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Fountain Pressure Measurements in Liquid He II†

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New observations of the fountain pressure P_f are described for slits of width 0.28 μ and 3.36 μ and for temperature differences up to 1.0°K. The measuring technique employs a commercially available strain gauge pressure transducer, the low-temperature characteristics of which are discussed. The results show that some of the fountain pressure phenomena previously observed by the authors and believed to be anomalous could be ascribed to experimental difficulties. In particular, in the present work no "excess P_f" (i.e., greater than given by the London equation) values were found and no hysteresis in P_f at large heat currents in the wide slit was observed. However, an anomalous lowering of the λ point with pressure and ideal P_f's for large heat currents have been obtained as previously. The validity of associating the heat of transport $Q^*=\rho ST$ in the thermomechanical effect with the calorimetric entropy is demonstrated up to 2.16°K.

I. INTRODUCTION

We have previously reported measurements[1] of heat conduction and fountain pressure in liquid He II in slits between 0.28μ and 3.36μ wide. At that time several apparently anomalous effects were observed for the fountain pressure P_f when measurements were made involving large temperature gradients. These observations were summarized as follows:

1. For the smallest slit P_f's are found larger than the maximum predicted by theory (integrated H. London equation).
2. For the larger slits maximum, or slightly larger than maximum P_f's are observed even for very high heat currents.
3. For the wider slits P_f shows a marked increase near T_λ.
4. For the 0.276-μ slit an anomalous $(dT_\lambda/dP)_{\rm slit}$ is observed.
5. For the 3.36-μ slit an hysteresis in P_f appears at high heat currents ($\dot{Q}$) with no discernible effect on (the) $\dot{Q}$ (vs ΔT curve).

Measurements of P_f which displayed the above anomalies were made by observing the volume change, of a given known volume of liquid He II, produced by the compressive action of the fountain pressure. At the time of publication of reference 1 no spurious effects attributable to the apparatus or to the method of data reduction were discernible, and the anomalies were believed to be manifestations of the non-linear nature of the thermohydrodynamics of liquid He II. In order to further test these findings a series of additional experiments has been performed using a different method to measure P_f, namely by introducing a pressure transducer directly in the heated reservoir in which the fountain pressure is developed. The purpose of the present communication is to describe these new experiments and, using the results therefrom, to clarify and amend some of the conclusions reached in the earlier work concerning the nature and the magnitude of the fountain effect.

II. APPARATUS AND PROCEDURE

Figure 1 shows schematically that part of the apparatus immersed in the liquid He bath. The copper can C is evacuated through a thin-walled stainless steel tube V, the lower 20 cm of which is copper and the bottom of which is capped off to form a radiation trap. Thermometers T_H and T_C probing the hot and cold cells, respectively, are $\frac{1}{2}$ w, 33ohms (nominal) Allen-Bradley resistors, one lead of each (ground) being soldered directly into the liquid reservoirs on either end of the superleak S. The filling tube F is of 1 mm i.d. glass

† Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] W. E. Keller and E. F. Hammel, Jr., Ann. Phys. 10, 202 (1960).

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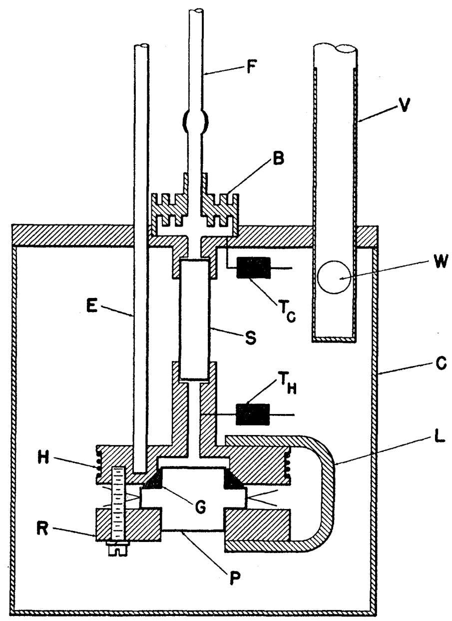

