as the experimental results. However, the curve computed from evaporation theory, just as in the case of the (α,n) and (α,p) excitation functions, attains its maximum value and then decreases rapidly; in contrast, the experimental curve appears to be just approaching its maximum at 42 Mev. Thus, again it appears that some non-compound nuclear process is in effect in the $(\alpha,\alpha n)$ reaction.

It could well be that the data, although not explicable in terms of the simple evaporation calculation described herein, may fit within the framework of the compound nucleus model. On the other hand, calculation of excitation functions based on some direct-interaction mechanism would be extremely useful in the analysis of the data; unfortunately, no such calculation is available at the present time.

V. CONCLUSIONS

The optical model calculation of the total reaction cross section appears to agree within 16% with the sum of experimental cross sections in the region of 36 Mev. In terms of the approximate form used for the continuum theory cross section, a value of $r_0 \approx 1.7$ fermis is indicated.

The statistical theory of nuclear reactions predicts values for the (α,p) and (α,n) cross sections which are in agreement with the experimental data in the energy range of 18–24 Mev. An $a=1.6$ Mev^{-1} appears to give the most satisfactory fit. However, the high-energy tails of the (α,p) and (α,n) excitation functions cannot be reproduced by the theory. Evaporation calculations of the $(\alpha,\alpha n)$ excitation function, using $a=1.6$, yield excitation functions of the required magnitude; however, the experimental data do not appear to be the result of compound nuclear processes.

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Nuclear Compressibility and Symmetry Energy*

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A modification and generalization of the Puff-Martin model for many-fermion systems is employed to calculate nuclear compressibility and symmetry energy in order to provide a practical test of the model and at the same time obtain useful information about these interesting quantities. An alternative, heuristic, derivation of the Puff-Martin equations is presented in order to exhibit the role of the exclusion principle. The condition stated for normal nuclear matter is that the mean binding energy be minimal (with respect to variation of the Fermi momentum) rather than the Puff-Martin condition that the mean binding energy equal the "single particle" energy at the Fermi surface. These two quantities differ from each other by the rearrangement energy, which is found to be 10 Mev. Employing Puff's potential (hard-shell potential plus a separable Yamaguchi potential, acting only in relative S states), satisfactory agreement is obtained with observed binding energy and density. The value of nuclear compressibility, 214 Mev, falls within the wide range of semi-empirical values. The symmetry energy coefficient, 43 Mev, is larger, by 40–80%, than those usually quoted in semiempirical mass formulas. However, our value of the symmetry coefficient is the same as that calculated by Brueckner and Gammel in the absence of odd-state forces; they found the coefficient to be reduced to 26 Mev when a more realistic potential, including odd-state contributions, is employed.

I. INTRODUCTION

A RELATIVELY simple procedure for calculating properties of nuclear matter has been devised by Puff and Martin,[1] based upon the formalism of Martin and Schwinger.[2] Their approximation may be arrived at in the following manner: First the two-body problem is solved as a function of energy ("off the energy shell") in the absence of other particles; the center-of-mass and relative motions separate in this case. Then the Fermi sea is filled loosely according to a prescription which satisfies the exclusion principle in an average way. The loose packing is essential, since two-body scattering is allowed to all final states.

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[1] R. D. Puff and P. C. Martin, Bull. Am. Phys. Soc. **5**, 30 (1960); R. D. Puff, Ann. Phys. **13**, 317 (1961).

[2] P. C. Martin and J. Schwinger, Phys. Rev. **115**, 1342 (1959).

The method treats "properly" two-body correlations, but approximates three- and four-body correlations. This is true also of other methods, such as that of Brueckner,[3] in which, to the same order, the Fermi sea is tightly packed but two-particle scattering is not allowed to states inside the sea. The difference between the methods arises from the treatment of higher correlations. An advantage of the Puff-Martin method lies in the simplicity with which the two-body reaction matrix can be obtained. The diagonal elements for S-wave scattering depend only on two parameters (relative momentum and energy) compared with four parameters (the magnitudes of the two momenta, the energy, and the density) in the Brueckner formulation. For special forms of potentials, analytic expressions for the reaction matrix can be derived.

It is difficult to make a careful estimate of the correction terms in the Puff-Martin method, although their investigation of the presumably most important omitted terms indicated an error of less than 10% in the energy per particle and mean separation distance. By using a simple, separable two-body potential which fits the low-energy nucleon-nucleon scattering data and the 310-Mev S-wave phase shift, they obtained the remarkably good value of 14.7 Mev binding energy per nucleon and a fair value (0.92 f compared with the Stanford value of 1.07 f) for the internucleonic distance. The potential contains no tensor or spin-orbit component, so it is not clear whether the interaction is adequate.

An important question arises concerning their treatment of rearrangement energy. It follows from the saturation condition alone that the separation energy of the "last" particle must equal the mean binding energy. It is not trivial to identify the separation energy, however, since one is not dealing with a truly independent particle model. The removal of a particle removes not only its kinetic and potential energy, but results in a rearrangement of the remaining particles which, in turn, affects their mutual interactions. Similar questions were raised by Hugenholtz and Van Hove[4] in the Brueckner theory, and the rearrangement terms were later identified by Brueckner.[5]

The Puff-Martin condition for the ground state is that the independent-particle energy (kinetic plus potential) of the "last" particle be equal to the mean binding energy. We discuss below why we believe the condition should be that the mean energy per particle is minimal. In re-solving their equations to minimize the energy, we found the rearrangement energy to be almost 10 Mev for this model.

In order to obtain further tests of the method, as well as for intrinsic interest, calculations were carried out to determine nuclear compressibility, symmetry energy, and other symmetry phenomena.

Empirical estimates of the symmetry energy are available from the coefficient of the $(N-Z)^2/A$ term in the semiempirical mass formula. The value is not well determined even when a good fit to nuclear masses near the stable valley is obtained. It would be desirable to fix the coefficient more precisely in order to predict nuclear masses far from stability. Such masses are seldom measurable, but are of importance in astrophysical problems and in the theory of nuclear fission.

Compressibility can be inferred only through model-dependent arguments. The phenomenon would be more interesting if nuclear matter were softer to compression. As it is, collective compressional oscillations lie too high for identification except in light nuclei, where surface and shell effects play an important role. Semiempirical estimates of the compressibility coefficient do give a lower bound to the coefficient and thus provide a meaningful comparison with the present calculations.

In Sec. II an heuristic derivation of the Puff-Martin equations is presented to give an alternative view of the method. A brief but generalized outline of the Puff-Martin derivation is also given. In Sec. III the results of numerical calculations are presented for the case of normal nuclear matter, $N=Z$, and also for the case of $N\neq Z$. From these calculations nuclear compressibility and symmetry energy are deduced, along with the dependence of other quantities on N and Z. In Sec. IV comparison of these calculations is made with experiment and other theories.

II. PUFF-MARTIN EQUATIONS

A. Heuristic Derivation

We present here an intuitive approach to the Puff-Martin equations which may help towards a better understanding of the method. Particular emphasis is laid on the role of the exclusion principle.

