Electron-Positron Colliding Beam Experiments

N. Cabibbo and R. Gatto
Istituti di Fisica delle Università di Roma e di Cagliari, Italy and
Laboratori Nazionali di Frascati del C.N.E.N., Frascati, Roma, Italy

(Received June 8, 1961)

Possible experiments with high-energy colliding beams of electrons and positrons are discussed. The role of the proposed two-pion resonance and of the three-pion resonance or bound state is investigated in connection with electron-positron annihilation into pions. The existence of a three-pion bound state would give rise to a very large cross section for annihilation into $\pi^0+\gamma$. A discussion of the possible resonances is given based on consideration of the relevant widths as compared to the experimental energy resolution. Annihilation into baryon-antibaryon pairs is investigated and polarization effects arising from the nonreal character of the form factors on the absorptive cut are examined. The density matrix for annihilation into pairs of vector mesons is calculated. A discussion of the limits from unitarity to the annihilation cross sections is given for processes going through the one-photon channel. The cross section for annihilation into pairs of spin-one mesons is rather large. The typical angular correlations at the vector-meson decay are discussed.

A neutral weakly interacting vector meson would give rise to a strong resonant peak if it is coupled with lepton pairs. Effects of the local weak interactions are also examined. The explicit relation between the e^2 corrections to the photon propagator due to strong interactions and the cross section for annihilation into strongly interacting particles is given.

INTRODUCTION

A PROPOSAL for electron-electron colliding beams was made some time ago at Stanford by Barber, Gittelman, O'Neill, Panofsky, and Richter, and an experiment on electron-electron scattering based on such a proposal is being carried out.[1] Projects for electron-positron colliding beams are also under development at Stanford[1] and at Frascati.[2] The project at Frascati is intended to obtain high-energy (>1 Bev) electron-positron colliding beams.[3] We have already discussed possible experiments with e^+-e^- colliding beams.[4] In this note we shall present a more detailed discussion of possible electron-positron experiments and of the theoretical questions connected with them.

Like electron-electron experiments, electron-positron experiments can test the validity of quantum electrodynamics at small distances.[4a] They present, however, some very typical features that sufficiently justify the effort to produce electron-positron colliding beams. Most of the annihilation processes of e^+-e^- take place through the conversion of the pair into a virtual photon of mass equal to the total center-of-mass energy. The photon then converts into the final products. These reactions proceed through a state of well-defined quantum numbers, and as consequence the possible initial and final states are essentially limited. The interaction of the final particles with the virtual photon is directly measured in the experiment. The virtual photon four-momentum is timelike in these experiments, in contrast, for instance, to electron scattering on nucleons where the four-momentum of the transferred virtual photon is spacelike. Form factors of strongly interacting particles can thus be measured for timelike values of the momentum, in a region where they have, in general, an imaginary part. Electron-positron annihilations in flight offer the possibility of carrying out a Panofsky program, of a systematic exploration of the spectrum of elementary particles by observing their production by the intermediate virtual gammas. Unstable particles with the same quantum numbers as the intermediate photon can be produced singly as resonant states that soon after decay. At the appropriate energy there would appear resonance peaks in the production cross section for the final decay products.

1. GENERAL CONSIDERATIONS

1.1. We consider a reaction of the kind

$$e^+ + e^- \rightarrow a+b+\cdots+c, \tag{1}$$

where a, b, $\cdots$, c are strongly interacting particles. At the lowest electromagnetic order we assume that the reaction goes through the one-photon channel represented by Fig. 1. In the figure, q_+ and q_- are the positron and electron four-momenta, respectively, $k=q_++q_-$ is the time-like four-momentum of the virtual photon, and a, b, $\cdots$, c are the four-momenta of the produced particles. The element of the S matrix is given by

$$\langle a,b\cdots c|S|e^+e^-\rangle = \frac{2\pi e}{k^2}(\bar{v}\gamma_\nu u)\langle a,b\cdots c;\text{out}|j_\nu(0)|0\rangle \times\delta(q_++q_--a-b-\cdots c), \tag{2}$$

[1] W. Barber, B. Gittelman, G. K. O'Neill, W. K. H. Panofsky, and W. C. Richter (to be published); G. K. O'Neill, *Proceedings of the International Conference on High-Energy Accelerators and Instrumentation, CERN, 1959* (CERN, Geneva, 1959), p. 125; W. K. H. Panofsky, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 769; G. K. O'Neill and E. J. Woods, Phys. Rev. **115**, 659 (1959).

[2] F. Amman, C. Bernardini, R. Gatto, G. Ghigo, and B. Touschek (unpublished). A smaller storage ring for electrons and positrons for maximum energy of 250 Mev is already at an advanced state of construction; see C. Bernardini, G. F. Corazza, G. Ghigo, and B. Touschek, Nuovo cimento **18**, 1293 (1960).

[3] Electron-positron colliding beams are also being considered at CalTech, Cornell, and Paris.

[4] N. Cabibbo and R. Gatto, Phys. Rev. Letters **4**, 313 (1960); Nuovo cimento **20**, 184 (1961).

[4a] See R. Gatto, Proceedings of the Aix-en-Provence Conference (1961) (to be published) for a discussion of the possible tests of electrodynamics with electron-positron beams.

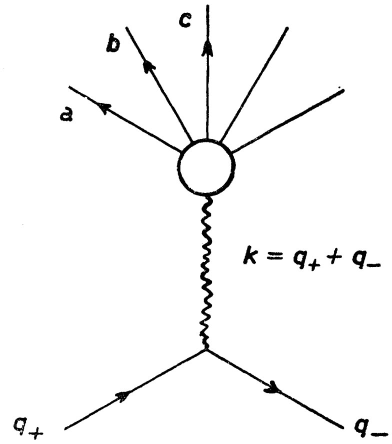

FIG. 1. Graph representing the one-photon channel. The symbols are defined in the text.

where v and u are Dirac spinors describing the positron and the electron, respectively, and $j_\nu(x)$ is the electromagnetic current operator. The relevant quantity is the matrix element of $j_\nu(0)$ between the vacuum and the final state of the produced particles. It will be convenient to define the four-vector

$$J_\nu=(2\pi)^{3n/2}\langle a,b,\cdots c;\,\mathrm{out}|j_\nu(0)|0\rangle, \tag{3}$$

where we have introduced for normalization purposes a factor $(2\pi)^{3n/2}$, where n is the number of the produced particles. From

$$\partial j_\nu(x)/\partial x_\nu=0,$$

which holds for the charge current $j_\nu(x)$, it follows that

$$k_\nu J_\nu=0. \tag{4}$$

1.2. It will be convenient to refer all the quantities to the center-of-mass system for the reaction. In a colliding beam experiment the center-of-mass system is actually the laboratory system itself.

We shall in the following neglect the electron mass. We call E the energy of each incident particle in the center-of-mass system. It follows that

$$k^2=(q_++q_-)^2=-4E^2. \tag{5}$$

Moreover, Eq. (4) in the center-of-mass system becomes

$$2iEJ_4=0. \tag{6}$$

Therefore, J_ν has no time-like component in this system. We shall call $\mathbf{J}$ its space-like component.

The total cross section for unpolarized initial and final particles is given by

$$\sigma=\frac{(2\pi)^{5-3n}\alpha}{16E^4}\int d^3ad^3b\cdots d^3c\,\delta(E_a+E_b+\cdots+E_c-2E)$$
$$\times\delta^3(\mathbf{a}+\mathbf{b}+\cdots+\mathbf{c})T_{mn}\sum_{a,b\cdots}R_{mn}, \tag{7}$$

where $\alpha=e^2/(4\pi)=(1/137)$; $\mathbf{a}$ and E_a are the momentum and energy of particle a, etc.; the tensor T_{mn} is given by

$$T_{mn}=\tfrac{1}{2}(i_mi_n-\delta_{mn}), \tag{8}$$

where $\mathbf{i}$ is the unit vector pointing along the direction of, say, the incoming positron; the tensor R_{mn} is defined as

$$R_{mn}=-J_mJ_n^*; \tag{9}$$

and the summation $\sum_{a,b\cdots c}$ is over the final spin states. Differential cross sections and cross sections for polarized final particles can be obtained from (7) by omitting the relevant integrations and spin summations.

1.3. For the production of two particles a, b of equal mass M, Eq. (7) gives

$$d\sigma=\frac{\alpha}{32}\frac{\beta}{E^2}(T_{mn}\sum_{a,b}R_{mn})d(\cos\theta), \tag{10}$$

where

$$\beta=[1-(M/E)^2]^{\frac{1}{2}} \tag{11}$$

is the velocity of the final particles. If the masses of a and b are different, (10) has to be replaced by

$$d\sigma=\frac{\alpha}{32}\left(\frac{p}{E}\right)\frac{1}{E^2}\frac{E_aE_b}{E^2}(T_{mn}\sum_{ab}R_{mn})d(\cos\theta), \tag{12}$$

where p is the final center-of-mass momentum and E_a and E_b are the energies of a and b. We have called θ the center-of-mass angle.

