMENTS

Colli *et al.*[7] and Allan,[10] in their analysis of 14-Mev (n,p) experiments, found their spectra consistent with a constant-temperature level density. However, discussion of the (p,α) work indicates the importance of using different bombarding energies in order to establish the applicability of the evaporation model. This was recognized by Gugelot[26] in his (p,p') measurements and emphasized by Cohen and Rubin[11] who investigated the (p,p') reactions in the Ti-Zn region with incident bombarding energies between 11.3 and 23.0 Mev.

We have tried to deduce the relative contributions of direct interaction and evaporation to the inelastic proton spectra of Cohen and Rubin.[11] In the following discussion, we consider only the portions of the spectra which correspond to $E_x=3$ to 12 Mev.

Figure 5 of reference 11 presents level density plots of Fe^{56} for bombarding energies of 11.3, 13.4, 14.9, 17.2, 20.2, and 23.0 Mev. The curves are based on inverse cross sections of the continuum model with a radius $R=2.0A^{\frac{1}{3}}$ fermis. At a bombarding energy of 11.3 Mev, a good fit is obtained with $T=1.5$ Mev, in agreement with the (p,α) data (Table II above). At higher bombarding energies the curves become progressively more concave upward and the average slope (over the interval $E_x=3$ to 12 Mev) decreases rapidly. The basic assumption of the present analysis is that this decrease in slope is caused by an increasing contribution of direct interaction to the observed spectra.

If the inelastic protons arose solely from an evaporation process, one would expect that for different incident energies the slopes of the level density plots at those values of ϵ corresponding to a particular value of E_x would be independent of incident energy. The value of the slope might depend on E_x, but cannot depend on ϵ_p. This property follows simply from Eq. (2), for on taking the partial derivative of Eq. (2), noting that $E_x=\epsilon_0-\epsilon$ and that $\epsilon_0=\epsilon_p$, one obtains

$$\left\{-\frac{\partial}{\partial\epsilon}\left[\ln\frac{N(\epsilon)}{\epsilon\sigma_c(\epsilon)}\right]\right\}_{\epsilon_0}=\left\{-\frac{\partial}{\partial\epsilon}[\ln\omega(E_x)]\right\}_{\epsilon_0}=\left\{\frac{\partial}{\partial E_x}[\ln\omega(E_x)]\right\}_{\epsilon_0}. \quad (5)$$

Since the level density depends only on E_x, it is evident that the term on the left, the slope of the level density curve, at ϵ corresponding to E_x, is also independent of ϵ_0. If, however, direct interactions also contribute to the spectrum, the value of the slope will not be expected to remain independent of ϵ_0.

The data of Cohen and Rubin[11] described above, suggested the possibility that the contribution of direct interaction was negligible at the lowest bombarding energies and became progressively more important as the incident energy increased. If this were the case, the values of the slopes of the level density curves, corresponding for example to $E_x=4$ Mev, would be constant at low ϵ_0, but then decrease at higher ϵ_0.

[31] I. Kumabe, J. Phys. Soc. Japan **13**, 325 (1958); I. Kumabe, E. Takekoshi, H. Ozata, Y. Tsumeoka, and S. Oki, *ibid.* **13**, 129 (1958), and Phys. Rev. **106**, 155 (1957).

[32] J. B. Mead and B. L. Cohen, Phys. Rev. Letters **5**, 105 (1960).

We have analyzed the 90° Fe spectra of Cohen and Rubin[11] (Fig. 3 of reference 11) in these terms, using $r_0=1.5$ f for the radius parameter. The level density curves for $\epsilon_0=11.3$, 13.4, 14.9, 17.2, 20.3, and 23 Mev were calculated. The slopes of these curves at value of ϵ appropriate to excitation energies of 4, 7, and 11 Mev were determined. These slopes were plotted against (ϵ_0-E_x) and are shown in Fig. 17; (ϵ_0-E_x) is used as the abscissa rather than ϵ_0 itself in order to give each set of points a common zero. The uncertainties in the points are estimated to be 5–10%. For purposes of comparison, an identical analysis of the (p,α) spectra would yield a horizontal line at an ordinate of ~0.67 corresponding to $T\approx1.50$ Mev.