We seek a self-consistent approximation which treats the detailed scattering between two particles exactly while representing the dynamic effect of all the other particles by some average potential. In the same manner we will treat the exclusion principle exactly for the two-body event (i.e., the wave function is antisymmetric in the two particles), but only in some average manner with respect to all the other particles (i.e., the two particles cannot, "on the average," scatter into already occupied states). We may consider this approach as an improvement to the Hartree-Fock approximation. In the latter method one considers each particle to be moving freely except for the effects of an external potential which represents the average effect of the other $N-1$ particles. This potential is then determined in a self-consistent manner. Here we shall consider two particles to interact only with each other

[3] A survey of recent progress in the field is given in *The Many-Body Problem*, Les Houches session of 1958 edited by DeWitt (John Wiley & Sons, Inc., New York, 1959). A list of references is given by K. A. Brueckner and J. L Gammel, Phys. Rev. **109**, 1023 (1958).

[4] N. M. Hugenholtz and L. Van Hove, Physica **24**, 383 (1958).

[5] K. A. Brueckner, Phys. Rev. **110**, 597 (1958).

in the presence of an external potential which represents the average effect of the other $N-2$ particles.

To see how this is achieved, consider the Hamiltonian ($\hbar=2m=1$):

$$H=H_0+V=-\sum_i \nabla_i^2+\sum_{i<j} V_{ij}. \tag{1}$$

If we only had two particles, their wave function, $\psi_{\mathbf{kl}}$, would be related to the plane-wave single-particle functions $\phi_{\mathbf{k}}$ (normalized in the total volume) by

$$V\psi_{\mathbf{kl}}=T\phi_{\mathbf{k}}\phi_{\mathbf{l}}, \tag{2}$$

where the scattering matrix, T, obeys[6] (symbolically)

$$T=V+V\frac{1}{\omega-H_0}T, \tag{3}$$

and is a function of the energy parameter ω. Here the inhomogeneous term, V, is considered to be multiplied by an antisymmetrical product of delta functions (see below).

The energy, ω, is the sum of the energies of the two particles,

$$\omega=\omega_{\mathbf{k}}+\omega_{\mathbf{l}}, \tag{4}$$

where, to include the effects of the external potential which is to represent the other N-2 particles, we write

$$\omega_{\mathbf{k}}=k^2+\mathcal{U}_{\mathbf{k}}. \tag{5}$$

To evaluate self-consistently the external potential, $\mathcal{U}_{\mathbf{k}}$, we note that the energy shift of a pair of particles is given by

$$\Delta_{\mathbf{kl}}=\langle\mathbf{kl}|V|\psi_{\mathbf{kl}}\rangle=\langle\mathbf{kl}|T|\mathbf{kl}\rangle. \tag{6}$$

The total shift per particle, which we equate to $\mathcal{U}_{\mathbf{k}}$, is then

$$\mathcal{U}_{\mathbf{k}}=\sum_{\mathbf{l}}\Delta_{\mathbf{kl}}\rho_{\mathbf{l}}=\sum_{\mathbf{l}}\langle\mathbf{kl}|T|\mathbf{kl}\rangle\rho_{\mathbf{l}}, \tag{7}$$

where $\rho_{\mathbf{l}}$ is the probability that level $\mathbf{l}$ is occupied.

Before discussing $\rho_{\mathbf{l}}$, it is well to note that the two-particle scattering matrix, T, considered as a function of ω, has a pole (i.e., a bound state) at $-E_B$, where E_B is the deuteron binding energy, and a branch cut, beginning at zero, along the real axis. Consequently we can only write (5) provided the maximum energy in the Fermi sea is less than $-\frac{1}{2}E_B$. Physically this means that single-particle levels are not stable at energies greater than $-\frac{1}{2}E_B$; they can decay to a state of lower energy by pairing off into deuterons which do not, in this approximation, interact with each other. We can, however, use (5) to describe the energy levels in the Fermi sea of nuclear matter where the Fermi energy is considerably less than $-\frac{1}{2}E_B$.

We must now turn to the problem of the statistics. In the Hartree-Fock approximation, one handles this by taking the total wave function antisymmetric in the particle variables. Here, while the two-body wave function is antisymmetric (because of the antisymmetrized product of delta functions in (3)), we must still antisymmetrize the total N-particle wave function. Equivalently, we must insure that the probability of there being a particle in any level $\mathbf{k}$ be less than or equal to unity. This is not trivially satisfied because (3) allows scattering into the Fermi sea. Thus the problem of the statistics becomes a problem of evaluating the density of states, $\rho_{\mathbf{k}}$.

Suppose we consider the Fermi sea to be filled except for one particular level, say $\mathbf{k}$ (described by the single-particle wave function $\phi_{\mathbf{k}}$). Now, this level is already "partially filled" corresponding to the finite probability that other pairs have scattered into it. We have called $\rho_{\mathbf{k}}$, the probability that level $\mathbf{k}'$ is occupied. Then $\rho_{\mathbf{k}'}\leqslant 1$, and $\rho_{\mathbf{k}'}=0$ for $k'>k_f$. The probability that level $\mathbf{k}$ is occupied due to scattering from other levels is given by

$$P_{\mathbf{k}}=\sum_{\mathbf{lk'l'}}{}' |\langle\mathbf{kl}|\psi_{\mathbf{k'l'}}\rangle|^2\rho_{\mathbf{k'}}\rho_{\mathbf{l'}}, \tag{8}$$

where the prime on the summation indicates that none of $\mathbf{l}$, $\mathbf{k}'$, $\mathbf{l}'$ is equal to $\mathbf{k}$.

In order to express $P_{\mathbf{k}}$ in terms of the single-particle plane wave functions, it is convenient to introduce the Ω matrix defined by

$$\psi_{\mathbf{kl}}=\Omega\phi_{\mathbf{k}}\phi_{\mathbf{l}}. \tag{9}$$

Comparison with (2) shows that

$$V\Omega=T, \tag{10}$$

and that, consequently, Ω obeys,

$$\Omega=1+\frac{1}{\omega-H_0}V\Omega, \tag{11}$$

where the 1 symbolizes the antisymmetric product of delta functions and corresponds to the fact that $\psi_{\mathbf{kl}}$ is an antisymmetric combination of plane waves in the absence of the interaction, V. Using (1), we may write $P_{\mathbf{k}}$ as

$$P_{\mathbf{k}}=\sum_{\mathbf{lk'l'}}{}' |\langle\mathbf{kl}|\Omega|\mathbf{k'l'}\rangle|^2\rho_{\mathbf{k'}}\rho_{\mathbf{l'}}. \tag{12}$$