1.4. The inclusion of radiative corrections to the next electromagnetic order brings about (through its interference with the lowest-order term) the two-photon channel for which most of the general considerations valid for the one-photon channel (such as, for instance, angular momentum, parity, and charge conjugation rules) do not apply, at least in the same form. However, for experiments which do not distinguish between a final state and its charge conjugate (such as a total cross-section measurement, or any measurement that treats symmetrically the produced charged particles) such an interference term with the two-photon channel vanishes. Radiative corrections for such experiments are

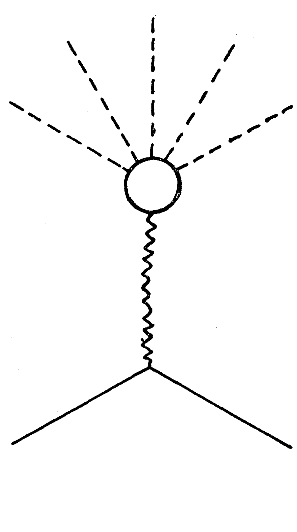

FIG. 2. Graph for the n-pion production reaction.

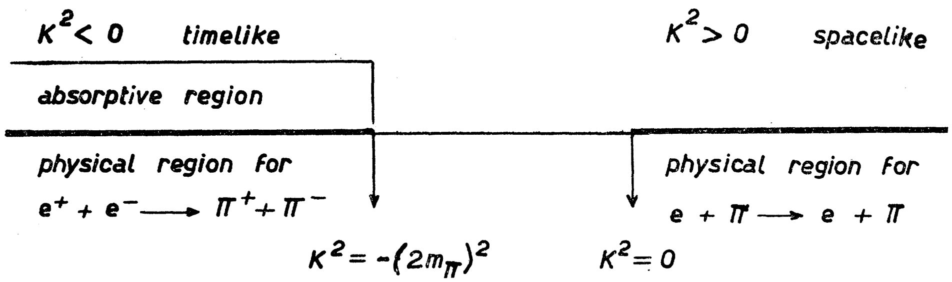

FIG. 3. The real k^2 axis for the pion form factor.

obtained by multiplying the expressions for the cross sections by an energy-dependent factor $\rho(E)$, and of course, by interpreting the matrix element J_ν as including the second-order radiative corrections.[5] The factor $\rho(E)$ is given by $1+\delta_{\text{s.e.}}+\delta_v+\delta_b$, where $\delta_{\text{s.e.}}$ is the percentage correction due to the photon self-energy graph, δ_v is the correction due to the vertex graph for the incoming electron and positron, and δ_b is the bremsstrahlung correction. The expressions for $\delta_{\text{s.e.}}$, δ_v, and δ_b can be found, for instance, in reference 5. These corrections take into account emission of soft photons. Before comparing with experiment one must, however, also add a correction for emission of hard photons under particular kinematical conditions that make them undetectable with the experimental apparatus employed (for an example see reference 6).

2. ANNIHILATION INTO PIONS AND K MESONS

2.1. Pion production in e^+-e^- collisions has already been discussed.[4,6] We shall here reproduce the main results and add some remarks. We consider the reaction

$$e^+ + e^- \longrightarrow n \text{ pions}, \tag{13}$$

occurring through a graph shown in Fig. 2. The relevant vertex is a γ-(n pions) vertex for a virtual γ of mass $k^2=-4E^2$. Such vertices are important for the theory of the nucleon structure.[7,8] For n even, they contribute to the isotopic vector part of the nucleon structure; for n odd, to the isotopic scalar part.

We consider reaction (13) in its center-of-mass frame. The final n-pion state produced by the virtual γ, according to the graph of Fig. 2, must have parity -1, charge conjugation quantum number -1, total angular momentum 1, and total isotopic spin 1 for n even, 0 for n odd. In particular it follows that reaction (13) cannot occur at the lowest electromagnetic order if all final pions are neutral. The space-like part of J_ν in the center-of-mass system, $\mathbf{J}$, must be formed out of the final pion momenta and must have the character of a polar vector for n even and of an axial vector for n odd. For two final pions $\mathbf{J}$ will thus be proportional to the final relative momentum; for three final pions $\mathbf{J}$ will be proportional to the only available axial vector, namely, the normal to the production plane. Inserting (9) and (8) into (7), one finds uniquely the form of the dependence of the cross section on the angle between $\mathbf{J}$ and the initial electron-positron relative momentum:

$$T_{mn} \sum_{a,b,-c} R_{mn} = \tfrac{1}{2}|\mathbf{J}|^2 \sin^2\theta, \tag{14}$$

where θ is the angle between $\mathbf{J}$ and $\mathbf{i}$, the unit vector along the initial positron momentum. Therefore, for two pions the angular distribution is $\sim\sin^2\theta$; for three pions the angle between the normal to the production plane and the initial line of collision is also distributed $\sim\sin^2\theta$.

2.2. The simplest pion production process is

$$e^+ + e^- \longrightarrow \pi^+ + \pi^-. \tag{15}$$

The matrix element of the current J_ν is written as

$$J_\nu = e(4\omega_+\omega_-)^{-\frac{1}{2}} F(k^2)(p_\nu^{(+)} - p_\nu^{(-)}), \tag{16}$$

where ω_+ and ω_- are the pion energies, and $p^{(+)}$ and $p^{(-)}$ are the pion momenta. The form factor $F(k^2)$ is taken at $k^2=-4E^2$. The cross section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\alpha^2 \frac{1}{E^2}\beta^3 |F(k^2)|^2 \sin^2\theta. \tag{17}$$

The dependence $\beta^3 \sin^2\theta$ is a direct consequence of angular momentum conservation that requires that the two final pions be produced in a p state and of our approximation of neglecting the electron mass. The total cross section is given by

$$\sigma_{\text{total}} = \frac{1}{m^2}(0.53\times 10^{-32}\ \text{cm}^2)\, b(x)\, |F(-4E^2)|^2, \tag{18}$$

where m (expressed in Bev) is the pion mass (or in general the mass of the produced boson) and

$$b(x) = (1/x^2)(1-1/x^2)^{\frac{3}{2}}, \tag{19}$$

with $x=E/m$.

To predict the absolute values of the cross section at

[5] G. Putzolu, Nuovo cimento **20**, 542 (1961).

[6] The same results as those of reference 4 have also been given by Yung Su Tsai, Phys. Rev. **120**, 269 (1960), and Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 771.

[7] G. F. Chew, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 775.

[8] Chew, Karplus, Gasiorowicz, and Zachariasen, Phys. Rev. **110**, 265 (1958), Federbush, Goldberger, and Treiman, *ibid.* **112**, 643 (1958).

the different energies, one should know the values of $|F(k^2)|$ for $k^2<-4m_\pi^2$. These values of k^2 lie in the absorption cut on the k^2 plane. In the graph of Fig. 3 we indicate the real axis of k^2 with a specification of the different regions.

The form factor at $k^2=0$ takes the value 1. The physical region for space-like k^2 can in principle be explored by pion-electron scattering. In such experiments k has the character of a momentum transfer. The physical region for negative k^2 can be explored with pair production in electron-positron collisions. The absorptive region starts at $k^2=-(2m_\pi)^2$. The lowest-mass intermediate state contributing in the $\gamma-2\pi$ vertex is the 2π state itself. The next state consists of four pions and its contribution to the absorptive part starts at $k^2=-(4m_\pi)^2$. An interpretation of $e^++e^-\rightarrow\pi^++\pi^-$ in the vicinity of its threshold can reasonably be given in terms of the two-pion intermediate states only, and therefore directly in terms of pion-pion scattering. This situation is indeed a very fortunate one and does not occur in other cases of pair production in e^+-e^- collisions. A pion-pion resonance with $T=1$, $J=1$ has been proposed by Frazer and Fulco[9] as a simple way of explaining the isotopic vector part of the nucleon structure. The form factor proposed by Frazer and Fulco can be approximated near the resonance by a resonant shape of the form

$$|F(k^2)|^2=[\beta^2+(k_0{}^2)^2]/[\beta^2+(k^2-k_0{}^2)^2],$$

where $\beta\cong2.65m_\pi{}^2$ and $k_0{}^2\cong10.4m_\pi{}^2$. At an energy $E=230$ Mev (total center-of-mass energy $2E=460$ Mev), near the maximum of the form factor, one finds a total cross section of 8.35×10^{-31} cm^2 for $e^++e^-\rightarrow\pi^++\pi^-$. Bowcock, Cottingham, and Lurié[10] suggest a resonance with the same quantum numbers but with rather different parameters. They propose a resonant shape

$$F_\pi(t)=\frac{t_r+\gamma}{t_r-t-i\gamma(t/4-1)^{\frac{3}{2}}},$$

with $t_r=22.4m_\pi{}^2$ and $\gamma=0.4m_\pi{}^{-1}$.

At an energy $E\approx330$ Mev (total center-of-mass energy ~660 Mev) near the maximum of the form factor the total cross section for $e^++e^-\rightarrow\pi^++\pi^-$ reaches a value of 6.6×10^{-31} cm^2.