The ordinates of the points in Fig. 17 decrease monotonically with increasing bombarding energy and there is no compelling evidence of an approach to constancy even at the lowest values of ϵ_0-E_x. It therefore appears

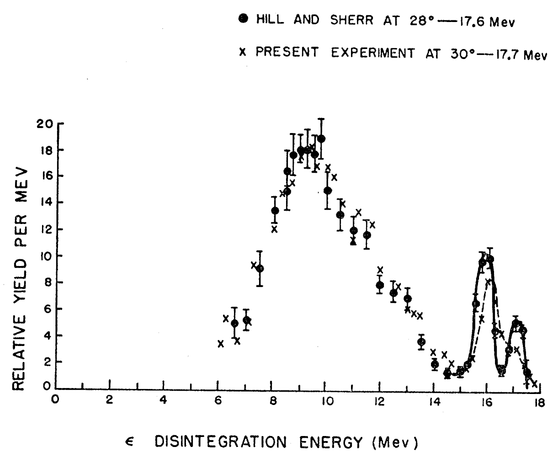

FIG. 15. Comparison of the 30° spectrum of Fig. 6 for the Ni(p,α) reaction with that of Hill and Sherr[29] at 28°.

that direct interaction may be significant at 11.3 Mev bombarding energy as well as at higher energy. However, considering the inherent difficulty in obtaining the slopes, one cannot exclude the possibility of an abrupt leveling off below $\epsilon_0-E_x\approx6.5$ Mev. It would clearly be desirable to have data at lower bombarding energies to determine if and where the slopes become constant.

All of the points in Fig. 17 seem to lie on a single curve. Since $\epsilon_0-E_x=\epsilon$, the energy of the emitted proton, one sees from Fig. 17 that the ordinates appear to depend only on ϵ, and are independent of E_x. An effect of this sort was noted and discussed by Gugelot.[26] It should be remarked that a different choice of r_0 would change the dependence of slope [Eq. (5)] on ϵ, but cannot affect this apparent lack of dependence of slope on E_x; the curve of Fig. 17 will simply look somewhat different. With $r_0=2.0$ f, the corresponding curve would be roughly the same at $\epsilon_0-E_x>13$ Mev and about 10% lower at 6 Mev.

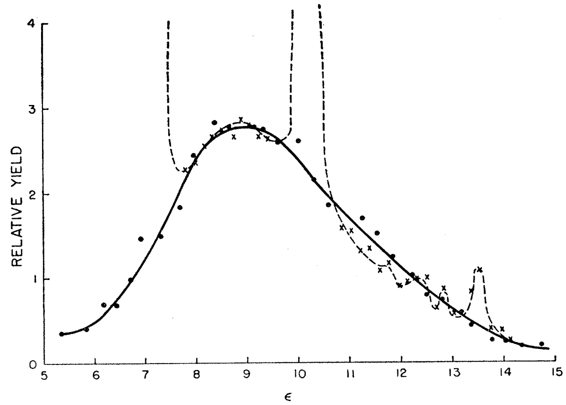

FIG. 16. Comparison of the 90° spectra ●—for $Co^{59}(p,\alpha)Fe^{56}$ at 17.6 Mev (from Fig. 5) and ×—for $Fe^{56}(\alpha,\alpha')Fe^{56}$ at 20 Mev of Lassen and Roy Poulsen.

The significance of the above observation that the slopes of the level density curves are independent of E_x is not at all clear. It is evident that the spectrum represented by Eq. (4) will give this result. So also will Eq. (4) multiplied by any arbitrary function of ϵ; indeed, Fig. 17 indicates that the (p,p') spectra require such a multiplying function, for otherwise the points of Fig. 17 would lie on a horizontal line. The implication of these remarks is that the relative shape of the (p,p') spectra is independent of bombarding energy.

In Fig. 18, the (p,p') spectra[11] for various elements are superimposed for best overlap on a semilog plot. Only the portions of the spectra corresponding to excitation energies in the interval 4–12 Mev were used in order to avoid confusion due to secondary protons from the (p,np) reaction. On the whole, with a few obvious exceptions, the spectra coincide to better than 10% from 3 to 14 Mev.