Since there is an *a priori* probability that level $\mathbf{k}$ is already occupied, the available phase space has been reduced, on the average, to $1-P_{\mathbf{k}}$, and so $\rho_{\mathbf{k}}$ must be restricted to satisfy

$$1-\rho_{\mathbf{k}}\geqslant P_{\mathbf{k}}. \tag{13}$$

The approximation will now be to satisfy (13) on the average, rather than for each $\mathbf{k}$. That is, we require only that

$$\sum_{k\leqslant k_f}(1-\rho_{\mathbf{k}})\geqslant\sum_{k\leqslant k_f}\sum_{\mathbf{l'k'l'}}{}' |\langle\mathbf{kl}|\Omega|\mathbf{k'l'}\rangle|^2\rho_{\mathbf{k'}}\rho_{\mathbf{l'}}. \tag{14}$$

This inequality can be satisfied by extending the summation on the right-hand side over all $\mathbf{k}$, and

[6] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950); M. Gell-Mann and M. L. Goldberger, *ibid.* **91**, 398 (1953).

changing the equation to an equality. This yields, with a change of dummy summation variables,

$$\sum_{k\leqslant k_f}(1-\rho_{\mathbf{k}})=\sum_{\mathbf{kl}}[\langle\mathbf{kl}|\Omega^\dagger\Omega|\mathbf{kl}\rangle-|\langle\mathbf{kl}|\Omega|\mathbf{kl}\rangle|^2]\rho_{\mathbf{k}}\rho_{\mathbf{l}}. \quad (15)$$

The second term in the square brackets is due to the fact that the summation in (14) is primed, and we have used the facts that Ω conserves total momentum and satisfies the exclusion principle for the two body problem (i.e., vanishes when operating on a state with two particles in the same level).

Now, using (11) and its adjoint, and (10), one may write

$$\Omega^\dagger\Omega=\Omega^\dagger+\Omega-1+(\Omega^\dagger-1)(\Omega-1)$$

$$=\Omega^\dagger+\Omega-1+\Omega^\dagger V\frac{1}{(\omega-H_0)^2}T. \quad (16)$$

On the other hand, if (3) is differentiated with respect to the parameter ω, one gets

$$V\frac{1}{(\omega-H_0)^2}T=-\left(1-V\frac{1}{\omega-H_0}\right)\frac{\partial T}{\partial\omega}. \quad (17)$$

Combining (17) with (16) and the adjoint of (11), one gets

$$\Omega^\dagger\Omega=\Omega^\dagger+\Omega-1-\frac{\partial T}{\partial\omega}, \quad (18)$$

so that

$$\langle\Omega^\dagger\Omega\rangle-|\langle\Omega\rangle|^2=-\left\langle\frac{\partial T}{\partial\omega}\right\rangle-|\langle 1-\Omega\rangle|^2. \quad (19)$$

Since both $\langle\partial T/\partial\omega\rangle$ and $\langle 1-\Omega\rangle$ vary inversely as the volume, the square of the latter can be neglected. Thus (15) may be written as

$$\sum_{k\leqslant k_f}(1-\rho_{\mathbf{k}})=-\sum_{\mathbf{kl}}\left\langle\mathbf{kl}\left|\left(\frac{\partial T}{\partial\omega}\right)_{\omega=\omega_{\mathbf{k}}+\omega_{\mathbf{l}}}\right|\mathbf{kl}\right\rangle\rho_{\mathbf{k}}\rho_{\mathbf{l}}$$

$$=-\sum_{\mathbf{k}}\frac{\partial\mathcal{U}_{\mathbf{k}}}{\partial\omega_{\mathbf{k}}}\rho_{\mathbf{k}}, \quad (20)$$

where use of (7) has been made. This equality may be satisfied term-by-term by setting

$$\rho_{\mathbf{k}}=\left[1-\frac{\partial\mathcal{U}_{\mathbf{k}}}{\partial\omega_{\mathbf{k}}}\right]^{-1}, \quad k\leqslant k_f,$$

$$=0, \quad k>k_f. \quad (21)$$

This completes the approximation. [One may note that if only the lowest-order approximation to (3) is used, one gets the Hartree-Fock approximation. Thus

$$\mathcal{U}_{\mathbf{k}}{}^{\mathrm{HF}}=\sum_{\mathbf{l}}[\langle\mathbf{kl}|V|\mathbf{kl}\rangle-\langle\mathbf{kl}|V|\mathbf{lk}\rangle]\rho_{\mathbf{l}}, \quad (22)$$

which is independent of ω. Consequently (21) reduces to the usual non-interacting Fermi gas distribution.]

The mean energy per particle is then given by

$$\frac{E}{A}=\frac{\sum_{\mathbf{k}}(k^2+\frac{1}{2}\mathcal{U}_{\mathbf{k}})\rho_{\mathbf{k}}}{\sum_{\mathbf{k}}\rho_{\mathbf{k}}}. \quad (23)$$

Given a two-body potential, the T matrix can be obtained from (3), once and for all, as a function of ω. Thus the statistics of the problem have been shifted from the T matrix to the density of states, $\rho_{\mathbf{k}}$. This latter is determined by solving Eqs. (5) and (21) simultaneously as a function of k_f. The Fermi momentum, k_f, is in turn determined by minimizing the mean energy per particle (23).

B. Green's Function Derivation

We present here a brief outline of a derivation of the approximation due to Martin and Schwinger,[2] and Puff and Martin.[1] We have extended it to include the case of unequal numbers of neutrons and protons. One advantage of this approach is that the approximation is more explicitly stated, and hence subject to numerical evaluation, a point to which we shall return. It also serves to point out other aspects of the approximation and to allow generalization to finite temperatures, unbound systems (e.g., superconductivity)[7] and to systems with Bose statistics.

Consider the Hamiltonian

$$\mathcal{H}=H-\mu A-\nu I, \quad (24)$$

where H is the usual Hamiltonian ($\hbar=2m=1$),

$$H=\int d\mathbf{r}_1\,\psi^\dagger(\mathbf{r}_1t_1)(-\nabla_1^2)\psi(\mathbf{r}_1t_1)+\tfrac{1}{2}\int d\mathbf{r}_1d\mathbf{r}_2d\mathbf{r}_3d\mathbf{r}_4$$

$$\times\psi^\dagger(\mathbf{r}_1t_1)\psi^\dagger(\mathbf{r}_2t_1)V(\mathbf{r}_1\mathbf{r}_2;\,\mathbf{r}_3\mathbf{r}_4)\psi(\mathbf{r}_2t_1)\psi(\mathbf{r}_1t_1). \quad (25)$$

The number of particles operator, A, is given by

$$A=\int d\mathbf{r}_1\,\psi^\dagger(\mathbf{r}_1t_1)\psi(\mathbf{r}_1t_1), \quad (26)$$

the proton excess $(Z-N)$ operator, I, is

$$I=\int d\mathbf{r}_1\,\psi^\dagger(\mathbf{r}_1t_1)\tau_3\psi(\mathbf{r}_1t_1), \quad (27)$$

and μ and ν are Lagrange multipliers. The anticommuting field operators, ψ, have four components, two in spin space and two in isotopic space (the latter will sometimes be indicated by a Greek subscript). The matrix τ_3 has only diagonal matrix elements, $\tau_\alpha=\pm1$. In (25) we have allowed for a nonlocal two-body potential.