These cross sections are much higher than the cross section calculated for $|F|=1$ (a factor ~17 in the Frazer-Fulco case and ~33 according to Bowcock, Cottingham, and Lurié).

Interpretations of the proposed $T=1$, $J=1$ resonance in terms of an unstable meson with $J=1$, $T=1$, and negative parity, which decays rapidly into $\pi^++\pi^-$ have been proposed.[11] The neutral meson of such a triplet has charge conjugation number $C=-1$. Electron-positron collisions offer a good way for detecting systematically neutral mesonic resonant states with $J=1$, $C=-1$, and negative parity. The $T=1$ resonant state discussed here belongs to such a class of states.

The netural production process

$$e^++e^-\rightarrow\pi^0+\pi^0,$$

does not occur at the lowest electromagnetic order (it requires the exchange of at least two photons).

2.3. The three-pion production process

$$e^++e^-\rightarrow\pi^++\pi^-+\pi^0, \tag{18}$$

can occur by the lowest-order graph (Fig. 2). If we call l the relative $\pi^+\pi^-$ angular momentum and L the angular momentum of π^0 relative to the $\pi^+\pi^-$ center of mass, we find that only the states $l=L=1$, $l=L=3$, $l=L=5$, etc., can be produced at the lowest electromagnetic order. This follows directly from parity, charge conjugation, and angular momentum conservation.

The matrix element J_ν of the current operator for three pions can be written[8]

$$J_\nu=-i(8\omega_+\omega_-\omega_0)^{-\frac{1}{2}}H^*(E,\omega_+,\omega_-)e^{\nu\rho\sigma\tau}p_\rho{}^{(+)}p_\sigma{}^{(-)}p_\tau{}^{(0)}. \tag{19}$$

The form factor H^* depends on three independent scalars that we have chosen as E, $\omega_+=$ energy of π^+, and $\omega_-=$ energy of π^-, all in the center-of-mass frame. The final momenta of π^+, π^-, and π^0 are $p^{(+)}$, $p^{(-)}$, and $p^{(0)}$. The differential cross section can then be written

$$\frac{d^2\sigma}{d\omega_+d\omega_-d(\cos\theta)}=\frac{\alpha}{(2\pi)^2}\frac{1}{64E^2}|H|^2\sin^2\theta(\mathbf{p}^{(+)}\times\mathbf{p}^{(-)})^2. \tag{20}$$

Here θ, as already explained, is the angle between the initial line of collision and the normal to the production plane. The absolute value of the cross section depends entirely on the form factor $|H|^2$. Knowledge of this form factor is very important for the theory of the isotopic scalar part of the nucleon structure.[8]

At present there is not much information available on $|H|^2$. There have been proposals for the existence of a three-pion resonance or bound state with $T=0$, $J=1$.[12] If one assumes the $T=1$, $J=1$ two-pion resonance, a three-pion state with $T=0$, $J=1$ may be formed in which all pairs of pions interact in the resonant state. The possibility of such a saturated structure might lead to the existence of a bound three-pion state with $T=0$, $J=1$. It has in fact been proposed to identify such a bound state with a possible resonant behavior observed by Abashian, Booth, and Crowe.[13] In this case its mass would be as low as 2.2 pion masses. A preliminary fit to the scalar part of the

[9] W. R. Frazer and J. R. Fulco, Phys. Rev. **117**, 1609 (1960).

[10] T. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters **5**, 386 (1960).

[11] J. J. Sakurai, Ann. Phys. **11**, 1 (1960); A. Salam, Revs. Modern Phys. **33**, 426 (1961); A. Salam and J. G. Ward, Nuovo cimento **19**, 167 (1961); M. Gell-Mann (to be published).

[12] G. F. Chew, Phys. Rev. Letters **4**, 142 (1960).

[13] A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters **5**, 258 (1960).

nucleon structure, according to the latest data,[14] has been attempted by Bergia *et al.* assuming the existence of such a three-pion bound state.[15] The existence of such a bound state may strongly influence the behavior of $|H|^2$ near the production threshold. Or, if a three-pion resonance exists in the physical region, it would be directly exhibited in the cross section for (18). A three-pion bound state with $T=0$, $J=1$ would decay mostly into $\pi^0+\gamma$ and $2\pi+\gamma$. It would lead to spectacular peaks in $e^++e^-\to\pi^0+\gamma$, and $e^++e^-\to 2\pi+\gamma$. This will be examined in more detail in the next sections. It is difficult to calculate its rate of decay. Its dominant modes of decay would involve the emission of a photon and the resulting lifetime may be relatively long as compared to typical nuclear times. A lifetime of the order of 10^{-21} sec would correspond to width of the order of a fraction of a Mev. The form factor H in (19) could then be approximated near threshold as

$$H=(\text{constant})\times[1/(k^2+M^2)],$$

where M is the mass of the bound state, whose contribution is assumed to dominate the behavior of H near the threshold.

Gauge theories of elementary particles[11] also lead to the prediction of a $J=1$, $T=0$ meson with negative parity and negative charge conjugation number. We have already mentioned that e^+-e^- collisions provide a systematical way of searching for resonant states with $J=1$, $C=-1$, and negative parity.

2.1. Production of four pions, five pions, etc., will become important at high k^2, as strongly suggested by the high pion multiplicity in nucleon-antinucleon annihilation. The direction of the current matrix element **J** cannot be specified in terms of the final pion momenta from parity considerations alone, as it could for production of two or three pions. Gauge invariance alone does therefore not lead to any simple geometrical consequence. The presence of two-pion and three-pion resonances will strongly affect the final state and could suggest models for a simplified treatment. Methods used in the analysis of the nucleon-antinucleon annihilation into pions[16] can be applied to the present problem, with the substantial simplification of the complete knowledge of the initial quantum numbers. In particular, assuming the dominance of the $T=1$, $J=1$ pion-pion resonance, there is the possibility of a "saturated" $T=0$, $J=1$ three-pion state of negative parity and charge conjugation number, with all pairs coupled by the resonant interaction.[12] No such "saturated" states can be formed with more than three pions. In fact, already with four pions, two pions must have the same charge and therefore their relative angular momentum must be even.

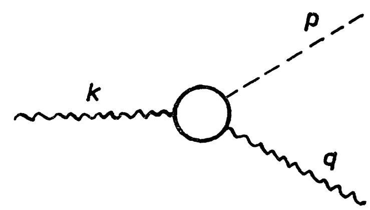

FIG. 4. Vertex for production of $\pi^0+\gamma$. The symbols are defined in the text.

2.5. Production of $K\bar{K}$ pairs can occur according to

$$e^++e^-\to K^++K^-,$$
$$e^++e^-\to K^0+\bar{K}^0,$$
$$e^++e^-\to K+\bar{K}+\pi,\quad \text{etc.}$$

Expression (17) applies for production of a $K-\bar{K}$ pair with $F(k^2)$ interpreted as the relevant K^+, or K^0, form factor. The charged K form factor is the sum of an isotopic vector form factor and an isotopic scalar form factor; the neutral K form factor is the difference of the isotopic vector and isotopic scalar form factors. Two-pion intermediate states and $K-\bar{K}$ states of isotopic spin one are among the contributors to the vector part, while three-pion states and zero isotopic spin $K-\bar{K}$ states are among the contributors to the scalar part. Presumably, $K-\bar{K}$ scattering will play a relevant role for production near threshold and the experiment will give information on its properties. Similarly $e^++e^-\to K+\bar{K}+\pi$ could give information on $K-\pi$ and $\bar{K}-\pi$ interactions.

In the $K^0-\bar{K}^0$ processes there is a very simple consequence of charge conjugation invariance that should be pointed out. The final $K^0\bar{K}^0$ pair must be produced in a state of $C=-1$. Therefore, the final amplitude is of the form

$$K^0\bar{K}^0-\bar{K}^0K^0.$$

In terms of the physical particles

$$K_1^0=(1/\sqrt{2})(K^0+\bar{K}^0),\quad K_2^0=(1/\sqrt{2})(K^0-\bar{K}^0),$$

the final amplitude can be written as

$$K_1^0K_2^0-K_2^0K_1^0.$$

It is now evident that only $K_1^0-K_2^0$ pairs can be produced (but not $K_1^0-K_1^0$ or $K_2^0-K_2^0$ pairs). This means that one particle must decay as a K_1^0 and the other as a K_2^0. It also follows that for a given configuration at production $K_1^0-K_2^0$ pairs are produced with the same probability as $K_2^0-K_1^0$ pairs. Note that the final amplitude maintains its form at any time in absence of interactions. The time development in fact is just given by

$$K_1^0\to K_1^0e^{-(\lambda_1+im_1)t},\quad K_2^0\to K_2^0e^{-(\lambda_2+im_2)t},$$

if K_1^0 and K_2^0 propagate through vacuum. Analogous conclusions apply if additional π^0's are produced

[14] D. N. Olson, H. F. Schopper, and R. R. Wilson, Phys. Rev. Letters **6**, 286 (1961); R. Hofstadter, C. De Vries, and R. Herman, *ibid.* **6**, 290 (1961); R. Hofstadter and R. Herman, *ibid.* **6**, 293 (1961).