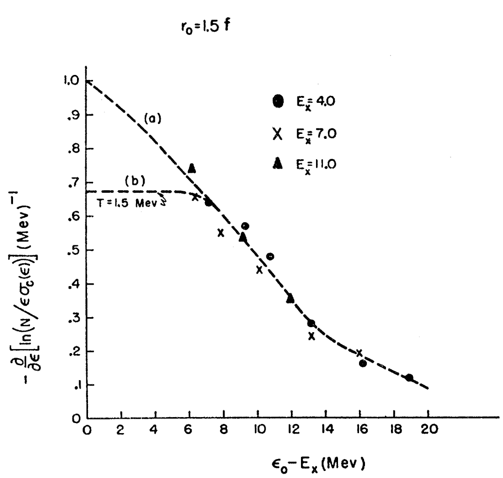

FIG. 17. Slopes of the level density curves for the 90° Fe(p,p') spectra[11] at selected excitation energies as a function of bombarding energy. If the spectra were due to evaporation, the points would lie on horizontal lines. The dashed lines near the origin are reasonable extrapolations.

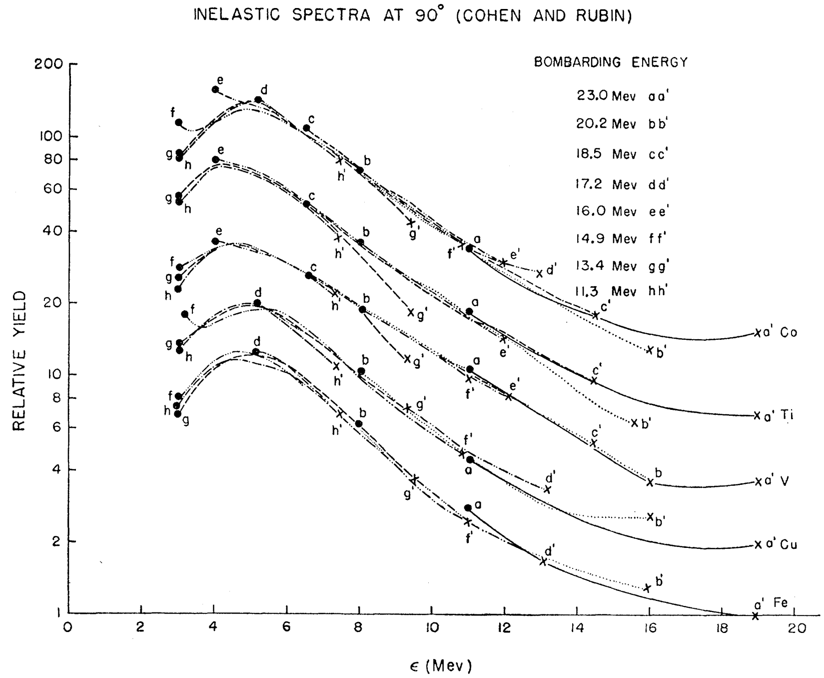

FIG. 18. Superposition of the (p,p') spectra of Cohen and Rubin[11] for Ti, V, Fe, Co, and Cu at various incident energies. At each energy only the section from $E_x=4$ to $E_x\leq 12$ Mev is shown. The curves for each element were arbitrarily displaced vertically.

Returning to Eq. (4), one notes that the equivalent expression for the Fermi gas level density would not factor into a similar product of a function of incident energy, and a function of outgoing energy. Therefore, if one wants to describe the (p,p') spectra by a product of functions in which the level density is a multiplying factor, only the constant temperature level density will be consistent with Figs. 17 and 18.

Thus Gugelot's observations[26] and their present amplification might be taken as evidence supporting the constant-temperature level density. However, it is more likely that for high bombarding energies where direct interactions are important, the apparent dependence of spectral shape on outgoing energy is an accident. It is difficult to see how the yield of protons from evaporation, direct, and intermediate processes could be combined to give a spectrum whose relative shape depends only on ϵ. Direct processes appear to be a function of momentum transfers which involve incident and emergent energies in functions which cannot usually be written as a product of the form $F(\epsilon_0)G(\epsilon)$ required to agree with Figs. 17 and 18.