The n-particle Green's operators are defined (for

[7] A. J. Cantor and P. C. Martin, Bull. Am. Phys. Soc. **3**, 202 (1958).

$n=1$ and 2) by

$$\mathcal{G}_{1;\alpha\alpha'}(1,1')=-i\langle\psi_\alpha(1)\psi_{\alpha'}^\dagger(1')\rangle_+\epsilon(t_1t_1'), \quad (28)$$

$$\mathcal{G}_{2;\alpha\beta,\alpha'\beta'}(1,2;1',2')=-\langle\psi_\alpha(1)\psi_\beta(2)\psi_{\beta'}^\dagger(2')\psi_{\alpha'}^\dagger(1')\rangle_+ \times\epsilon(t_1t_2t_2't_1'), \quad (29)$$

where the subscript + means positive time ordering, and the ϵ's are antisymmetrical in their arguments and equal to $+1$ when the time order is as written. We write $(1')$ for $(\mathbf{r}_1't_1')$, etc. Averaged Green's functions are defined by

$$G_n=[AEI|\mathcal{G}_n|AEI]=\frac{\sum_\xi\langle AEI\xi|\mathcal{G}_n|AEI\xi\rangle}{\sum_\xi\langle AEI\xi|AEI\xi\rangle}, \quad (30)$$

where $|AEI\xi\rangle$ is a state of a given A, E, and I (eigenvalues of the operators A, H, and I), and any other quantum numbers ξ.

The G_n obey a set of linked equations, the first few of which may be written as

$$\left[i\frac{\partial}{\partial t_1}+\nabla_1^2+\mu+\nu\tau_\alpha\right]G_{1;\alpha\beta}(1,1')=\delta_{\alpha\beta}\delta(1,1'), \quad (31)$$

$$G_1(1,1')=G_1^0(1,1')+G_1^0(1,2)V(23;45)G_2(45;3^+1'), \quad (32)$$

and (combining several equations),

$$\begin{aligned}G_2(12,1'2')=&[G_1(1,1')G_1(2,2')-G_1(1,2')G_1(2,1')]\\&+G_1^0(1,3)G_1^0(2,4)V(34;56)G_2(56,1'2')\\&+G_1^0(1,3)G_1^0(2,5)V(34,3'4')V(56,5'6')\\&\times\{G_4(3'4'5'6';6^+4^+1'2')\\&-[G_2(3'4',4^+1')G_2(5'6';6^+2')\\&\quad-G_2(3'4';4^+2')G_2(5'6';6^+1')]\},\end{aligned} \quad (33)$$

where (1^+) means we are to evaluate at a time infinitesimally greater than t_1. Neglecting all but the first term in (33) leads to the Hartree-Fock approximation. Corresponding to our desire to treat two-particle correlations exactly, but to treat higher order correlations in some average manner, we terminate the series by neglecting the four-particle correlation term in (33), i.e., we set

$$\begin{aligned}G_4(3'4'5'6';6^+4^+1'2')=&G_2(3'4';4^+1')G_2(5'6';6^+2')\\&-G_2(3'4';4^+2')G_2(5'6';6^+1').\end{aligned} \quad (34)$$

The fact that this choice of G_4 is not completely antisymmetric means that the statistics, as well as the dynamics, are being treated in some average manner. The resulting asymmetrical appearance of unperturbed and perturbed Green's functions in the equation for G_2 reflected in the appearance of the exclusion principle not in the T matrix but in the loose packing of the Fermi sea. We see this by noting that the T matrix, defined by

$$VG_2=TG_1G_1, \quad (35)$$

obeys, in momentum space,

$$\begin{aligned}&\langle\xi\mathbf{k}|T_{K;\alpha\beta,\alpha'\beta'}(\omega)|\xi\mathbf{k}'\rangle\\&=[\langle\xi\mathbf{k}|V|\xi\mathbf{k}'\rangle\delta_{\alpha\alpha'}\delta_{\beta\beta'}-\langle\xi\mathbf{k}|PV|\xi-\mathbf{k}'\rangle\delta_{\alpha\beta'}\delta_{\beta\alpha'}]\\&\quad+\int\frac{d\mathbf{k}''}{(2\pi)^3}\langle\xi\mathbf{k}|V|\xi\mathbf{k}''\rangle\Lambda_{K;\alpha\beta}(k'',\omega)\\&\qquad\times\langle\xi\mathbf{k}''|T_{K;\alpha\beta,\alpha'\beta'}(\omega)|\xi\mathbf{k}'\rangle,\end{aligned} \quad (36)$$

where ξ refers to the spin state of the two scattering particles; P is $+1$ for the spin triplet state and -1 for the singlet state; the momenta are written in terms of the center-of-mass momentum, $\mathbf{K}=\mathbf{k}_1+\mathbf{k}_2$, and the relative momentum, $\mathbf{k}=\frac{1}{2}(\mathbf{k}_1-\mathbf{k}_2)$; and Λ is the factor resulting from the product $G_1^0G_1^0$ in (33). For a finite temperature, signified by $\beta=1/kT$,

$$\begin{aligned}\Lambda_{K;\alpha\gamma}(k'',\omega)=&\left[\frac{1}{1+\exp[-\beta(k_1''^2-\mu-\nu\tau_\alpha)]}\right.\\&\left.+\frac{1}{1+\exp[-\beta(k_2''^2-\mu-\nu\tau_\gamma)]}-1\right]\\&\times\frac{1}{\omega-k_1''^2-k_2''^2}.\end{aligned} \quad (37)$$

Thus Λ is a product of a statistical factor (in brackets) and a two-particle noninteracting propagator $1/(\omega-H_0)$. The factor $\mu+\nu\tau_\alpha$ may be shown to be the Fermi energy of particles of type α, and is, consequently, negative for a bound system such as nuclear matter for sufficiently small $N-Z$. Hence in the zero-temperature limit $(\beta\to\infty)$, Λ reduces simply to $1/(\omega-H_0)$ and we see that (3) is a direct consequence of the approximation (34) for a bound system.

The statistics, of course, reappear in the density of states. The one-particle Green's function for particle of type α may be written in the spectral form[2]

$$\begin{aligned}iG_\alpha(\mathbf{r}t,\mathbf{r}'t')=&\pm\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{d\omega}{2\pi}e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')-i\omega(t-t')}\\&\times\frac{A_\alpha(\mathbf{k},\omega)}{1+e^{\mp\beta(\omega-\mu-\nu\tau_\alpha)}},\end{aligned} \quad (38)$$

the upper sign corresponding to $t>t'$ and the lower sign to $t<t'$. With this form, Eq. (32) becomes an equation for the spectral densities $A_\alpha(\mathbf{k},\omega)$, which, with (36) and (35), may be written in the form[8]

$$\begin{aligned}A_\alpha(\mathbf{k},\omega)=i&\left[\frac{1}{\omega+i\epsilon-k^2-\mathcal{U}_\alpha(\mathbf{k},\omega+i\epsilon)}\right.\\&\left.-\frac{1}{\omega-i\epsilon-k^2-\mathcal{U}_\alpha(\mathbf{k},\omega-i\epsilon)}\right],\end{aligned} \quad (39)$$

