Stimulated Emission into Optical Whispering Modes of Spheres

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Stimulated emission into optical whispering modes of a spherical sample of $CaF_2:Sm^{++}$ has been observed. Light produced by the stimulated emission is radiated tangentially form each point on the surface of the sphere. The experimental results have been interpreted in terms of the electromagnetic analog of the Rayleigh theory of the whispering gallery.

WE have investigated the emission of fluorescent light from spherical samples of $CaF_2:Sm^{++}$ under high exciting light intensity. The concentration of Sm^{++} was about the same as that used previously for the investigation of optical maser effects in cylindrical rods.[1,2] The crystals were grown by Guggenheim.[3] The spheres used in the present investigation were between one and two mm in diameter, uniform to about 1 part in 10^4. The surfaces were highly polished.[4]

In the experiment, the sample was allowed to rest on the bottom of a Pyrex Dewar containing liquid hydrogen. The sample could be illuminated by focusing on it the light from a high-pressure xenon flashtube, by means of four large-aperture spherical mirrors. An image of the sphere was focused on the entrance slit of a small grating spectrometer. By this means an image of the sphere was formed in the plane of the exit slit by the monochromatic fluorescent light emitted by the sample. The resolution of the instrument was sufficient to allow the entrance slit to be opened to a width greater than the diameter of the sphere, so that the whole of the subtended surface of the sphere could be seen simultaneously. Arrangements were then made to photograph the image of the sphere formed at the exit slit, using either low-intensity continuous illumination or the xenon flash lamp.

Figure 1 shows a result obtained with low-intensity continuous illumination. A typical photograph obtained with the flash lamp is shown in Fig. 2; the flash

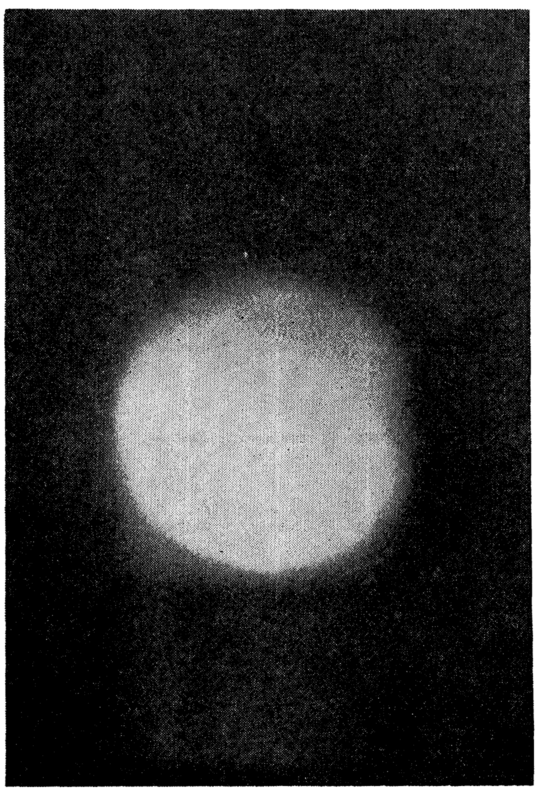

FIG. 1. Photograph of the sphere by its fluorescent light. Low-intensity illumination, long exposure.

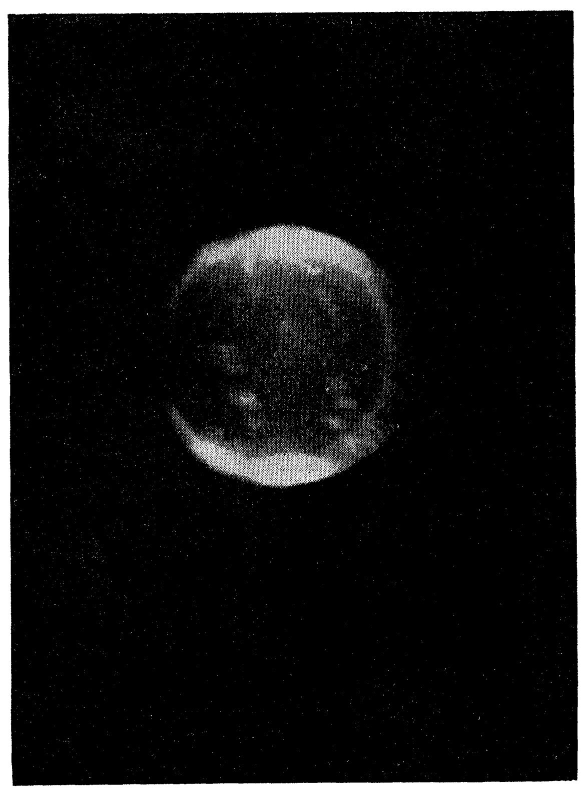

FIG. 2. Photograph of the sphere by its fluorescent light. High-intensity flash illumination.