FIG. 1. Schematic diagram of the low temperature section of the apparatus used for measuring fountain pressures with a pressure transducer.

tubing and terminates via a glass-Kovar seal at the cold cell. The cap B is finned to provide a large area for heat transfer, thereby ensuring that the cold-cell temperature is that of the bath. The heater H is wound around the circumference of the hot cell and is of #36 (B&S) manganin wire with a resistance of 100 ohms. Two power-input and two potential leads are fixed to the heater. Since heat conduction by the stainless steel slit assembly is poor, a liquid He II exchange tube[2] E connecting the bath and the copper plate of the hot cell is employed for initial cool-down and for calibration of the thermometers. The bottom of the hot cell is formed by the pressure sensitive diaphragm of the pressure transducer P (Consolidated Electrodynamics Corporation pressure pickup type 4–325, 0–15 psia.) Two power-input and two voltage-output leads are attached to the transducer. A seal leak-tight to He II is effected with a Cerroseal-35 O ring G compressed in place by 8 screws around the retaining ring R. To ensure that the transducer assembly reaches isothermal conditions rapidly, a flexible copper rod L of 3 mm diam is soldered to the top of the hot cell and to the retaining ring. Electrical lead wires of #40 copper from the heater, transducers and thermometers are taken to a terminal strip (not shown) inside the can; from the strip the insulated wires (#40 constantan except the input leads to the heater and the transducer, which are #40 copper) are wound several times around the base of tube V and attached with G. E. 7031 adhesive, thence led through port W up the center of tube V and finally to a ten-prong Stupakoff seal at room temperature.

The slits used in these experiments had gap widths of 0.276μ and 3.36μ and were the same ones designated in reference 1 as slits II and III′, respectively.

The C. E. C. pressure transducer is a strain-sensitive electrical bridge entirely encapsulated in a stainless steel pill-box, the cover of which is a thin diaphragm. Attached to this diaphragm is a four-member yoke upon which the identical arms of the bridge are wound. Movement of the diaphragm deforms the yoke in such a way as to place two arms of the bridge in tension and the other two in compression. The total impedance of the bridge remains essentially constant (350 ohms) during deformation, but the voltage output E_0 for a given voltage input E_i, varies linearly with the pressure upon the diaphragm. According to the manufacturer the pickup was tested and temperature-compensated only to $-60°C$ with 5.0 v dc or ac rms excitation recommended.

For use in the present experiments the power dissipation by the transducer was required to be maintained below about 50 μw. Hence excitation voltages of 0.03 to 0.12 v dc were employed, yielding sensitivities dE_0/dP of the order of 1 μv/mm Hg. For given E_i the sensitivity of the transducer increased by about 3% on reducing the temperature from 300° to 2°K, and once

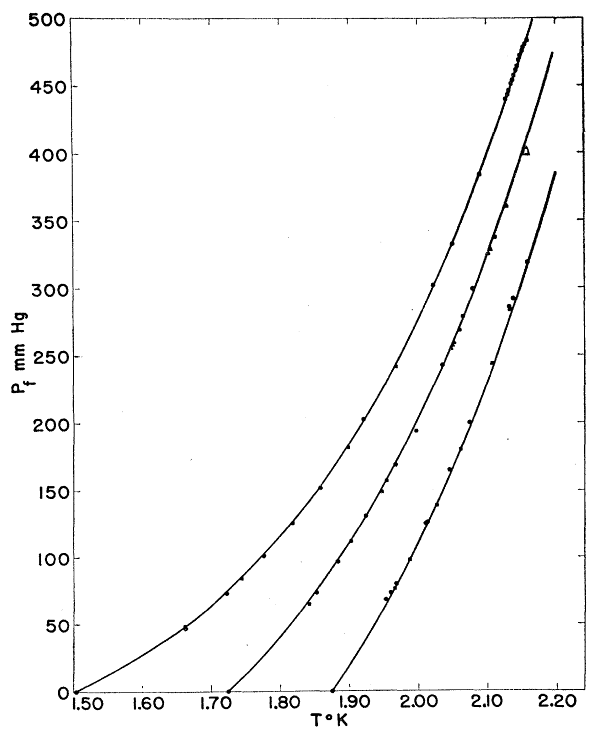

FIG. 2. Integrated fountain pressure P_f vs temperature T obtained with slit II (gap width 0.276 μ) for three different T_0's. Solid curves are calculated from the H. London equation.