[8] See reference 2, particularly Eqs. (5.86) to (5.90) and the ensuing discussions.

where

$$\mathcal{U}_\alpha(\mathbf{k}_1,\omega_1)=\int\frac{d\mathbf{k}_2}{(2\pi)^3}\frac{d\omega_2}{2\pi}\sum_{\gamma,\xi}\langle\mathbf{k}\xi|\,T_{K;\alpha\gamma,\alpha\gamma}(\omega_1+\omega_2)\,|\,\xi\mathbf{k}\rangle\times\frac{A_\gamma(\mathbf{k}_2,\omega_2)}{1+e^{\beta(\omega_2-\mu-\nu\tau_\gamma)}}+[\text{terms tending to zero as }\beta\to\infty]. \quad (40)$$

As noted in Sec. II-A, because the T matrix is the ordinary two-particle scattering matrix, in the limit of zero temperature, it is continuous across the real ω axis provided both particles are in the Fermi sea. In this limit then,

$$\mathcal{U}_\alpha(\mathbf{k},\,\omega+i\epsilon)=\mathcal{U}_\alpha(\mathbf{k},\,\omega-i\epsilon),\quad \omega<\mu+\nu\tau_\alpha<-\tfrac{1}{2}E_B. \quad (41)$$

Equation (39) may then be rewritten, under these conditions, as

$$A_\alpha(\mathbf{k},\omega)=2\pi\delta(\omega-k^2-\mathcal{U}_\alpha(\mathbf{k},\omega))=2\pi\rho_\alpha(\mathbf{k})\delta(\omega-\omega_\alpha(\mathbf{k})), \quad (42)$$

where

$$\omega_\alpha(\mathbf{k})=k^2+\mathcal{U}_\alpha(\mathbf{k},\omega_\alpha(\mathbf{k})), \quad (43)$$

$$\rho_\alpha(\mathbf{k})=\left[1-\frac{\partial\mathcal{U}_\alpha(\mathbf{k},\omega)}{\partial\omega}\bigg|_{\omega=\omega_\alpha(k)}\right]^{-1}, \quad (44)$$

and

$$\mathcal{U}_\alpha(\mathbf{k}_1,\omega)=\sum_\beta\int_{k_\beta\leqslant k_{f\beta}}\frac{d\mathbf{k}_2}{(2\pi)^3}\rho_\beta(\mathbf{k}_2)\times\sum_\xi\langle\xi\mathbf{k}|\,T_{K;\alpha\beta,\alpha\beta}(\omega+\omega_\beta(\mathbf{k}_2))\,|\,\xi\mathbf{k}\rangle. \quad (45)$$

The density, the proton excess, and the energy are then given as

$$\rho=2\sum_\alpha\int_{k\leqslant k_{f\alpha}}\frac{d\mathbf{k}}{(2\pi)^3}\rho_\alpha(\mathbf{k}), \quad (46)$$

$$\frac{I}{A}=\frac{2}{\rho}\sum_\alpha\int_{k\leqslant k_{f\alpha}}\frac{d\mathbf{k}}{(2\pi)^3}\tau_\alpha\rho_\alpha(\mathbf{k}), \quad (47)$$

and

$$\frac{E}{A}=\frac{2}{\rho}\sum_\alpha\int_{k\leqslant k_{f\alpha}}\frac{d\mathbf{k}}{(2\pi)^3}[k^2+\tfrac{1}{2}\mathcal{U}_\alpha(\mathbf{k},\omega_\alpha(\mathbf{k}))]\rho_\alpha(k). \quad (48)$$

These equations are the same as those in Sec. II-A, and, as indicated there, one solves (36) as a function of ω. One then simultaneously solves (43), (44) and (45) as a function of the proton and neutron Fermi levels, $k_{f\alpha}$. These latter are determined by equating (47) to one's choice of I/A, and minimizing (48) with respect to the density (46).

The Lagrange parameters, although not needed for the above, may be determined by the conditions

$$\omega_\alpha(k_{f\alpha})=\mu+\nu\tau_\alpha. \quad (49)$$

The separation, or binding, energy of the "last" particle of the system is defined by

$$E_B=\left(\frac{\partial E}{\partial A}\right)_I=\frac{E}{A}+\frac{P}{\rho}, \quad (50)$$

where

$$P=-\left(\frac{\partial E}{\partial V}\right)_{A,I}=\rho^2\frac{\partial}{\partial\rho}\left(\frac{E}{A}\right)_{V,I}, \quad (51)$$

since E/A is a function only of ρ. A system which is held together by its own forces, such as nuclear matter, will achieve a finite density in the absence of any external pressure. This density may then be determined by $P=0$, i.e.,

$$\frac{\partial}{\partial\rho}\left(\frac{E}{A}\right)_{V,I}=0. \quad (52)$$

As suggested by the heuristic derivation of II-A, (52) may be viewed as the self-consistent solution for the single-particle wave functions $\phi_\mathbf{k}$, which, for an infinite medium, depend only on the density.

If we had a truly independent-particle model (I.P.M.) or a "normal system,"[9] then it would follow that

$$E_B=\mu, \quad \text{(I.P.M.)} \quad (53)$$

in which case one would also have as the condition when $P=0$

$$\mu=E/A. \quad \text{(I.P.M.)}. \quad (54)$$

In fact (54), rather than (52), is the condition used by Puff and Martin in solving for the case of normal nuclear matter with $I=0$.[1] Unfortunately, despite the appearance of (42), we do not have a completely independent-particle model, as can be checked by comparing E_B and μ at the energy minimum (52). The difference between these two quantities, referred to as the "rearrangement energy," is nearly 10 Mev in this model. On the other hand, even if nuclear matter is actually an example of the elusive normal system,[9,10] the approximation (34) destroys the validity of (53). The application of $\psi_\alpha^\dagger(\mathbf{k})$ will not actually change the energy by $\omega_\alpha(\mathbf{k})$. The independent-particle character of (42) cannot be valid for the exact solution which will exhibit the more general form (39). We believe there is no basis for using (54) as the normal density condition.

As to the question of the error introduced in (33) by the approximation (34), an estimate of this may be obtained by evaluating the contribution to G_2 of the last term in (33) using a *fully* antisymmetrized combination of G_2's for G_4 (18 terms). The terms which might be expected to give the largest contribution are those which would ordinarily cancel the terms allowing scattering into the Fermi sea; that is, the important

[9] This result, (54), is evident for an independent-particle model [see V. F. Weisskopf, Nuclear Phys. **3**, 423 (1957)], but has also been proven in the more general case of the "normal system" [see references 4 and 2, and also A. Klein, Phys. Rev. **121**, 950 (1961)].