[15] S. Bergia, A. Stanghellini, S. Fubini, and C. Villi, Phys. Rev. Letters **6**, 367 (1961).

[16] A. Pais, Ann. Phys. **9**, 548 (1960).

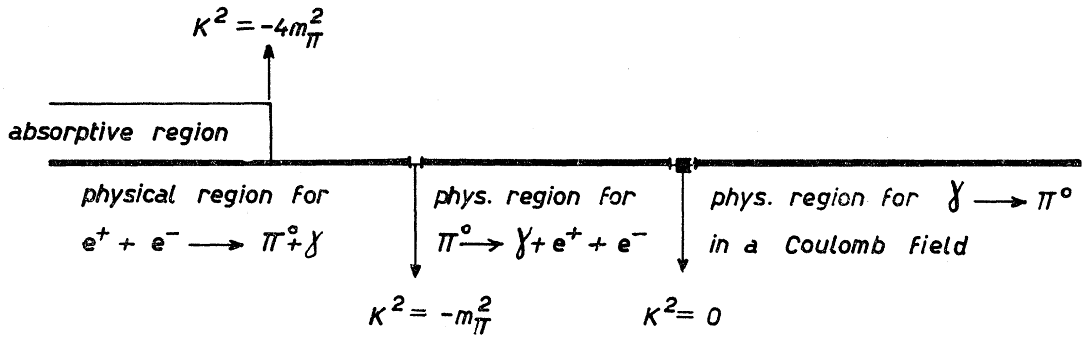

FIG. 5. The real k^2 axis for the π^0 form factor.

together with the $K^0\bar{K}^0$ pair. If, however, $\pi^+\pi^-$ pairs are also produced the $K^0\bar{K}^0$ pair can be produced in a $C=+1$ combination and the correlation would be different.

3. ANNIHILATION INTO $\pi^0+\gamma$

3.1. The process

$$e^+ + e^- \rightarrow \pi^0 + \gamma \tag{21}$$

is very interesting from a theoretical point of view as it is directly related to the properties of the vertex shown in Fig. 4. Here k and q are the photon momenta and p is the π^0 momentum. In the electron-positron annihilation process (21), the photon momentum k is off the mass shall, corresponding to a virtual photon mass of $(-k^2)^{\frac{1}{2}}=2E$. For $k^2=0$, the vertex describes π^0 decay into two photons and can be computed in terms of the π^0 lifetime. There exist various experimental possibilities for exploring the above vertex for the different ranges of values of k^2, as illustrated in Fig. 5. In the figure we have exhibited the real k^2 axis and indicated the various physical regions. We have also indicated the absorptive region, whose threshold starts at two pion masses.

The decay of a free π^0 into two photons occurs at $k^2=0$. The region at the right of $k^2=0$ can in principle be explored through the so called Primakoff effect.[17] In the Primakoff effect an incident real photon produces a π^0 through the interaction with the Coulomb field of a nucleus. The vertex of Fig. 4 can be held responsible for such a process with q taken as the incident photon momentum and k as the virtual photon momentum. The Dalitz process $\pi^0 \rightarrow \gamma + e^+ + e^-$ may be used to investigate the $(\pi^0\gamma\gamma)$ vertex for small negative values of k^2.[18]

The physical region for (21) starts at $k^2=-m_\pi^2$. The absorptive region starts only at $k^2=-4m_\pi^2$, with the possibility of two-pion intermediate states. The $T=1$, $J=1$ two-pion resonance would produce a strong resonant-like behavior of the vertex in the physical region for (21). The next absorptive threshold due to three-pion continuum starts at $k^2=-9m_\pi^2$. If there is a bound $T=0$, $J=1$, 3π state at some $k^2>-9m_\pi^2$, which decays only through electromagnetic interaction, its presence would lead to a pole contribution to the vertex at the relevant value of $k^2=-M^2$, where M is the mass of the bound state. Of course, the pole would occur strictly at a complex value of k^2, due to the finite lifetime of the bound state. The associated width is, however, presumably only a fraction of a Mev, and the description by a pole may be safely applied except for the immediate vicinity of the resonance peak. The contribution from the peak to the cross section may turn out to be effectively very big, possibly of the order of $10^{-29}-10^{-30}$ cm^2. This can be seen from simple considerations based on a Breit-Wigner description of the resonance. The $T=0$, $J=1$ three-pion bound state would presumably decay into $\pi^0+\gamma$, or $2\pi+\gamma$, with a lifetime $\sim 10^{-20}$ sec. The corresponding width Γ is then between 0.06 Mev and 0.6 Mev. The experimental energy resolution is given by $2\Delta E$, where ΔE is the energy resolution for each colliding beam (positrons or electrons). If the energy spread of the incident beams is larger than a few Mev, as it will presumably be, the measured quantity will be the integral of the cross section in a region comprising the peak. We therefore estimate the average cross section as

$$\bar{\sigma}=\frac{1}{2\Delta E}\int_{2E=M-\Delta E}^{2E=M+\Delta E}\sigma d(2E)=\frac{1}{\Delta E}\int_{E=\frac{1}{2}M-\frac{1}{2}\Delta E}^{E=\frac{1}{2}M+\frac{1}{2}\Delta E}\sigma dE. \tag{22}$$

We use a Breit-Wigner formula for the cross section near the resonance. For production through a $J=1$ resonant state of mass M, total disintegration rate Γ and partial rates Γ_i and Γ_f for decay, respectively, into the initial e^++e^- channel and final $\pi^0+\gamma$ (or $2\pi+\gamma$) channel, we have

$$\sigma=\tfrac{3}{4}\pi\lambda\!\!\!^{-2}\Gamma_i\Gamma_f/[(2E-M)^2+\Gamma^2/4]. \tag{23}$$

The contribution to $\bar{\sigma}$ from the peak is then given by

$$\bar{\sigma}=\tfrac{3}{2}\pi^2\lambda\!\!\!^{-2}B_iB_f(\Gamma/2\Delta E), \tag{24}$$

[17] H. Primakoff, Phys. Rev. **81**, 899 (1951); C. Chiuderi and G. Morpurgo, Nuovo cimento **19**, 497 (1961); V. Glaser and R. A. Ferrell, Phys. Rev. **121**, 886 (1961); S. Berman (to be published).

[18] S. M. Berman and D. A. Geffen, Nuovo cimento **18**, 1192 (1960); How Sen Wong, Phys. Rev. **121**, 289 (1961).

where B_i and B_f are the branching ratios Γ_i/Γ and Γ_f/Γ, respectively. In performing the integration we have assumed $\Gamma \ll 2\Delta E$ but the result can be applied for an estimate if $\Gamma \lesssim 2\Delta E$. Let us assume $M \lesssim 3m_\pi$ and a branching ratio into $e^+ + e^-$, B_i, of the order of 10^{-3}. One obtains $\bar{\sigma} = 1.3\times 10^{-28}(\Gamma/2\Delta E)$ cm^2, or, with $\Gamma \sim 10^{20}$ sec^{-1}, $\bar{\sigma} = 0.8\times 10^{-29}(2\Delta E$ in Mev$)^{-1}$ cm^2.

3.2. In the following we shall discuss in detail the process $e^+ + e^- \to \pi^0 + \gamma$ assuming the dominance of the resonant 2π state and of the 3π bound state. From what we have seen in the previous paragraphs, if the 3π $T=0$, $J=1$ bound state exists, its contribution is likely to be very important.

Presumably also a $T=0$, $J=1$ three-pion resonance (as opposed to a bound state) would have a very important effect in $e^+ + e^- \to \pi^0 + \gamma$. A limitation of the theory to the $T=1$, $J=1$ 2π resonance[19] would be rather artificial also in view of the remark by Chew[12] that a $T=1$, $J=1$ 2π resonance could generate a $T=0$, $J=1$ 3π resonance or bound state. The experimental result by Samios[20] on internal conversion of gamma rays in π^0 decay indicates a negative value for the derivative of the π^0 form factor at the origin. This result would be difficult to understand if only the 2π resonant state is kept in the calculations. A possible explanation could be to include contributions from the intermediate nucleon-antinucleon pairs. A theory of π^0 decay based on keeping only contributions from nucleon-antinucleon pairs has been published by Goldberger and Treiman.[21] The negative coefficient of the Samios experiment could be possibly understood through the coherent contribution of the 2π resonant state and of the nucleon-antinucleon states. Such a point of view has been proposed by Berman and Geffen in their discussion of internally converted pairs.[18] The point of view that we follow here by including two- and three-pion states is closer to that of Wong[18] in his discussion of π^0 decay.