We turn now to the problem of estimating the relative contribution of direct and compound nucleus reactions to the (p,p') spectra. To estimate an upper limit for the evaporation yield, we assume that at very low ϵ, the protons come entirely from evaporation. An evaporation spectrum, assumed to be given by Eq. (4), is compared with the actual spectrum with the proviso that the former never exceeded the latter. This requires normalization of the calculated to the experimental spectra near $\epsilon=4.5$ Mev. The ratio of the evaporation spectrum to the experimental spectrum, designated by f in Table IV, was evaluated from the ratio of the areas under the two curves.

In order to carry out these computations, values for r_0 and T have to be assumed. In Fig. 17, the dotted curve (a) is merely a conceivable extrapolation; if this corresponded to reality, one would have to conclude that direct interactions contribute significantly to the spectra down to quite low bombarding energies. The values of T at $\epsilon_0-E_x=0$ is 1.0 Mev for curve (a). Curve (b), on the other hand, corresponds to the level density slope found for Fe^{56} in the $Co^{59}(p,\alpha)Fe^{56}$ reaction ($T\approx 1.5$ Mev according to Table II). It was noted earlier that the 11.3-Mev level density curve of Fig. 5 of reference 11, was in good agreement with $T=1.5$ Mev, with $r_0=2.0$ f. Both the 11.3- and 13.4-Mev spectra are moderately well reproduced with $T=1.5$ Mev and $r_0=1.75$ f. Thus it seems reasonable to assume values of $r_0=1.5$ to 2.0 f, and $T=1.0$ to 1.5 Mev, for the present calculations. The results are given in Table IV. Here ΔE_x is the range of excitation energy for which f is evaluated. For $\epsilon_0\geq 17.2$ Mev, there is an increasing amount of secondary protons at $\epsilon=4.5$ Mev (the energy used for normalization), so that the corresponding values of f are increasingly overestimated.

The values of f in Table IV depend appreciably on the parameters selected. If the spectrum at 11.3 Mev were, in fact, predominantly due to evaporation, these calculations indicate that r_0 is in the neighborhood of 2.0 f, and $T\approx 1.5$ Mev. This value of r_0 ($R=r_0A^{\frac{1}{3}}$) is considerably larger than that deduced from electron scattering or the optical model calculations of nuclear scattering. However, in the present context, r_0 is a parameter in the continuum model calculations of inverse cross sections and its precise meaning depends on the validity of this model. This question is discussed in detail in the following section (VII).

It would be desirable to set lower limits to the cross section for evaporation of protons. If the Maxwellian portion of the (p,α) spectrum is due to evaporation for proton bombarding energies of 15–19 Mev, one must expect a corresponding evaporation of protons. From the $\sigma(p,\alpha)$ cross section for Fe at 17.6 Mev (see Table III), one can, in principle, calculate the evaporation cross section for inelastic protons from Fe. However, there is not sufficient theoretical or experimental information to allow one to make more than an estimate. Such a calculation is sensitive to choices of parameters. Assuming $r_0=1.5$ f and $T=1.0$ for protons, we find $f\approx 3.5\%$ at $E_0=17.2$ Mev, while for $r_0=2.0$ f and $T=1.5$, we obtain $f=63\%$. These estimates are consistent with the upper limits of f given in Table IV.

Further analysis of these data would be greatly helped by a "statistical" model for direct interactions to highly excited states. The small values of f for 20.2 and 23.0 Mev show that the spectra at these energies would be suitable for comparison with such a model. It should be remarked that the small values of f at these energies disagree with the results of Monte Carlo calculations. According to Lane's review,[33] only few direct protons should be observed for incident energies below 20 Mev, whereas the present analysis (Table IV) suggests that even at 13.4 Mev, 9 to 35% of the protons may result from direct interaction.

TABLE IV. Upper limits for the ratio f of the evaporation process to the observed (p,p') spectra for Fe at 90°.