[10] c.f. L. N. Cooper, Phys. Rev. **122**, 1021 (1961).

terms should be those which involve exchange of the scattered particles with Fermi sea particles. There are four such terms, and indeed Puff found that they gave the dominant contribution in his error estimate. Unfortunately he only considered 6 of the 18 terms, including only two of the four "important" terms. His choice was dictated by the fact that, since the potential is large only at small distances, the neglected term in (33) gives its major contribution for $3'$ close to $4'$ and $5'$ close to $6'$. He therefore selected the six terms with $3'$ and $4'$ correlated (and therefore with $5'$ and $6'$ correlated). As the full 18 term G_4 has an overall factor of $1/3$ which Puff's 6 term G_4 does not, his estimate may be expected to be fairly good. This estimate indicates an error in the mean energy per particle of less than about 2 Mev, and, if we write the density as

$$\rho=(\tfrac{4}{3}\pi r_0^3)^{-1}, \tag{55}$$

then the error in r_0 due to (34) should be less than about 10%. Any additional error would be due to the choice of two-particle potentials.

III. NUMERICAL RESULTS

A. Puff-Martin Potential

We adopt for our calculations the potential used by Puff and Martin. This consists of the sum of a Yamaguchi potential,[11] which is operative only in relative S states, and the S-wave part of a hard-shell potential. The potential was chosen of this form primarily because it allows an analytic solution for the T matrix. The parameters were adjusted to give the experimental singlet and triplet scattering lengths and effective ranges, the deuteron binding energy, and the singlet phase shift at 310 Mev.[12] This potential does not contain spin-orbit or tensor components. The latter appear to contribute significantly (perhaps a few Mev) to binding energies, so the absolute energies which emerge from this calculation may not be too significant. However, because the net binding energy is only about $\frac{1}{2}$ or $\frac{1}{3}$ of the individual kinetic and potential terms, for our purposes (namely relative energies as a function of density and neutron-proton difference) we believed the potential to be adequate.

The Fourier transform of the potential is given by

$$\frac{1}{(2\pi)^3}\langle \xi k|V|\xi k'\rangle=\lim_{\lambda_c\to\infty} 2\lambda_c\frac{\sin kr_c}{k}\frac{\sin k'r_c}{k'} -2\lambda_\xi\frac{1}{(k^2+\beta_\xi^2)}\frac{1}{(k'^2+\beta_\xi^2)}, \tag{56}$$

each term of which is separable in the relative momenta, k and k'. Here ξ labels the spin state (singlet or triplet). The potential depends only on the momenta magnitudes, since S-state interactions lead to isotropic scattering.

The experimental data are fit by choosing

$$\begin{gathered} r_c=0.45\text{ f},\\ \beta_s=2.004\text{ f}^{-1},\quad \beta_t=2.453\text{ f}^{-1},\\ \beta_s^3/\pi^2\lambda_s=0.224,\quad \beta_t^3/\pi^2\lambda_t=0.172. \end{gathered} \tag{57}$$

From (45) we see that we only need the T matrix summed over spins. It is easily seen from (36) and (56) that

$$\begin{aligned}\textstyle\sum_\xi \langle \xi k|T_{K;\alpha\beta,\alpha\beta}(\omega+\omega_\beta(k_2))|\xi k\rangle &=\tfrac{1}{2}\langle sk|T(\omega+\omega_\beta(k_2)-\tfrac{1}{2}K^2)|sk\rangle(1+\delta_{\alpha\beta})\\ &\quad+\tfrac{3}{2}\langle tk|T(\omega+\omega_\beta(k_2)-\tfrac{1}{2}K^2)|tk\rangle(1-\delta_{\alpha\beta}),\end{aligned} \tag{58}$$

where

$$\begin{aligned}&\langle \xi k|T(-2\gamma^2)|\xi k\rangle\\ &=-2(2\pi)^3\Big\{\frac{\lambda_\xi}{(k^2+\beta_\xi^2)^2}-\frac{\gamma\sin kr_c}{k(1-e^{-2\gamma r_c})}\Big[\frac{\sin kr_c}{k\pi^2}\\ &\quad\times\Big(1-\frac{\pi^2\lambda_\xi}{\beta_\xi(\beta_\xi+\gamma)^2}\Big)+\frac{4\lambda_\xi}{(k^2+\beta_\xi^2)}\frac{e^{-\gamma r_c}-e^{-\beta_\xi r_c}}{(\beta_\xi^2-\gamma^2)}\Big]\Big\}\\ &\quad\times\Big\{1-\frac{\pi^2\lambda_\xi}{\beta_\xi(\beta_\xi+\gamma)^2}\Big[1-\frac{4\gamma\beta_\xi}{(\gamma-\beta_\xi)^2}\\ &\qquad\times\frac{(e^{-\gamma r_c}-e^{-\beta_\xi r_c})^2}{(1-e^{-2\gamma r_c})}\Big]\Big\}^{-1}.\end{aligned} \tag{59}$$

B. Nuclear Compressibility

The Puff-Martin equations (43), (44), and (45) were solved numerically on an IBM-709 computer and the results used to calculate the mean energy per particle (48) as a function of internucleonic separation r_0. The calculation was first performed for the special case $I/A=0$. Under these circumstances, $k_{f\alpha}$, $\rho_\alpha(k)$, and $\omega_\alpha(k)$ are all independent of α. The results are plotted in Fig. 1. The error due to computational methods is probably less than 0.1 Mev. A least-squares fourth order polynomial has been fit to the nine points calculated and is drawn as a smooth curve in the figure. Expressed as an expression about the minimum, r_{00}, the polynomial is given

$$\begin{aligned}E/A=&-17.58+105.01(r_0-r_{00})^2-291.61(r_0-r_{00})^3\\ &+387.03(r_0-r_{00})^4\text{ Mev},\end{aligned} \tag{60}$$

with r_0 and r_{00} measured in fermis. The energy minimum occurs at

$$r_{00}=(1.01_3\pm0.01)\text{ f}, \tag{61}$$

$$E/A=(-17.5_8\pm0.1)\text{ Mev}. \tag{62}$$

[11] Y. Yamaguchi, Phys. Rev. **95**, 1629 (1954).

[12] This was done prior to the fits at 310 Mev by M. H. MacGregor, M. J. Moravscik and H. P. Stapp [cf. P. S. Signell, Phys. Rev. Letters **5**, 475 (1960)]; this is probably as good as anything else. However, it may be expected to have some effect on the higher energy dispersive properties of the medium, and hence on the results calculated here, which are fairly sensitive to the behavior at the Fermi surface.

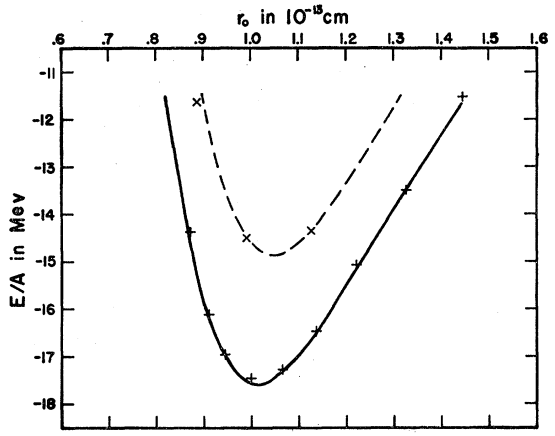

FIG. 1. Mean energy per particle in Mev as a function of interparticle spacing $r_0=(4\pi\rho/3)^{-\frac{1}{3}}$. The crosses are calculated points and the solid curve is a fourth-order polynomial least-squares fit for the case $N=Z$. The $\times$ points are calculated for $\Delta k_f/k_f=0.08$. The broken curve is merely a translation of the solid curve.