[1] P. Sorokin and M. Stevenson, IBM J. Research and Develop. 5, 56 (1961).
[2] W. Kaiser, C. G. B. Garrett, and D. L. Wood, Phys. Rev. 123, 766 (1961).
[3] H. Guggenheim, J. Applied Phys. 32, 1337 (1961).
[4] W. L. Bond, Rev. Sci. Instr. 25, 401 (1954).

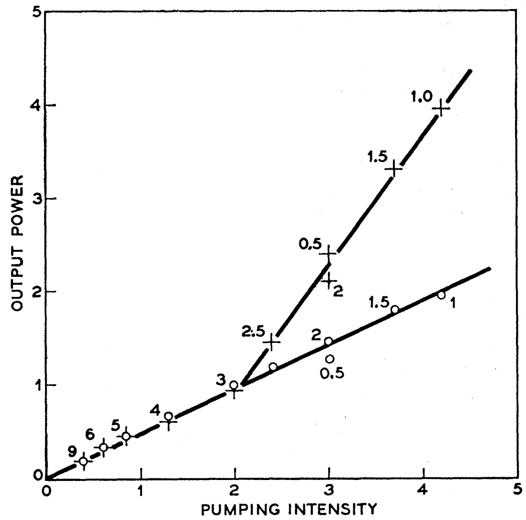

FIG. 3. Dependence on pumping intensity of the emission from the disk (circles) and from the rim (crosses). Numbers indicate intervals of time from the beginning of the flash in microseconds.

duration was about 3 μsec and the peak input power (in the range 5500–6500 A) of the order of 50 w/cm^2. Note that Fig. 1 indicates a disk the brightness of which is (within a factor of the order of π) uniform, while in Fig. 2 most of the disk is surrounded by a narrow rim of a brightness substantially greater than that of the rest of the disk. In addition, various spots on the surface of the disk light up quite brilliantly during the high-intensity illumination.

By means of a sheet of polarizing filter, it was established that the light from the rim was plane polarized with the E vector radial, while the light from the rest of the disk was unpolarized.

The time dependence of the light output from the rim and the light from the rest of the disk was determined by allowing the emerging light to fall on a photomultiplier. The entrance slit was narrowed and set successively on one edge of the disk and down the center. The light from the center of the disk followed fairly closely the pumping light; that from the rim, however, showed behavior typical of the stimulated emission phenomenon found with rods.[2] When the pumping light exceeded some threshold ($\sim$20 watts cm^{-2}) the output began to increase much faster with time. Figure 3 shows a typical plot of rim output power against lamp brightness, with the time at which each point was obtained indicated on the graph. Figure 3 reproduces almost exactly the form of similar curves obtained with rods. It was even found that, at sufficiently high light intensities, the similarity extended to the appearance of a heating effect.

We believe that these observations, and especially the existence of a sharp threshold (see Fig. 3), indicate that maser action is taking place. Since the refractive index of the sphere (1.45) is greater than that of the liquid hydrogen (1.11), elementary considerations from geometric optics lead to the expectation that some of the fluorescent light emitted from the Sm^{++} will always be totally reflected and so remain permanently entrapped. In fact, however, reflexion is not quite total for any angle of incidence if the surface is not plane. Nevertheless, light traveling roughly circumferentially just inside the surface of the sphere will only leak out very slowly. This phenomenon is well-known in the analogous problem in acoustics—that of the "whispering gallery"—which was first analyzed quantitatively by Rayleigh in 1914.[5]

For the electromagnetic case, the formal solution of the boundary value problem for the modes of a dielectric sphere is discussed in standard texts.[6] Using the analytic methods given by Rayleigh, we have explored the whispering modes for the electromagnetic case, and arrived at the following conclusions:

(i) The modes of highest Q have Q's of $(n^2-1)^{\frac{1}{2}} \times(2\pi a/\lambda)e^{2t}$ (magnetic modes) and $n^{-2}(n^2-1)^{\frac{1}{2}}(2\pi a/\lambda)e^{2t}$ (electric modes). Here n stands for the ratio of the refractive indices of the sphere and its surroundings; a for the radius of the sphere, λ for the free-space wavelength, and $t=(2\pi a/\lambda)[\cosh^{-1}n-\{(n^2-1)/n\}^{\frac{1}{2}}]$. Since $t\sim 10^4$ in the present case, the Q's of the modes of highest Q are enormous.