ϵ_0 (Mev)	ΔE_x (Mev)	f: $r_0=1.5$ f, $T=1.0$ Mev	f: $r_0=1.5$ f, $T=1.5$ Mev	f: $r_0=2.0$ f, $T=1.5$ Mev
11.3	3–8	0.75	0.91	0.96
13.4	3–10	0 65	0.91	0.85
14.9	3–12	0.65	0.83	0.90
17.2	3–11	<0.33	<0.83	<0.76
20 2	3–11	<0.10	<0.51	<0.44
23.0	3–11	<0.014	<0.22	<0.18

VII. LEVEL DENSITY OF Fe^{56}

In the previous sections, the level density of Fe^{56} was obtained from $Fe(p,p')$ reaction at 11.3 Mev, from the $Co^{59}(p,\alpha)$ reaction for incident energies of 15–19 Mev and from $Fe(\alpha,\alpha')Fe$ at 20 Mev. The value of T for Fe^{56} extracted from the (p,α) spectra was found to be 1.46 Mev or 1.54 Mev (when the cross sections of the continuum model or of the optical model, respectively, were used). We have seen that the $Fe^{56}(\alpha,\alpha')$ and $Co^{59}(p,\alpha)$ spectra are in excellent agreement, so that the values of T are the same for both reactions. For the (p,p') reaction on Fe^{56} a value of $T=1.5$ Mev was found to be in fair agreement with the (p,p') spectra[11] at 11.3 Mev using the inverse cross sections of the continuum model with $r_0=2.0$ f, while a choice of $r_0=1.70$ f yielded $T=1.42$ Mev[9] for the same data. Thus, the level densities obtained from the (α,α') (p,α), and (p,p') reactions are found to be in satisfactory agreement for the particular choices of parameters and inverse cross sections used.

There is also experimental information on the $Fe(n,n')Fe$ reaction. Thomson[34] has measured the inelastic neutron spectra for a number of elements. His spectra for Cu and In for different bombarding energies (5, 6, and 7 Mev) show the same independence of shape on bombarding energy as was found for the (p,p') spectra discussed in Sec. VI above. The level density plots are linear for the lower part of the spectra, but decrease in slope at the upper end, as in the case of the (p,p') spectra. Thomson has analyzed his spectra assuming that the inverse cross section is constant for neutron energies in excess of 0.5 Mev.

The level density plot for Fe^{33} is linear between 0.5 and 3 Mev and corresponds to $T=0.95$ Mev. This value for T is completely inconsistent with $T\approx 1.5$ Mev obtained from the (p,α), (α,α'), and (p,p') reactions. This disagreement illuminates the basic difficulty of the evaporation model [Eq. (1)], namely, the lack of precise knowledge of the inverse cross section. This cross section, $\sigma_c(\epsilon)$, is the cross section for forming the original compound nucleus by bombarding the excited residual nucleus with particles of energy ϵ. Clearly, one cannot obtain these cross sections by direct measurements. Therefore, the usual procedure is to use inverse cross sections calculated on the basis of a theoretical model such as the continuum model, or to use semitheoretical cross sections based on optical model parameters obtained from elastic scattering. In either case, the assumption has to be made that the cross section for an excited nucleus is identical with that for the same nucleus in its ground state.

The parameters of the optical model have been interpreted in terms of multiple scattering of incident nucleons by the target nucleons.[35] In its simplest form this analysis relates the absorption potential W_0 directly to the nucleon-nucleon scattering cross sections. In order to calculate W_0, only those scatterings which do not violate the Pauli principle are to be considered. If the nucleus in its ground state is assumed to be a degenerate Fermi gas, both nucleons involved in the scattering must have final states above the top of the Fermi sea. This restriction leads to small values of W_0 at low bombarding energies; W_0 increases at higher energy where the effect of the Pauli principle is less important.

[33] A. M. Lane, Revs. Modern Phys. **29**, 191 (1957).

[34] D. B. Thomson, Doctoral thesis, University of Kansas (unpublished).

[35] J. P. Elliott and A. M. Land, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 385.