[2] W. E. Keller, H. S. Sommers, Jr., and J. G. Dash, Rev. Sci. Instr. **29**, 530 (1958).

at the low temperature dE_0/dP remained constant to better than 1%. Temperature cycling had no adverse effect upon the mechanical or electrical performance of the pickup except that the zero of the instrument shifted irreproducably from cycle to cycle. At the lowest pressures and power-inputs some erratic response of the transducer was encountered; these effects while not understood were easily recognized and the results discarded.

Before and after each run the transducer was calibrated for the zero as well as for dE_0/dP. Changing the temperature between 1°K and 2°K did not alter the electrical response, so that calibration of the transducer was generally carried out at the reference temperature T_0 of each run, even though during the run the transducer was operating at $T>T_0$. The pressure standards for determining the P–E_0 relation were (1) the vapor pressure of the bath (as the temperature was lowered from T_λ to T_0) as measured on an oil manometer with a cathetometer; and (2) a Wallace and Tiernan gauge (either 0–100 mm or 0–800 mm range, previously calibrated against a mercury manometer) in communication with the filling line—used generally with the filling line pressured above the vapor pressure. For all pressure readings appropriate corrections were made for the hydrostatic head of liquid He above the transducer diaphragm as determined by the liquid level viewed in the glass filling tube.

Various excitation voltages were obtained from a voltage divider powered by a 2-v storage battery. Drift in E_i during a given run remained less than 0.2%. E_0 was read on a Leeds and Northrup type K-3 potentiometer which is capable of being read to 0.1 μv.

Temperatures were obtained by measuring the resistance of T_C and T_H on separate dc bridges employing Hewlett–Packard null-indicators (sensitivity 10^{-13} amp/mm). In all cases thermal emf's were balanced out by reversing the direction of the current. The thermometers were calibrated against the vapor pressure of the He bath at the same time the transducer was calibrated.

Following calibration of the thermometers and the transducer, the liquid was pumped from the liquid exchange tube (thermal switch). Successive increments of heat were then applied to the hot cell, pressure and temperature measurements being made at each step. Equilibrium times varied from 3 to 15 min, but reliable results could be obtained from the system at near-equilibrium conditions since P_f was found to follow temperature changes very closely. This was established from data taken with rising and falling hot-cell temperatures, the results lying on the same curve.

It is difficult to assign to each experimental point a precise figure for the probable error arising from calibration, nonlinearity, and hysteresis in the response of the transducer. However, a figure of 1 to 2% of P seems reasonable, the percent error at high pressures being less than at low pressures.

III. RESULTS

The results of experiments with the smaller slit run at $T_0=1.502$°K, 1.724°K, and 1.875°K are shown in Fig. 2. For comparison the integrated London equation

$$P_f=\int_{T_0}^{T}\rho\, SdT$$

is drawn as the solid curve for each T_0. All the measured points, except the triangles, were obtained with a transducer sensitivity of 0.50 μv/mm Hg. The triangles for $T_0=1.724$°K were obtained with a sensitivity of 1.04 μv/mm Hg to test the effect upon the results of changing the input voltage. The agreement between the two sets of data for this T_0 is well within the limits expected to arise from calibration errors. The run designated by squares for $T_0=1.502$°K showed in the beginning an erratic zero; for this run only, the zero was determined by normalizing the data for the lowest pressure point ($P_f=48$ mm) to the theoretical curve.

A detailed examination of the fountain pressure behavior near the λ point was made for $T_0=1.502$°K. The measurements for three separate runs are shown in Fig. 3.