[The results of Puff and Martin, obtained using (54), are 0.92 f and -14.9 Mev.]

The nuclear compressibility at $I=0$, determined from the polynomial is then

$$K\equiv r_0^2\frac{\partial^2}{\partial r_0^2}\left(\frac{E}{A}\right)_{V,I}\Bigg|_{P=0}=214 \text{ Mev}. \tag{63}$$

The solutions for $\rho(k)$ and $\omega(k)$ at the minimum (by interpolation between nearest calculations) are given in Fig. 2.[13] Least-squares polynomial fits for these functions are

$$\rho(k)=0.87746+0.01422(k/k_f)-0.0264(k/k_f)^2, \tag{64}$$

and

$$\omega(k)=-116.406-47.285(k/k_f)+183.283(k/k_f)^2 -47.362(k/k_f)^3 \text{ Mev}, \tag{65}$$

where

$$k_f=1.575 \text{ f}^{-1}, \tag{66.a}$$

and

$$\omega(k_f)=-27.7 \text{ Mev}. \tag{66.b}$$

(For the same density, a noninteracting four-component Fermi gas has a Fermi momentum of 1.50 f^{-1}.)

The effective mass at the Fermi surface is given by

$$\frac{m}{m^*}=\frac{1}{2k}\frac{\partial\omega}{\partial k}\Bigg|_{k=k_f}=\frac{1}{0.58}. \tag{67}$$

C. Symmetry Energy

Equations (43) through (45) were also solved for the case $I/A\neq 0$ in order to investigate the dependence of nuclear binding energy on neutron-proton difference. This was accomplished in the following manner. The Fermi momenta were written as

$$k_{f\alpha}=k_f+\tau_\alpha\Delta k_f. \tag{68}$$

The parameters k_f and $\Delta k_f/k_f$ were then taken as the independent variables instead of ρ and I/A. Thus k_f was determined by minimizing E/A [as in Eq. (52)], and the arbitrarily chosen $\Delta k_f/k_f$ then determined I/A. The symmetry energy, C_{sy}, is then determined as the coefficient of the quadratic symmetry term,

$$C_{\text{sy}}(N-Z)^2/A, \tag{69}$$

in the expression for the energy. The results, displayed in Fig. 1, correspond to

$$\Delta k_f/k_f=0.08,$$
$$(N-Z)/A=0.249,$$
$$(k_f)_{\min}=1.51 \text{ f}^{-1},$$
$$(r_0)_{\min}=1.05 \text{ f},$$

and yield for the symmetry coefficient,

$$C_{\text{sy}}=43\pm 2 \text{ Mev}. \tag{70}$$

The results suggest setting

$$r_0(N,Z)=r_{00}[1+\alpha_r(N-Z)^2/A^2], \tag{71}$$

with

$$r_{00}=(1.01_3\pm 0.01) \text{ f}, \tag{72a}$$

$$\alpha_r=0.6\pm 0.2. \tag{72b}$$

The indicated error limits are our estimates of the accuracy of the numerical calculations. There appeared to be only a weak dependence of compressibility on $(N-Z)/A$.

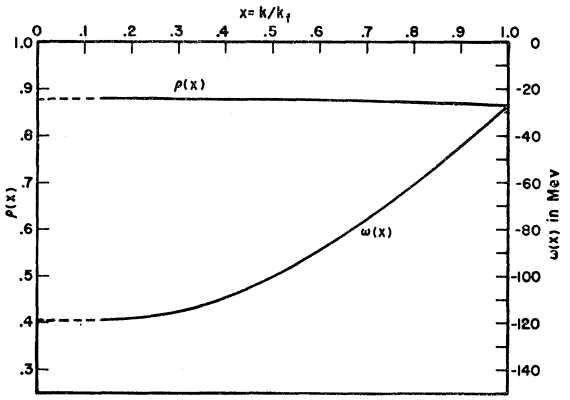

FIG. 2. Density of states $\rho(x)$ [cf. Eq. (44)], and the single-particle energies $\omega(x)=k^2+v(k)$ [cf. Eq. (43)] as functions of $x=k/k_f$.

[13] Hugenholtz and Van Hove[4] have indicated why the calculation of the mean energy per particle should be more accurate than that of the single-particle energies. See also P. Mittelstaedt, Nuclear Phys. **17**, 499 (1960), for a general discussion of the rearrangement energy.

IV. COMPARISON WITH EXPERIMENTS AND OTHER THEORIES

A. Energies

From the Bethe-Weizsäcker semiempirical mass formula, the Stanford electron-nuclear scattering experiments, and semiempirical models of the nucleus, experimental information on nuclear energies and densities can be extracted. The usual Weizsäcker mass formula contains terms which describe volume, symmetry, surface, and Coulomb effects. A rather general form of the formula is given by

$$\text{Binding energy}=\left[C_v+C_{sy}\frac{(N-Z)^2}{A^2}\right]A+\left[C_{surf}+C_{surf,sy}\frac{(N-Z)^2}{A^2}\right]A^{\frac{2}{3}}+\text{Coulomb energy}+O(A^{\frac{1}{3}})+\text{pairing and shell corrections.}\quad(73)$$

Values given by Green[14] and Cameron[15] for the constants C_v, C_{sy}, C_{surf}, and $C_{surf,sy}$ are listed in Table I. It is clear from this table that there is considerable latitude in the determination of the constants, even though both authors claim good fits to observed binding energies over a considerable range of nuclear masses. The non-uniqueness in the results arises from a variety of effects, among which are: (1) uncertainties in the empirical data; (2) treatment of specific "shell" effects; (3) treatment of the Coulomb (ordinary and exchange) energy; and (4) the handling of terms of order $A^{\frac{1}{3}}$ (curvature effects). In regard to (4), Cameron[15] proposed a formula with explicit $A^{\frac{1}{3}}$ terms. Green[14] fit Cameron's formula (by least squares) to a formula without $A^{\frac{1}{3}}$ terms; the result (column labeled "Cameron-Green" in Table I) shows the sensitivity of the coefficients of A and $A^{\frac{2}{3}}$ terms to the inclusion or noninclusion of $A^{\frac{1}{3}}$ terms.

Furthermore, the formulas are fit to data near the rather narrow valley of stable nuclides. It is not clear, for example, whether the quadratic dependence $(N-Z)^2/A^2$ is adequate as far from the region of normal nuclear matter ($N=Z$) as the stable valley in fact lies.

The uncertainties in the coefficients of the mass formula are not only of importance for comparison with theory, but are also of importance in problems of stellar structure and evolution, and nuclear fission where one is dealing with short-lived species whose masses are not now measurable. With these uncertainties in mind, we now compare the calculated energies with experiment:

TABLE I. Comparison of results. All numbers except r_0 are in Mev; see Eqs. (63) and (73) for definitions. r_0 is in fermis.