The general form for the $\pi^0\gamma\gamma$ vertex as determined from invariance requirements is

$$\tfrac{1}{4}G(-k^2,-q^2,-p^2)\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}(q)F_{\lambda\rho}(k), \tag{25}$$

where k and q are the four-momenta associated to the photon lines, $F_{\mu\nu}(q)$ and $F_{\lambda\rho}(k)$ are Fourier components the electromagnetic tensor, p is the π^0 four-momentum, $\epsilon_{\mu\nu\lambda\rho}$ is the isotropic antisymmetric tensor, and G is a form factor. In our case one photon and the π^0 are on the mass shell. We shall denote by $G(-k^2)$ the value of the form factor for $-p^2=m_\pi^2$ and $q^2=0$. The π^0 lifetime, τ, depends on $G(0)$ according to the relation

$$1/\tau = (m_\pi^3/64\pi)\,|G(0)|^2. \tag{26}$$

From (7), (8), and (9) we can derive an expression for the cross section for $e^+ + e^- \to \pi^0 + \gamma$. The matrix element of the current is given in the center-of-mass system for the reaction by

$$\mathbf{J} = -[E/(\omega|\mathbf{p}|)^{\frac{1}{2}}]G(-k^2)\boldsymbol{\varepsilon}\times\mathbf{p},$$

where ω is the π^0 energy, $\mathbf{p}$ its momentum, and $\boldsymbol{\varepsilon}$ the polarization vector of the emitted γ ray. The tensor R_{mn} is then given by

$$(E^2/\omega|\mathbf{p}|)\,|G(-k^2)|^2(p_np_m - p^2\delta_{mn}),$$

and the differential cross section is given by

$$d\sigma = (\alpha/64)\beta^3|G(-k^2)|^2(1+\cos^2\theta)d(\cos\theta).$$

It will be convenient to express the cross section in terms of the π^0 lifetime through (26):

$$d\sigma = \frac{\pi\alpha}{m_\pi^3}\frac{1}{\tau}\beta^3(1+\cos^2\theta)\left|\frac{G(-k^2)}{G(0)}\right|^2 d(\cos\theta). \tag{27}$$

The total cross section is given by

$$\sigma = \frac{8\pi}{3}\frac{\alpha}{m_\pi^3}\frac{1}{\tau}\beta^3\left|\frac{G(-k^2)}{G(0)}\right|^2$$

$$\cong 2.75\times 10^{-35}\ \text{cm}^2\left|\frac{G(-k^2)}{G(0)}\right|^2(1-x^{-2})^3, \tag{28}$$

where $x=2E/m_\pi$, and the numerical factor, inversely proportional to the π^0 lifetime, has been calculated for a lifetime of 2.2×10^{-16} sec.

The form factor $G(k^2)$ is assumed to satisfy a dispersion relation of the form

$$G(-k^2) = \frac{1}{\pi}\int_4^\infty \frac{\mathrm{Im}G(t)dt}{t+k^2-i\epsilon} \tag{29}$$

(we are using $m_\pi=1$). The imaginary part has contributions from the absorptive region as indicated in Fig. 5. If a three-pion bound state is present an additional pole contribution should be added to (29). We first evaluate the contribution to the dispersion integral from the $T=1$, $J=1$ two-pion intermediate state, assuming a resonant pion-pion amplitude as proposed by Frazer and Fulco[9] and by Bowcock, Cottingham, and Lurié.[10] The contribution from the resonant $T=1$, $J=1$ two-pion intermediate state can be expressed, following Wong,[18] as

$$\frac{e}{48\pi^2}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}}{t^{\frac{1}{2}}(t+k^2-i\epsilon)}F_\pi^*(t)M_1(t)dt, \tag{30}$$

where $F_\pi(t)$ is the pion electromagnetic form-factor and $M_1(t)$ is the amplitude for pion photoproduction on pions $(\gamma+\pi\to\pi+\pi)$ in p wave. This amplitude has been studied by Wong[22] who has shown that under the assumption of a resonant $T=1$, $J=1$ π-π interaction,

$$M_1(t) = \Lambda(1+a)D_1(1)/(t+a)D_1(t),$$

[19] G. Furlan, Nuovo cimento **19**, 840 (1961).
[20] N. P. Samios, Phys. Rev. **121**, 265 (1961).
[21] M. L. Goldberger and S. Treiman, Nuovo cimento **9**, 451 (1958).
[22] H. Wong, Phys. Rev. Letters **5**, 70 (1960).

where $D_1(t)$ is the denominator function, as defined by Chew,[23] a is a positive constant, and Λ is an unknown parameter. In this scheme also $F_\pi(t)$ is related to $D(t)$ by

$$F_\pi(t)=D_1(0)/D_1(t).$$

One sees that $F_\pi^*(t)M_1(t)$ is essentially proportional to the square of the absolute value of the pion form factor. The two-pion contribution $G_2(-k^2)$ is therefore proportional to the integral

$$\frac{1}{\pi}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}|F_\pi(t)|^2dt}{t^{\frac{1}{2}}(t+a)(t+k^2-i\epsilon)}. \tag{31}$$

We shall evaluate the above integral by assuming for $|F_\pi(t)|^2$ the form proposed by Bowcock, Cottingham, and Lurié[10]

$$F_\pi(t)=(t_2+\gamma)/[t_2-t-i\gamma(t/4-1)^{\frac{3}{2}}], \tag{32}$$

where t_2 and γ are the parameters that characterize the resonance. The form factor $F_\pi(t)$ satisfies the nonsubstracted dispersion relation

$$F_\pi(-k^2)=\frac{1}{\pi}\int_4^\infty \frac{\mathrm{Im}F_\pi(t)dt}{t+k^2-i\epsilon}. \tag{33}$$

Normally a subtracted dispersion relation is used; however, with the form (32) for $F_\pi(t)$ also a nonsubtracted relation like (33) is convergent. The absorptive term in (33) as derived from (32) can be written in the form

$$\mathrm{Im}F_\pi(t)=\frac{\gamma}{4(t_2+\gamma)}(t-4)^{\frac{3}{2}}|F_\pi(t)|^2\theta(t-4). \tag{34}$$

The comparison with (33) and (34) will permit a direct evaluation of the integral (31). The factor $|F_\pi(t)|^2$ in the integral (31) represents a very sharp resonance. It will therefore be possible to neglect the energy variation of the slowly varying terms in the denominator of (31) and obtain

$$G_2(-k^2)\propto\frac{1}{\pi}\int_4^\infty \frac{(t-4)^{\frac{3}{2}}|F_\pi(t)|^2dt}{t+k^2-i\epsilon}.$$

By comparing with (33) and (34) we then find directly

$$G_2(-k^2)=c_2/[t_2+k^2-i\gamma(-\tfrac{1}{4}k^2-1)^{\frac{3}{2}}], \tag{35}$$

where c_2 is a constant.

We shall now approximate $G(-k^2)$ for not very large values of $|k^2|$ as

$$G(-k^2)=\frac{c_2}{t_2+k^2-i\gamma(-\frac{1}{4}k^2-1)^{\frac{3}{2}}}+\frac{c_3}{t_3+k^2-i\Gamma}. \tag{36}$$

The first term in the right-hand side of (36) represents the contribution from the 2π resonant state. The second term represents the "pole" contribution from the proposed 3π bound state. The mass M of the bound state is given by $t_3^{\frac{1}{2}}$ and Γ is its total decay rate, corresponding to a lifetime presumably of the order of 10^{-20} sec. The imaginary part $-i\Gamma$ is very small, producing a very narrow peak in $G(-k^2)$ in the neighborhood of $k^2=-t_3$. The constant c_3 is the residuum of the pole corresponding to the 3π bound state. We shall not try any theoretical determination of c_2 and c_3 but rather try to evaluate them on the basis of experimental information. $G(0)$ is related to the π^0 lifetime through (26). This gives approximately

$$|c_2/t_2+c_3/t_3|^2=(64\pi/m_\pi^3)\tau^{-1}. \tag{37}$$

A second piece of information is given by the Samios experiment on internally converted electron pairs from π^0 decay.[20] The experiment gives some information on the derivative of $G(-k^2)$ at $k^2=0$. With the definition

$$-\frac{1}{G(0)}\left(\frac{\partial G(-k^2)}{\partial k^2}\right)_0=\alpha m_\pi^2, \tag{38}$$

Samios finds $\alpha=-0.24\pm0.16$. From (36) and (38) one finds directly $(m_\pi=1)$

$$\frac{c_3}{c_2}=-\left(\frac{t_3}{t_2}\right)^2\frac{1-\alpha t_2}{1-\alpha t_3}. \tag{39}$$

We now use for t_2 the value suggested by Bowcock, Cottingham, and Lurié, $t_2=22.4$, and for t_3 the value suggested from the identification of the bound state with the resonance observed by Abashian, Booth, and Crowe,[13] namely $t_3=5$. From (39) and from the experimental value of α there follows a value for c_3/c_2 between -0.22 and -0.13. Expressed in terms of τ and α, the π^0 form factor becomes

$$|G(-k^2)|^2=\frac{64\pi}{m_\pi^3}\tau^{-1}\frac{1}{(t_2-t_3)^2}\left|\frac{t_2^2(1-\alpha t_3)}{t_2+k^2-i\gamma(-\frac{1}{4}k^2-1)^{\frac{3}{2}}}-\frac{t_3^2(1-\alpha t_2)}{t_3+k^2-i\Gamma}\right|^2. \tag{40}$$