For the 3.36-μ slit data obtained at $T_0=1.143$°K, 1.414°K, 1.582°K, 1.767°K, 1.924°K, and 2.048°K are given in Fig. 4, where the heavy solid curves represent the integrated London relation. During the course of the experiments, after runs at $T_0=1.414$°, 1.767°, 1.924°, and 2.048°K were completed, the transducer was irreparably damaged by an inadvertently applied overpressure. The transducer was replaced by a new one and the $T_0=1.767$°K run repeated, with the result that the former measurements were reproduced. Sensitivities employed in experiments with slit III′ ranged from 1.06 μv/mm Hg to 3.39 μv/mm Hg.

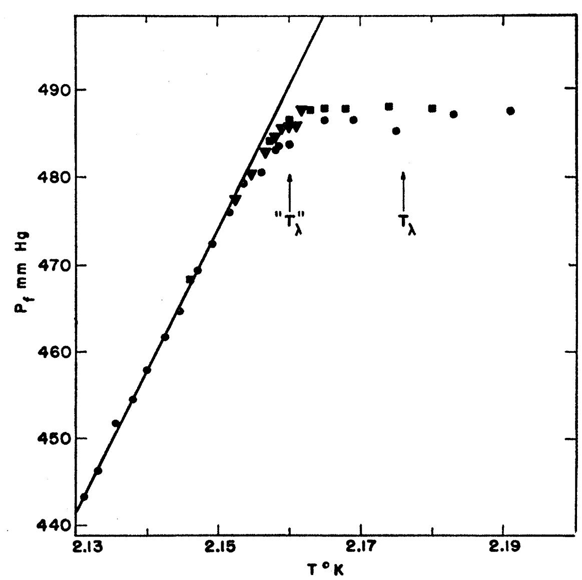

FIG. 3. Details of the P_f vs T measurements near the λ point for slit II, $T_0=1.503$°K, showing an apparent λ point shift to lower temperatures of about 13 mdeg.

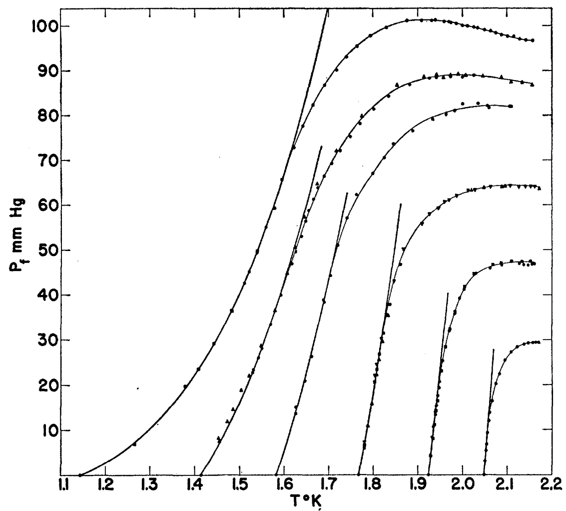

FIG. 4. Integrated fountain pressure P_f vs temperature T obtained with slit III′ (gap width 3.36 μ) for six different T_0's. Heavy solid curves are calculated from the London equation.

IV. DISCUSSION AND CONCLUSIONS

With reference to the five anomalous characteristics enumerated in Sec. I, the present results appear to corroborate some of the previous findings and seriously disagree with others. Since the transducer experiments are direct and give results more in agreement with expectation than the volume-change measurements, there is the temptation to give preference to the former. However, up to this point a reasonable basis is lacking for considering either set of data more reliable. As an attempt to provide such a basis a new set of experiments was performed using a modification of the volume-change apparatus described in reference 1.