	Semiempirical			Theoretical	
	Green[b]	Cameron-Green[c]	Cameron[d]	Brueckner-Gammel[e]	Present work
C_v	−15.83	−16.34	−17.04	−15.2	−17.5
C_{sy}	23.52	30.34	31.45	26–43[f]	43
C_{surf}	17.97	20.96	25.84		
$C_{surf,sy}$	0	−36.35	−44.24		
K[a]	175	218	302	172	214
r_0		1.07[g]		1.02	1.01

[a] Nuclear compressibility K [Eq. (58)] was determined semiempirically from the model of Berg and Wilets (reference 16). In addition to the parameters listed directly above, the Stanford data (reference 17), $r_0=1.07$ f and surface thickness (90%−10%) =2.4 f, were employed.
[b] See reference 14.
[c] This column represents a least-squares fit by Green (reference 14) to Cameron's (reference 15) mass formula so that $A^{\frac{1}{3}}$ terms do not appear explicitly.
[d] See reference 15.
[e] See reference 19, Table II, column C.
[f] The value of 26 Mev was obtained[e] with the "best' Gammel-Thaler potential. The value of 43 Mev was obtained with another potential which also yields acceptable values of the density and binding energy but, like the one used in the present work, contains no odd-state terms (reference 19, Table II, column B).
[g] See reference 17.

1. Volume energy. In Table I are given the results of the present calculation for comparison with the mass formulas. The mean binding energy, $(E/A)_{min}=C_v$ is in surprisingly good agreement with the semiemprical value. Because the potential employed contains no tensor, spin-orbit or odd-state forces, the absolute energies could well be in error by several Mev from this source. There appears to be no violation of the 10% estimate by Puff of the accuracy of the model.

2. Symmetry energy. Our value of the coefficient of symmetry energy is larger than any of the semiempirical values found in Table I. The deviation from the $(N-Z)^2/A$ law is too weak (15% in Pb^{208}) to account for the discrepancy. In view of the uncertainties quoted above, it is by no means clear that this is a failure of the model.

3. Compressibility. In order to obtain comparison with experiment, it is necessary to appeal to semiempirical models. The model of Berg and Wilets[16] correlates *observed* nuclear mean binding energy, density, surface energy and surface thickness with nuclear compressibility K [cf. (63)]. The values predicted by that model from the various mass formula parameters are given in Table I. The semiempirical values of K thus range over nearly a factor of two, with our value (214 Mev) included. The primary point of interest of these numbers is that K is large, and thus nuclear matter is relatively incompressible. Compressional modes of nuclear excitation are expected to lie considerably higher than particle modes except perhaps in the lightest nuclei, where, unfortunately, surface phenomena complicate the analysis.

[14] A. E. S. Green, Revs. Modern Phys. **30**, 569 (1958); Phys. Rev. **95**, 1006 (1954); also private communication.
[15] A. G. W. Cameron, Can. J. Phys. **35**, 1021 (1957).
[16] L. Wilets, Revs. Modern Phys. **30**, 542 (1958); R. A. Berg and L. Wilets, Phys. Rev. **101**, 201 (1956); L. Wilets, *ibid.* **101**, 1805 (1956).

B. Densities

Analyses of the Stanford electron-nuclear scattering experiments[17] [assuming a $\rho=\rho_0(1+e^{(r-R)/a})^{-1}$ distribution] yield a central charge density for intermediate and heavy nuclei corresponding to

$$r_0=(1.07\pm0.02)\text{ f}\quad\text{(exptl)}. \tag{74}$$

The present calculations give $r_0=1.01_3$ f for normal nuclear matter. At least two corrections must be made for real nuclei: (1) As indicated in Eq. (71), the nuclear radius expands with increasing $(N-Z)^2/A^2$ for fixed A. Thus for gold (which has been particularly well studied), this effect would increase the calculated value by 0.02_4 f. (2) The Coulomb field exerts a (negative) pressure on the nucleus tending to produce expansion. This causes an increase in radius given by[18]

$$\delta r_0/r_0\simeq E_c/KA, \tag{75}$$

where E_c is the Coulomb energy of the nucleus. For gold, this yields an increase of 0.03_1 f. The two effects give a total increase of 0.05_5, for a final value of

$$r_0=1.06_8\text{ f}\quad\text{(Au,calc)}. \tag{76}$$

The agreement between theory and experiment is far better than either warrants. However, the very reasonable values obtained both for C_v and r_0 lend support to the calculated values of compressibility and symmetry (see "Discussion" below).

C. Comparison with Bruckner and Gammel

The most detailed calculations using the most "realistic" potential have been carried out by Brueckner and Gammel[19] employing the Gammel-Thaler[20] potential. Their results are given in Table I. Our calculations are in reasonable agreement with theirs.

Brueckner and Gammel find that the symmetry coefficient is very sensitive to the potential used. Employing the "best" Gammel-Thaler potential, they obtained $C_{sy}=26$ Mev. However, using another potential which contained no odd state interactions but which also yielded "acceptable" values of the mean binding energy, density and compressibility, they obtained $C_{sy}=43$ Mev. They attributed the reduction of the coefficient from 43 to 26 Mev to the inclusion of odd-state forces, where both the repulsive singlet and attractive triplet components act to lower the symmetry energy. Since no odd-state forces have been included in our work, we anticipate that their inclusion would also lower our value for the symmetry coefficient.

[17] A review and further references is given by D. G. Ravenhall, Revs. Modern Phys. **30**, 430 (1958).
[18] E. Feenberg, Phys. Rev. **59**, 149 (1941); L. Wilets, D. L. Hill, and K. W. Ford, *ibid.* **91**, 1488 (1953).
[19] K. A. Brueckner and J. L. Gammel, Phys. Rev. **109**, 1023 (1958).
[20] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 and 1337 (1957).

V. DISCUSSION

The main object of this work has been to test the validity of the very simple Puff-Martin model (regarded as a "variational" method) and to obtain useful information about nuclear compressibility and symmetry energy. The good agreement obtained between the calculations and experiment for mean binding energy and radius must be in large part fortuitous, since the interaction used contains no tensor, spin-orbit, or odd-state components (indeed, only S-state interactions are considered). The tensor force, in particular, was found by Brueckner and Gammel[19] to contribute 6 Mev to the binding energy per particle; its neglect also led to a lowering of the equilibrium density.

Regarded from another point of view, it would be a reasonable procedure to adjust the potential parameters to fit not only two-particle data, but to yield the correct binding energy and radius in this model, and then calculate with it such properties as compressibility and symmetry energy. This would constitute a semi-phenomonological approach to the determination of the parameters. We are spared this added procedure. However, fortuitous or not, the agreement attained for the binding energy and radius lends credibility to the other results. This is somewhat tempered by the apparent sensitivity of the symmetry coefficient on the odd-state interactions found by Brueckner and Gammel.

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