The form factor (40) is composed by two resonant terms. The first term due to the resonant $T=1$, $J=1$ π-π interactions reaches its maximum around $k^2=-t_2$ and has a width given by $\gamma(-\frac{1}{4}t_2-1)^{\frac{3}{2}}t_2^{-\frac{1}{2}}$. With the Bowcock, Cottingham, and Lurié values for the pion-pion form factor, $\gamma\cong0.4m_\pi^{-1}$, and the resonance width is then about $0.8m_\pi$. The second term, due to the proposed 3π bound state, reaches its maximum at $k^2=-t_3=-(\text{mass of the bound state})^2$. Its width, presumably determined by the decay rate of the bound state into $\pi^0+\gamma$, is expected to be only a fraction of a Mev. According to (40) with the values $t_2=22.4$ and $t_3=5$, the contribution from the 3π resonant term is negligible for values of k^2 about -22.4, in comparison with the 2π resonant term that reaches its maximum

[23] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

in that region. The enhancement factor we find at $k^2=-t_2$ is

$$|G(t_2)/G(0)|^2 \cong 250. \tag{41}$$

The reason for the enormous enhancement can be traced back in the required compensation of the two resonant contributions near $k^2=0$ to produce a small negative derivative. The enhancement factor (41) multiplies the perturbation-theory (i.e., constant form factor) cross section of 2.75×10^{-35} cm$^2\times(1-t_2^{-1})^3\cong2.4\times10^{-35}$ cm^2. The resulting cross section may thus become observable in the region of the 2π resonance maximum.

The resonance peak due to the 3π resonance is very narrow and only an average cross section, integrating the contributions from the peak, will be measurable. On the basis of the expression (40) for the form factor we can again estimate the value of the average cross section $\bar{\sigma}$, as defined by (22). Near the 3π resonance the cross section can be approximated as

$$\sigma=2.75\times10^{-35}\text{ cm}^2(1-t_3^{-1})^3\left|\frac{t_3-\alpha t_2t_3}{t_3-t_2}\right|^2\left|\frac{t_3}{t_3+k^2+i\Gamma}\right|^2.$$

The resulting average $\bar{\sigma}$ is

$$\bar{\sigma}\cong3.5\times10^{-29}\left(\frac{t_3-\alpha t_2t_3}{t_3-t_2}\right)^2\frac{1}{(2\Delta E)}\text{ cm}^2,$$

with ΔE expressed in Mev. With the proposed values for the parameters, $\bar{\sigma}$ can become $\cong10^{-28}/(2\Delta E)$ cm^2, a value about a factor of 10 higher than what we obtained before on the basis of assumptions on the decay rates. The considerations that we have developed strictly apply to the explicit case of a 3π bound state. In the case of a 3π resonance there might occur an important change in the conclusion if the decay rate of the resonant state into three pions is strong enough. As no selection rules would be expected to prevent such a decay mode, its rate is expected to be rather big and might become much bigger than the rate for decay into $\pi^0+\gamma$ as soon as the energy release is large enough to overcome the effects of the smaller statistical weight and of the centrifugal barrier. In this case the resonance width would be much larger and a better dispersion theoretical approach would be required to obtain reliable estimates. Apart from the difficulties met in obtaining a precise estimate one can see that a very big cross section for $e^++e^-\to\pi^0+\gamma$ could be obtained if there is an intermediate bound state with spin one and charge conjugation number -1, which mainly decays into $\pi^0+\gamma$. We obtained the first indication for such a big cross section on the basis of a Breit-Wigner formula for the resonance with suitable choices of the relevant partial widths. The second indication is based on a theory for the π^0 form factor on the assumption that only the 2π resonant amplitude and the three-pion bound state contribute, with a relative weight determined in such a way as to give the value for the derivative at $k^2=0$ required by the distribution of internal converted electrons, in π^0 decay. Of course both approaches are rather tentative and subject to criticisms. For an energy resolution $\Delta E\sim5$ Mev one expects a cross section, on the resonance peak, of the order of $10^{-30}-10^{-29}$ cm^2, tremendously big if compared to the perturbation-theory (i.e., constant form factor) values, always smaller than 2.75×10^{-35} cm^2 for a π^0 lifetime of 2.2×10^{-16} sec. Verification of the existence or nonexistence of such big cross section should be a feasible though probably very delicate task.

3.3. In general the reaction (21) would be observed as an annihilation into three gammas of the initial electron-positron pair. A close examination of the competing electromagnetic process,

$$e^++e^-\to3\gamma, \tag{42}$$

will therefore be necessary. The process (42) occurs at the same order in the fine structure constant as the $\pi^0+\gamma$ process.

A relevant contribution to π^0 production in e^++e^- collisions will also come from a process first discussed by Low,

$$e^++e^-\to e^++e^-+\pi^0. \tag{43}$$

Low calculates the leading term of the cross section using a Weizsäcker-Williams method.[24] Such a leading term corresponds to a forward scattering pole in (43) and its value depends only on the value of the π^0 form factor at $k^2=0$. For $E=150$ Mev, Low finds a total cross section for (43) of about 10^{-33} cm^2 with a π^0 lifetime 10^{-18} sec. With the value for the lifetime that we have used above, 2.2×10^{-16} sec, the cross section would be of the order 10^{-35} cm^2. A recent re-evaluation by Chilton[25] has lead to essentially similar results. The cross section, as calculated from the pole term, increases linearly with energy already at $E\gtrsim m_\pi$ and, for $\tau\cong10^{-16}$ sec, can be approximated as $\sigma=2.2\times10^{-35}(E/m_\pi)$. At the same electromagnetic order of (43) a double bremsstrahlung process can occur, and the two emitted photons may simulate the photons from π^0 decay; Low suggests discrimination between the two processes by the different spread of the photon angular distributions.[24] However, a detailed calculation of the double bremsstrahlung process should be carried out for an accurate discrimination.

4. GENERAL DISCUSSION OF THE POSSIBLE RESONANCES

4.1. In the previous sections we have discussed the possibility of resonances due to the contribution of pion-pion real or virtual bound states. In particular, we have examined the role of the proposed $T=1$, $J=1$ pion-pion resonance and of the proposed $T=0$, $J=1$

[24] F. E. Low, Phys. Rev. **120**, 582 (1960).
[25] F. Chilton (to be published).

3π bound state in reactions such as $e^+ + e^- \to 2\pi$, or 3π, or $\pi^0+\gamma$. Both the π-π resonant state and the 3π bound state or resonant state that we have considered have angular momentum $J=1$, and charge conjugation number, $C=-1$. Electron-positron collisions offer indeed a very suitable means for exploring the properties of intermediate neutral states with $J=1$, $C=-1$, $P=-1$, zero nucleonic number, and zero strangeness. Such states can transform into a single virtual gamma and this is in fact what selects them among all the other states accessible only through the exchange of more virtual gammas. However other quantities, such as the experimental energy resolution ΔE, and the partial decay rates from the intermediate state, are relevant to the discussion, and a more detailed examination is necessary. In the following we shall illustrate our statement by employing a simplified description of the resonant reaction based on a Breit-Wigner formula. We consider a resonant channel of the type

$$e^+ + e^- \to B_J \to \text{(final state)}, \qquad (44)$$

where B_J represents an intermediate state of zero strangeness and nucleonic number, spin J, and mass M. In the vicinity of the resonance we assume that a Breit-Wigner description holds. The resonance cross section for (44) at a total energy $2E$ around M will then be approximated by

$$\sigma_R(E)=\pi\lambda\!\!\!^{-2}\frac{2J+1}{4}\frac{\Gamma_i\Gamma_f}{(2E-M)^2+\Gamma/4}, \qquad (45)$$

where Γ_i is the rate for $B_J \to e^+ + e^-$ and Γ_f the rate for $B_J \to$ (final state). The total rate is given by Γ. In any actual measurement the measured quantity is the integrated product of $\sigma(E)$ with the experimental resolution curve. For our purposes it will be enough to approximate the resolution curve with a rectangle of width $2\Delta E$. It will be necessary to distinguish among three cases: case (a): The resonance is very narrow, with a width much smaller than the experimental energy resolution, $\Gamma \ll 2\Delta E$; case (b): The resonance is wide, with a width much larger than the experimental energy resolution, $\Gamma \gg 2\Delta E$; and case (c): The resonance has a width comparable to the experimental energy resolution, $\Gamma \sim 2\Delta E$. The contribution to $e^+ + e^- \to$ (final state), from the resonance, in an experiment carried at an energy $2E=M$ with an energy spread given by $2\Delta E$ will be given by

$$\bar{\sigma}_R=\frac{1}{\Delta E}\int_{\frac{1}{2}(M-\Delta E)}^{\frac{1}{2}(M+\Delta E)}\sigma(E)dE.$$

This quantity is given in case (a) by

$$\bar{\sigma}_R=2\pi\lambda\!\!\!^{-2}(\pi/4)(2J+1)B_iB_f\Gamma/(2\Delta E), \qquad (46)$$

with $B_i=\Gamma_i/\Gamma$ and $B_f=\Gamma_f/\Gamma$. In case (b) it is simply given by

$$\sigma(2M)=\pi\lambda\!\!\!^{-2}(2J+1)B_iB_f. \qquad (47)$$

For the intermediate case (c) both (46) or (47) can be applied for order-of-magnitude estimates, since they only differ by a factor $\pi/2$ if $\Gamma \cong 2\Delta E$.