For an excited nucleus there can be nucleon-nucleon collisions with final states below as well as above the top of the Fermi sea. Therefore, the Pauli principle will not be as restrictive as it is for the nucleus in its ground state. Thus, excitation of the nucleus would lead to cross sections which are higher at low particle energy, but not too different at high particle energy, relative to the cross sections for the unexcited nucleus. If one uses optical model cross sections based on elastic scattering experiments for analysis of evaporation spectra, one may therefore expect that the optical model cross sections are relatively too low at low energies.

In addition, the inverse cross sections for compound nucleus formation at high energy must be smaller than the optical model absorption cross section. The latter includes direct interactions as well as compound nucleus formation, and experiments indicate that direct interactions become increasingly important at higher energy. To summarize this discussion, the inverse cross sections are larger than the optical model cross sections at low energy, and smaller at high energy. The magnitude of these effects remains to be estimated. Qualitatively, such modifications of the optical model cross sections would lower the level density plots at small ϵ relative to large ϵ, and therefore act in the direction of increasing T.

In the case of optical model calculations for α particles, the absorption is so large that the modifications suggested above are probably negligible. If this were correct, then the modified cross sections would increase T for the (n,n') spectra without affecting the T for the (p,α) spectra.

In order to obtain $T=1.5$ Mev from Thomson's[34] neutron spectrum for Fe, one has to assume that the inverse cross section is not constant, but varies as $\sim e^{-\epsilon/3}$ between 0.5 and 4.0 Mev. This implies a decrease by a factor of ~ 4 in cross section between these energies. Optical model calculations by Beyster *et al.*[36] show a decrease by a factor of $\frac{4}{3}$ for Fe^{56} over this interval; thus, the effects discussed above would have to yield an additional factor of 3.

Ericson[6] has suggested that the continuum model cross sections should be reasonably good for excited nuclei. However, one would expect that the fraction of the reaction cross section which goes into direct interaction will increase with the particle energy. The corresponding reduction of the compound nucleus cross section would lead to an increased value of T for the (n,n') spectrum. Because of the relative unimportance of direct interactions in the $Co^{59}(p,\alpha)$ reaction, T for this reaction should be unaffected. This type of correction might also lead to T in the neighborhood of 1.5-Mev for the (p,p') reaction with a value of r_0 smaller than 2.0 and perhaps bring the latter into agreement with accepted nuclear radii.

[36] J. R. Beyster, M. Walt, and E. W. Salmi, Phys. Rev. **104**, 1319 (1956).

Unfortunately, these are at present speculative arguments, and probably one could in the same vein think of reasons why the (n,n') temperature is correct and why the analysis of the (p,α) spectrum should be modified. Ericson[37] has determined nuclear temperatures from the distribution of known (resolved) energy levels for Fe^{55}, Fe^{57}, and Fe^{58}, for which he finds $T=1.2$ Mev. If a similar distribution of levels holds for Fe^{56}, both the neutron and the α-particle inverse cross sections would require modification.

VIII. SUMMARY

The present experiments on (p,α) reactions of 15–19 Mev protons on targets in the V-Ni region have yielded spectra which can quite easily be separated into two parts. Groups of α particles leading to low residual states are strongly peaked in the forward direction and are attributed to direct interactions. The major part of the spectra has a Maxwellian shape and is essentially independent of angle observation. The isotropic component can be satisfactorily described in terms of the evaporation model if one assumes a constant-temperature level density, $\exp(E_x/T)$, for excitation energies in the interval 3–12 Mev. Using the inverse cross sections of the continuum model with the Blair-Kerlee-Farwell[23] nuclear radius, one finds values of T varying between 1.2 and 1.5 Mev for the nuclei investigated in the V-Ni region. When the inverse cross sections of the optical model are used, the values of T are about 10% larger. In either case, T is independent of bombarding energy (15–19 Mev). The evaporation spectra can also be successfully described with a Fermi-gas level density, $\exp[2(aE_x)^{\frac{1}{2}}]$, with a suitable choice of nuclear radius. However, since the experimental spectrum shape is independent of bombarding energy, either the nuclear radius or the parameter a would vary with bombarding energy for this form of the level density.