The slit assembly was replaced by a thin-wall inconel tube 0.3 cm i.d., 12 cm long, all other components of the apparatus remaining the same. The liquid level in the glass capillary in the filling line was again observed as the heat current was varied. In the wide heat conduction tube both ΔT and P_f were expected to be small, even with large $\dot{Q}$, so that an appreciable change in liquid level could not be associated directly with the fountain pressure. We recall that a drop of the meniscus was interpreted as an increase in P_f. At low $\dot{Q}$ no effect was discerned; but for a given reference temperature at a certain power, which seemed reproducible, the liquid level began to fall, and continued to do so in increments corresponding to steps of increased power. Sometimes the change in meniscus position was instantaneous, sometimes delayed. This behavior, together with that when the power was interrupted, reduced, or shut off entirely was indeed very reminiscent of the hysteresis effects mentioned in item 5 Sec. I. Considerable time was spent in studying the characteristics of this behavior until a very careful observation of the filling tube revealed that a small drop of liquid was collecting at the point where the viewing capillary (i.d. = 0.05 cm) flared out into the wider filling tube (i.d. = 0.4 cm); this drop increased in size as the liquid meniscus in the capillary fell.

These observations clearly indicate the source of the "excess" fountain pressure anomalies observed previously with the volume-change apparatus at the larger heat currents. Since no ambiguous effects were obtained using the transducer, we now have good reason to prefer the results from the present experiments wherever the two sets of data diverge. In particular, we conclude from the data shown in Figs. 2 and 4 that (1) nowhere are P_f values observed larger than those predicted by the London relation; (2) no hysteresis effects are obtained for the 3.36-μ slit; and (3) near T_λ the fountain pressure does not show a marked increase for the wider slit, but instead P_f flattens off and even decreases slightly for runs at the lower T_0's. On the other hand, Fig. 3 demonstrates that for the 0.276-μ slit the apparent anomalous shift of λ-point temperature with pressure is still observed, the value of $(dT_\lambda/dP)_{\rm slit}$ being in reasonable agreement with the earlier measurements. Furthermore, the data on the 3.36-μ slit again indicate that even for very large heat flows and temperature gradients the ideal integrated fountain pressure is obtained. This phenomenon is especially striking for the run at $T_0 = 1.143$°K where P_f remains ideal for ΔT's almost as large as 500 mdeg. Under these conditions the heat current density is about 1.5 w/cm^2 corresponding to relative normal fluid and superfluid velocities $\bar{V}_n - \bar{V}_s$ as high as 230 cm/sec at the cold end of the slit.

In the analysis of the present data we are primarily concerned, as in reference 1, with determining the extent of agreement of the measurements with the linear thermohydrodynamic theory of liquid He II, leaving for a later paper a treatment of nonlinear effects. However, it is useful to interject into the following discussion pertinent results of some calculations we have made based on the nonlinear theory of Gorter and Mellink[3] in which the values of the mutual friction parameter A are taken from the work of Vinen.[4]

The nonlinear calculations for the smallest slit indicate that for the $T_0 = 1.502$°K curve the deviation from London relation due to dissipative effects should be less than 0.5% even for T near the λ point. Hence the experimentally observed fountain effect for this slit can be used as a critique of the calorimetrically determined entropy $S_{\rm cal}$. A paper by Brewer and Edwards[5] summarizes the previous comparisons between the thermomechanical effect and $S_{\rm cal}$ and provides new data for the heat of transport Q^*, which by London's theory[6] is equivalent to ρST. The result is that the correspondence

[3] C. J. Gorter and J. H. Mellink, Physica **15**, 285 (1949).
[4] W. F. Vinen, Proc. Roy. Soc. (London) **A240**, 114 (1957); **240**, 128 (1957).
[5] D. F. Brewer and D. D. Edwards, Proc. Phys. Soc. (London) **71**, 117 (1958).
[6] H. London, Proc. Roy. Soc. (London) **A121**, 484 (1939).

between Q^* and S_{cal}, as determined by Kramers, Wasscher, and Gorter[7] and by Hill and Lounasmaa,[8] is well established up to 1.8°K. Above this temperature the Q^* measurements of Brewer and Edwards are about 5% higher than expected from the calorimetric data. The indirect measurements of Q^* by van den Meydenberg *et al.*[9] deviate up to about 7% above 2.0°K. In Fig. 5 we have plotted the percent deviation of the observed P_f's from those calculated, using the London relation and the Kramers entropy values,[10] versus the temperature for the three curves shown in Fig. 2. It is seen that within our expected experimental error the agreement is good up to 2.16°K and indicates that the London relation between Q^* and S_{cal} is valid to better than 2% above 1.8°K as well as below (as shown previously by others), when the comparison is made using Kramers' values of entropy.