4.2. For purposes of comparison, one can consider a typical cross section, such as that for $e^+ + e^- \to \mu^+ + \mu^-$, which for $E \gg m_\mu$ is given by $\frac{1}{3}\pi\alpha^2\lambda\!\!\!^{-2}$. It will also be sufficient to limit the discussion to the most important final channels, for which B_f is of the order unity. The important quantity is then $\Gamma_i/\Delta E$ in case (a); Γ_i/Γ in case (b); and any of these two quantities in case (c). We can then examine what important factors will appear in Γ_i, the rate for the transition $B_J \to e^+ + e^-$. We make use of gauge invariance and of the charge conjugation selection rules. For $J=0$, $C=1$, Γ_i is proportional to $\alpha^4 m_e^2$, and for $J=0$, $C=-1$, it will be proportional to $\alpha^6 m_e^2$. The rates vanish for $m_e=0$ because the final electron and positron should be emitted in configurations with parallel spiralities thus violating angular momentum conservation. For $J=1$, $C=1$, Γ is proportional to α^4, but for $J=1$, $C=-1$ it is proportional to α^2. For $J=2$, $C=1$, Γ_i is proportional to α^4, for $J=2$, $C=-1$ it is proportional to α^6; and, similarly, higher powers of α appear when J is increased.

It is now important to state that the energy resolution ΔE will presumably not be smaller than ~ 1 Mev. An energy spread of this order corresponds to rates Γ of the order of 10^{21} sec^{-1}. Therefore, in case (a) only intermediate states with $J=1$, $C=-1$ will produce comparatively large effects. In fact, rather large effects will be expected if $\Gamma_i/\Delta E \gg \alpha^2$. Next in importance are states with $J=1$, $C=+1$, and $J=2$, $C=1$ with an additional factor α^2. In case (b) the relevant quantity is Γ_i/Γ, and Γ is supposed to be much bigger than $2\Delta E$. Therefore the same conclusions apply as for case (a), and, of course, they hold also in case (c). If B_J can decay through strong interactions, Γ is expected to be of the order of 10^{23}–10^{22} sec^{-1} and thus much bigger than $2\Delta E$. The $T=1$, $J=1$ pion-pion resonance belongs to this class of resonances, case (b). For the 3π bound state, which decays slowly into $\pi^0+\gamma$ or $2\pi+\gamma$, Γ is presumably of the order 10^{20} sec^{-1}, rather smaller than $2\Delta E$, case (a) or case (c). For a narrow energy resolution, the factor $\Gamma_i/\Delta E$ is expected to be big enough to give large resonance peaks. In general, the occurrence of case (a) or (c) requires some inhibition of a fast decay via strong interactions and would in fact correspond to a rather exceptional situation (such as a bound state of very low mass).

5. ANNIHILATION INTO BARYON PAIRS

5.1. We shall discuss in this section electron-positron annihilation in flight into a fermion-antifermion pair according to

$$e^+ + e^- \to f + \bar{f}. \qquad (48)$$

Pairs of strong interacting fermions can be produced according to

$$e^+ + e \to p + \bar{p},\ n + \bar{n},\quad e^+ + e^- \to \Sigma + \bar{\Sigma},\quad (49)$$
$$e^+ + e^- \to \Lambda + \bar{\Lambda},\qquad e^+ + e^- \to \Xi + \bar{\Xi},$$

all of the type (48). The final pair is produced in the states $^3S_1$ and $^3D_1$ as follows directly from angular momentum and parity considerations (charge conjugation does not add anything new to this case). The cross section near the threshold is thus expected to grow up proportional to the velocity of the final particles in the centers-of-mass system, and the threshold angular dependence is expected to be isotropic. The matrix element (3) of the electromagnetic current operator between the vacuum and the state containing the fermion-antifermion pair can be written in the form

$$J_\mu = e\bar{u}(p)[F_1(k^2)\gamma_\mu - (\mu/2m)F_2(k^2)\sigma_{\mu\nu}k_\nu]v(\bar{p}),\quad (50)$$

where p_ν and $\bar{p}_\nu$ are the four-momenta of the produced fermion and antifermion, respectively; $\bar{u}(p)$ and $v(\bar{p})$ are their Dirac spinors; and $F_1(k^2)$, $F_2(k^2)$ are the analytic continuations of the electric and magnetic form factors of the fermion for the values of k^2 relevant in (48), namely $k^2 < -4m^2$, where m is the mass of the produced fermion. In (50), μ is the static anomalous magnetic moment of the produced fermion. The form (50) for J_ν follows from Lorentz and gauge invariance. The form factors are normalized in such a way that $F_1(0)=1$ and $F_2(0)=1$ if the fermion is charged; and that $F_1(0)=0$ and $F_2(0)=1$ if it is neutral. The current matrix element J_ν can be decomposed, as usual, as the sum of an isotopic vector part and of an isotopic scalar part (for Λ and Σ^0 there is only the scalar part). This decomposition brings about a considerable simplification for the three processes leading to Σ-$\bar{\Sigma}$ production which are described in terms of four independent form factors. On the basis of the presently assumed mass spectrum, we expect for the isotopic vector form factors the absorptive cut in the k^2 plane to start at $k^2 = -4m_\pi^2$ and for the isotopic scalar form factors to start at $k^2 = -9m_\pi^2$, except for the possible presence of a 3π bound state, producing a pole contribution at a lower $|k^2|$. The above consideration does not hold for the Σ which can transform into an intermediate Λ by pion emission, giving rise to a lowering for the absorptive cut[26] (for instance, for the charged Σ's the isotopic vector amplitude has a threshold at

$$-4m_\pi^2\left[1-\left(\frac{m_\Sigma^2-(m_\Lambda^2+m_\pi^2)}{2m_\pi m_\Lambda}\right)^2\right]\Bigg).$$

5.2. The form factors are in general complex along the absorptive cut. In particular, they are complex for the physical values of k^2 in reaction (48), $k^2 < -4m^2$. Thus in $e^+ + e^- \to f + \bar{f}$ there can be a polarization of f normal to the production plane, already at the lowest electromagnetic order. The polarization will be proportional to the sine of the phase difference between the electric and the magnetic form factors. The situation here is different from that of the scattering process $e + f \to e + f$, occurring at positive k^2, where the form factors are real. In the scattering process there can be no polarization of f normal to the scattering plane, except for higher order electromagnetic corrections. This follows from usual time-reversal arguments.

In calculating $\sum R_{mn}$ according to (7) and (9) we sum over the polarization states of $\bar{f}$, but we introduce a spin projection operator before summing on the polarization states of f. The spin projection operator is $\frac{1}{2}(1+i\gamma_5\gamma_\mu s_\mu)$ where s_μ is the covariant polarization vector for f. We know that the polarization of f will be transverse and normal to the production plane; therefore s_μ will be of the form $(\boldsymbol{\zeta},0)$ where $\boldsymbol{\zeta}$ is a unit vector normal to the production plane.

The cross section can be expressed in the form

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{8}\alpha^2\lambda\!\!\!^-{}^2\beta\left[|F_1(k^2)+\mu F_2(k^2)|^2(1+\cos^2\theta) + \left|\frac{m}{E}F_1(k^2)+\frac{E}{m}\mu F_2(k^2)\right|^2 \sin^2\theta\right],\quad (51)$$

where $\lambda\!\!\!^- = E^{-1}$.

The form factors are taken at $k^2 = -4E^2$. Near the threshold $E \cong m$ the cross section (51) is proportional to β, the velocity of the final fermion, and is isotropic, in accordance with production in the $^3S_1$ state.

As we have already remarked, the fact that the form factors have an imaginary part for k^2 in the physical region for reaction (48) implies the possibility of a polarization of f normal to the plane of production and proportional to the sine of the phase difference between the electric and the magnetic form factors. The polarization of f along the normal to the production plane is given by $p(\theta)$, defined from

$$\frac{d\sigma}{d(\cos\theta)}p(\theta) = -\frac{\pi}{8}\alpha^2\lambda\!\!\!^-{}^2\beta^3\mu\frac{E}{m} \times \mathrm{Im}[F_1(k^2)F_2^*(k^2)]\sin(2\theta).\quad (52)$$

The normal to the plane here has been defined as the unit vector pointing in the direction of $\mathbf{p}\times\mathbf{q}_+$, where $\mathbf{p}$ is the momentum of the final fermion and $\mathbf{q}_+$ that of the incoming positron. We note that, by direct application of the TCP theorem, the polarization of the produced antifermion, $\bar{f}$, is equal but opposite in sign to that of the produced f.