(ii) The small amount of energy which does leak out of these modes of highest Q does so nearly tangentially.

(iii) There are $\sim(2\pi a/\lambda)$ modes of highest Q. In the case of exact spherical symmetry these will be degenerate. The modes of the same frequency and next lowest Q, again $\sim(2\pi a/\lambda)$ in number, have Q's smaller by a factor $\exp(2\cosh^{-1}n)$, which is about 5 in the present case. Of course, in practice the actual Q must be limited by something other than the radiative losses, so that the difference in calculated Q between this and the first set may not be significant, and several sets may in fact be excited. Thus, in practice, the mode pattern is likely to be upset by the existence of surface scratches and other imperfections, some of which are undoubtedly responsible for the spots visible within the rim in Fig. 2.

(iv) Electric modes will lead to radiation having the E vector radial; magnetic modes to radiation having the E vector tangential. The experimental observations show that it is the electric modes that are excited. Now it has been found that the maser transition is forced electric dipole rather than magnetic dipole.[7] Thus the condition for oscillation to occur first in the rth mode is $4\pi\overline{\chi''}_rQ_r=1$, where Q_r is the Q of that mode and $\overline{\chi''}_r$ is the peak negative imaginary *electric* susceptibility, weighted by multiplying its value at each point by the ratio of the electrostatic to the total field energy and integrating over space. The experimental finding that oscillations occur in the electric modes therefore demonstrate that $\overline{\chi''}_rQ_r$ is higher for the electric modes, in

[5] Lord Rayleigh, Phil. Mag. **27**, 100 (1914).
[6] See, e.g., J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, New York, 1941), p. 554.
[7] D. L. Wood (private communication).

spite of the fact that the Q's themselves are slightly lower.

(v) Since the field associated with the whispering modes inside the sphere is limited to the vicinity of the surface, it should suffice if the luminescent centers are present in this surface layer.

(vi) Since the degeneracy of the whispering modes can be removed by introducing any departure from spherical symmetry, further mode selection ought to be possible. This could be done, e.g., by use of a spheroidal shape, by employing a uniaxial host lattice, or by applying a magnetic field.

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Theory for the Photoemission from a Space-Charge Region of a Semiconductor

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A theory is developed for evaluation of the effect of a space-charge region on photoemission from semiconductors. Such effect is shown to be significant in many cases and leads to the conclusions: (1) a "tail" should extend beyond the normal threshold region; (2) the energy of the valence band edge cannot, in general, be inferred from the observed threshold; (3) positive and negative space-charge regions cause effects which may not be symmetrical; (4) power-law fits to photoemission data near the threshold are of doubtful validity. It is shown that several anomalous results appearing in the literature can be explained by the space-charge effect.

INTRODUCTION

IT has long been realized that surface effects might influence the external photoelectric behavior of semiconductors. Such influence may arise from the surface states themselves or from the adjacent space-charge region in which an electrostatic potential alters some of the bulk properties of the material. The latter aspect of the problem will be treated here and the surface states enter only as they affect the space-charge region. The purpose of the work is to evaluate the effect of the presence of a space-charge region on photoemission from semiconductors. It is thus concerned with what is generally called the "volume photoelectric effect," so that the conclusions reached also have a bearing on the question of the relative importance of the volume vs surface photoeffect.