We have already pointed out two striking effects displayed by the curve for $T_0 = 1.143$°K obtained with the 3.36-μ slit, namely the large ΔT over which P_f remains ideal and the maximum in the curve. The former effect implies that for these conditions a large critical velocity exists which must be exceeded before dissipation effects appear. The Gorter–Mellink type calculations made without inclusion of a critical velocity give results which at small ΔT slowly diverge from the linear theory and at $\Delta T = 500$ mdeg lie only 5% below the linear theory and the measurements. This seems to indicate that the effect of the critical velocity, when included in the calculations, should enter in a rather subtle and complex manner. It is rather disturbing in this connection that the heat flow measurements[1] were observed to follow the linear London theory over a considerable range of temperatures, but for $T_0 < 1.5$°K the $\dot{Q}$ vs ΔT data showed as much as 20% *larger* heat conduction than predicted by the theory. As yet we have found no satisfactory explanation for this behavior, nor for the difference in character between the heat flow and fountain pressure measurements when both are compared to the linear theory below 1.5°K. It should be noted, however, that the heat conducted by the stainless steel of the slit becomes large in this temperature region and at about 1.25°K exceeds that carried by the liquid helium (cf. Table II of reference 1). It is possible that errors in accounting for the heat flow through the stainless steel provide the source of the above-mentioned effect. The fountain pressure measurements are independent of such corrections.

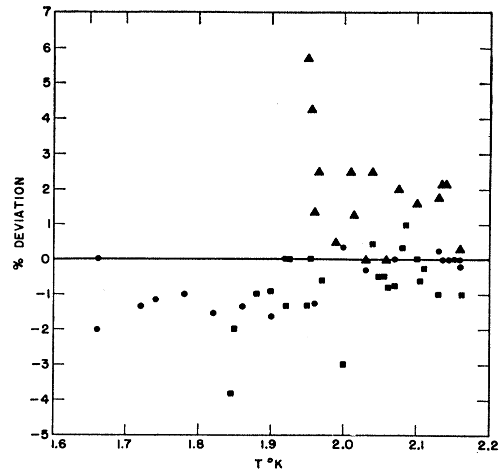

FIG. 5. Percent deviation of experimental points plotted in Fig. 2 from the theoretical curves obtained using the entropy values of Kramers *et al.* ● $T_0 = 1.502$°K; ■ $T_0 = 1.724$°K; ▲ $T_0 = 1.875$°K.

With respect to the maximum in the P_f curves for low T_0 and the 3.36-μ slit, sufficient care was exercised in the obtaining of the data to give confidence that the effect is indeed real. Additional support for the possibility of such a maximum is given by some of the Gorter–Mellink type calculations, which exhibit maxima and place them at T's and P_f's approximately where they are observed. However, these effects are not as yet understood.

[7] H. C. Kramers, J. D. Wasscher, and C. J. Gorter, Physica **18**, 329 (1951).
[8] R. W. Hill and O. V. Lounasmaa, Phil. Mag. **2**, 1943 (1957).
[9] C. J. N. van den Meydenberg, K. W. Taconis, J. J. M. Beenakker, and D. H. N. Wansink, Physica **20**, 157 (1954).
[10] The values actually used here are those of Kramers corrected to the temperature scale T_{L55} [H. van Dijk and M. Durieux, "Helium Vapor Pressure Scale T_{L55}," Kamerlingh Onnes Laboratory, Leiden, Holland (1955)]. The data of Hill and Lounasmaa are based on the scale T_{55E}. The difference between the two sets of data is not sufficient to be distinguishable in comparison with the present P_f results.