We have compared our experimental results with the predictions of the evaporated model using the two forms of level density discussed above. While our analysis indicates a preference for the constant temperature level density, the differences in the spectra based on these two forms, for a particular choice of radius, is not large. Thomas and Grover[38] have pointed out that spectra based on the Bethe-Bardeen[39] level density, $(E_x)^{-2} \times\exp[2(bE_x)^{\frac{1}{2}}]$, would be nearly the same as those based on $\exp(E_x/T)$. Indeed this third form, with $b=7.0$ Mev^{-1}, would agree within 10% with $\exp(E_x/1.5)$ between $E_x=4.5$ and 13 Mev. Thus the experimental results we have presented above could not distinguish between these two prescriptions for the level density.

[37] T. Ericson, Nuclear Phys. **11**, 481 (1959).

[38] T. D. Thomas and J. R. Grover, Brookhaven National Laboratory (private communication).

[39] H. A. Bethe, Revs. Modern Phys. **9**, 79 (1937); J. Bardeen, Phys. Rev. **51**, 799 (1937); T. Ericson, *Proceedings of the International Conference on Nuclear Structure, Kingston, 1960* (University of Toronto Press, Toronto, 1960), p. 704.

The Maxwellian portion of the α spectra obtained in the present (p,α) reactions are very similar to those observed in (α,α') and (d,α) reactions with similar targets, for bombarding energies of 10 to 20 Mev. This agreement is considered to be strong evidence in favor of the evaporation model as a valid model for describing the emission of α particles to the region of high level density in the residual nucleus.

Direct interaction α particles are prominent in our (p,α) spectra only for $\theta < 60°$. The strong forward peaking suggests that direct interactions occur only at the nuclear surface, either by pickup or knockout. It appears that quasi-tritons or α particles are available only on the surface and that in the interior such clusters are not likely to occur. This picture is consistent with the sharp-cutoff and diffraction inelastic scattering models of Blair[22,40] and the optical model calculations of Igo,[24] in which the nucleus is found to be "black" for α particles.

The comparative roles of direct interaction and evaporation in the inelastic scattering of protons is not as clear as it is in the (p,α) reactions. For targets in the V-Zn region, and for excitation energies in the interval 3–11 Mev, it appears that evaporation may predominate at 11.3-Mev bombarding energy, while at incident energies ≥ 20 Mev, at least 50% of the yield is due to direct interaction. In order to reach quantitative conclusions, more experimental data is needed at bombarding energies between 8 and 12 Mev. The 90° proton spectra for a given target obtained with incident proton energies of 11.3 to 23 Mev were found to have a shape independent of bombarding energy (for excitation energies from 3 to 11 Mev). This feature of the spectra is consistent with a constant temperature level density, but not with a Fermi gas level density. However, since the spectra contain an admixture of protons from evaporation and direct interaction it may be that the universal shape found for each element is only a peculiar accident of no particular significance. Our analysis of the (p,p') reaction data indicates the importance of careful investigation of the dependence of spectra on bombarding energy for other reactions such as (n,p) and (p,n) in order to determine the nature of the reaction mechanisms involved. It is felt that there is at present insufficient information for an adequate understanding of reactions involving the emission of single nucleons.

On comparing the values of T deduced from experimental results for the (n,n') reaction[34] with those obtained in the (p,α) reaction, one finds appreciable differences. In particular, for Fe^{56}, the present work on the $Co^{59}(p,\alpha)Fe^{56}$ reaction indicates a value of $T \approx 1.5$ Mev, while Thomson's analysis[34] of the (n,n') spectrum of Fe yields $T = 0.95$ Mev. The level density should not depend on the nature of the particle used to reveal this property of the nucleus. In order to resolve this discrepancy within the framework of the evaporation mode, appreciable modifications in our present models for inverse cross sections would be required. Such modifications might alter the conclusions stated above with regard to a constant-temperature level density in the interpretation of the (p,α) spectra. Until this disagreement is resolved, one cannot consider the evaporation model in the form given by Eq. (1) as more than qualitatively useful; in particular, calculations of competing reactions (e.g., Table III) have questionable significance.

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[40] J. S. Blair, Phys. Rev. **115**, 928 (1959).