CONVENTIONAL TREATMENT OF PHOTOEMISSION

Because there exists no established theoretical model for photoemission from semiconductors, these calculations are based on the most general expressions which can usefully represent the fundamental processes. The essential conclusions can thus be applied to any of the theories found in the literature. A simple, nondegenerate semiconductor is assumed throughout with Fermi energy E_f, valence band edge E_v, and work function ϕ. It is further assumed that all photoelectrons originate in the valence band so that the photoelectric threshold energy $(h\nu)_t$ corresponds to emission of electrons whose initial energy was E_v. The photocurrent may be written as

$$i=\int_E I(\nu)\sigma(\nu,E)f(E)n(E)t(E+h\nu)dE, \qquad (1)$$

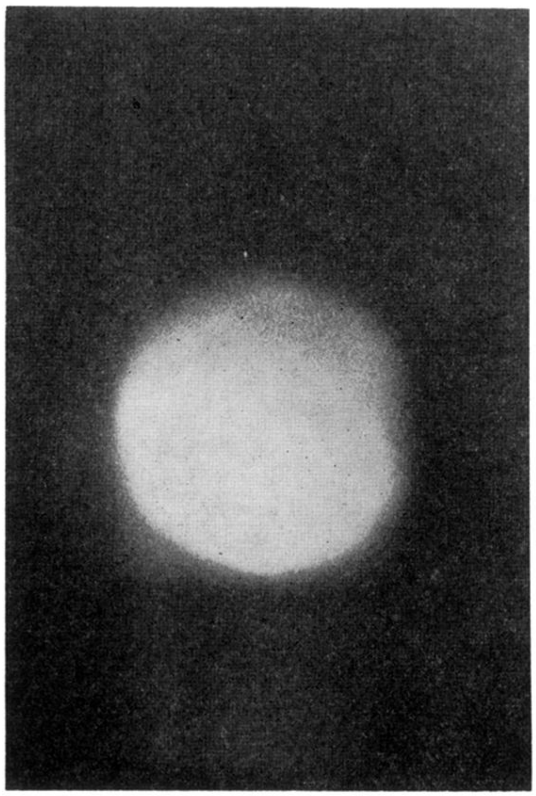
where E is the initial energy of the electron, $h\nu$ the photon energy of the light whose intensity is I, σ the cross section for an absorption capable of producing a photoelectron, f the Fermi occupation function, n the density of states in the semiconductor, and t an escape probability for an excited electron. In keeping with the assumption of nondegeneracy, it will be assumed $f(E)\simeq 1$ in the valence band, although this restriction can be relaxed if necessary.

In the various theories of photoemission, different models are advanced for the dependence on energy of the absorption and escape probabilities. The density of states is normally taken as $n(E)\propto(E-E_v)^{\frac{1}{2}}$ if energy increases for levels lying deeper in the valence band. This approximation should be satisfactory in most cases, particularly for energies near the band edge where surface effects should be most important. Combining the energy dependences of the terms in Eq. (1) results in all cases in a functional dependence on energy having the form

$$i\propto(E-E_v)^l \qquad (2)$$

near the photoelectric threshold. Apker *et al.*[1] used a value

[1] L. Apker, E. Taft, and J. Dickey, Phys. Rev. 74, 1462 (1948).



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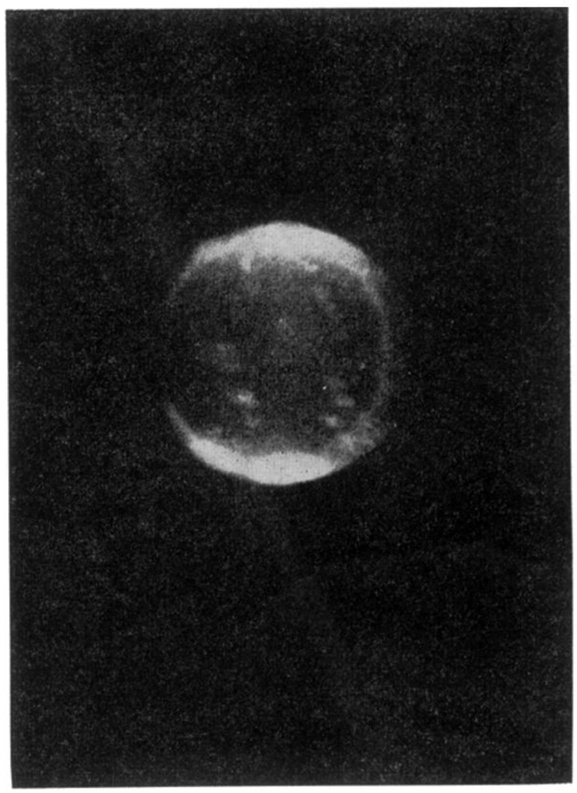

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