With $F_1=1$ and $F_2=0$ the total cross section, as obtained from (51), is

$$\sigma = m^{-2}(2.1\times10^{-32}\ \mathrm{cm}^2)u(1-u)^{\frac{1}{2}}(1+\tfrac{1}{2}u),\quad (53)$$

with m in Bev and $u=(m/E)^2$. Of course, there is no reason whatsoever why the position $F_1=1$, $F_2=0$ should

[26] R. Karplus, C. M. Sommerfield, and E. H. Wichmann, Phys. Rev. **111**, 1187 (1958).

have any reliability in the physical region for the production process, which is far away from the limit $k^2=0$.

5.3. It is at present difficult to decide whether the form factor in (56) should strongly decrease or increase the value of the cross sections over the perturbation theory value given by (53). At present there is no information available on the form factors of the hyperons. Electron scattering on nucleons has provided reliable information on the nucleon form factors for positive values of k^2. It will not be easy, however, to extract information from the form factor at positive k^2, as determined from electron scattering experiments, about the values for large negative k^2 relevant to the production experiments. The recent indication of a core term in the nucleon structure[14] could eventually be related to the presence of singularities for large negative values of k^2, but the location and the nature of such singularities cannot be determined at present. To show a kind of science-fiction argument that one can use to relate the information from the scattering experiments to possible guesses on the pair production reactions, we shall make some (completely arbitrary) hypotheses on the origin of the core term in the nucleon structure and see what consequences it leads to for pair production. Suppose, for instance, that the core terms in the form factors given by Hofstadter and Herman[14] come from a big absorptive term concentrated around, say, $k^2=-(3m)^2$. This choice is quite arbitrary and, as far as the experiments tell us, there is no reason why the core term should not originate from singularities at much lower values of k^2, say, $k^2\cong -m^2$, and furthermore, it is very likely that it merely results from contributions of singularities extending all over the absorptive region. Let us also, for definiteness, assign some small width Γ to the states originating the singularities. For $k^2\cong-(3m)^2$, the nucleon form factors should then be approximated, using the Hofstadter[14] results, as $F_{1p}=1.2/D$, $F_{2p}=-3.4/D$, $F_{1n}=3.2/D$, and $F_{2n}=0$, where the common denominator is given by $D=20-2E+i(\Gamma/2)$, and we have expressed all energies in units of the pion mass. Inserting into (54) we find for the cross sections near the singularity $\sigma\cong(\pi/3)\alpha^2\lambda^2\beta\times 3.6\times(m_\pi/\Gamma)^2$ for p-$\bar{p}$ production and $\sigma\cong(\pi/3)\alpha^2\lambda^2\beta\times 50\times(m_\pi/\Gamma)^2$ for n-$\bar{n}$ production. If, for instance, $\Gamma\cong m_\pi$, these values are about 3.6 and 50 times bigger than the perturbation theory value for $e^++e^-\rightarrow f^++f^-$ (the p-$\bar{p}$ cross section is smaller because of an accidental cancellation). The above considerations have admittedly little value, except that they may serve to illustrate the hope that the cross sections, at least in same energy intervals, might come out rather bigger than what expected an the basis of (53).

5.4. Besides the reactions (49) one should also list

$$e^++e^-\rightarrow\Sigma^0+\bar{\Lambda},\ \Lambda+\bar{\Sigma}^0, \tag{54}$$

which involve a fermion-antifermion pair, but not charge conjugate of each other. The expression for the cross section of (54) depends on the relative Σ-Λ parity and, actually, if an experiment like (48) could be carried out, it would provide a good mean for measuring the relative Σ-Λ parity. That the cross section for (54) has a strong dependence on the relative Σ-Λ parity can also be seen directly by examining the threshold behavior. If the relative Σ-Λ parity is positive, the final $\Sigma\bar{\Lambda}$ (or $\bar{\Sigma}\Lambda$) will be produced in $^3S_1$, and $^3D_1$, as follows from parity and angular momentum conservation. If the relative Σ-Λ parity is negative, the accessible final states are instead $^1P_1$ and $^3P_1$. Therefore, the cross section near the threshold increases linearly with the final momentum p in the center-of-mass system for even parity, and it is also isotropic. For odd parity it increases as p^3 and will contain in general a $\cos^2\theta$ term.

The general form of the matrix element J derived from the requirements of Lorentz and gauge invariance is different from the case considered in the preceding section of a self-conjugate fermion-antifermion pair. For even relative parity we can write

$$J_\nu=\bar{u}_\Lambda[f_1(k^2)\gamma_\nu+f_2(k^2)\sigma_{\nu\mu}k_\mu+f_3(k^2)k_\nu]v_\Sigma, \tag{55}$$

subject to the condition $k_\nu J_\nu=0$ which gives

$$f_3(k^2)=i\frac{f_1(k^2)}{k^2}(m_\Sigma-m_\Lambda). \tag{56}$$

For odd relative parity

$$J_\nu=\bar{u}_\Lambda[f_1(k^2)\gamma_\nu+f_2(k^2)\sigma_{\nu\mu}k_\mu+f_3(k^2)k_\nu]\gamma_5 v_\Sigma, \tag{57}$$

and $k_\nu J_\nu=0$ gives

$$f_3(k^2)=-i\frac{f_1(k^2)}{k^2}(m_\Sigma+m_\Lambda). \tag{58}$$

The form factors $f_1(k^2)$, $f_2(k^2)$, $f_3(k^2)$ are the analytic continuations of the form factors describing, for positive k^2, a virtual transition $\Sigma\rightarrow\Lambda+\gamma$. The correspondence is correct, provided k_μ is defined in the $\Sigma\rightarrow\Lambda+\gamma$ transition as $k_\mu=\Sigma_\mu-\Lambda_\mu$, where Σ_μ and Λ_μ are the Σ and Λ four-momenta. One notices that $f_3(k^2)$ will not enter in the description of the production process (54), as can be seen by specializing (55) or (57) in the center-of-mass system where k_ν has only the time-like component, but J_4 is zero because of $k_4J_4=0$.

The cross section is given by

$$\frac{d\sigma}{d(\cos\theta)}=\frac{\pi}{8}\alpha^2\lambda^2\beta\bigg\{\beta^2\cos^2\theta[|f_1(k^2)|^2+k^2|f_2(k^2)|^2]+[|f_1(k^2)|^2-k^2|f_2(k^2)|^2]\frac{E_\Lambda E_\Sigma\pm m_\Lambda m_\Sigma}{E^2}-4\frac{m_\Lambda E_\Sigma\pm m_\Sigma E_\Lambda}{E}\mathrm{Re}[f_1(k^2)f_2^*(k^2)]\bigg\}, \tag{59}$$

where the plus sign refers to even relative Σ-Λ parity and the minus sign to odd relative parity, and $\beta=p/E$. For production near the threshold, the cross sections become

$$\frac{d\sigma}{d(\cos\theta)}=\frac{\pi}{4}\alpha^2\lambda^2\beta\frac{m_\Lambda m_\Sigma}{E^2}|f_1-2Ef_2|^2 \tag{60}$$

for even relative parity, and

$$\frac{d\sigma}{d(\cos\theta)}=\frac{\pi}{8}\alpha^2\lambda^2\beta^3\{A+B\cos^2\theta\} \tag{61}$$

for odd relative parity, with

$$A=(|f_1|^2-k^2|f_2|^2)\frac{1}{2}\left(\frac{m_\Lambda}{m_\Sigma}+\frac{m_\Sigma}{m_\Lambda}\right)+2E\left(\frac{m_\Sigma}{m_\Lambda}-\frac{m_\Lambda}{m_\Sigma}\right)\mathrm{Re}[f_1f_2^*]$$

$$B=|f_1|^2+k^2|f_2|^2.$$

The decay process,

$$\Sigma^0\to\Lambda^0+\gamma,$$

is expected to be essentially determined by $f_2(0)$ (proportional to the so-called transition magnetic moment between Σ and Λ), for each case of relative parity. In fact, for a real γ, terms proportional to k_ν in both (55) and (57) do not contribute because of the transversality condition $k_\nu\epsilon_\nu=0$. Similarly, $f_1(k^2)$ should presumably vanish at $k^2=0$ as suggested from (56) or (58). The same should apply to the quasi-real gammas in $\Sigma^0\to\Lambda^0+e^++e^-$. The physical values of k^2 in the production process $e^++e^-\to\Sigma^0+\Lambda^0$ lie very far from $k^2\cong 0$ so that a direct connection with Σ^0 decay seems unjustified.

6. ANNIHILATION INTO POSSIBLE VECTOR MESONS

6.1. Vector mesons have been discussed recently[11] because of their formal connection with local conservation laws.[27] We have already discussed in some detail the possibility of detecting neutral unstable vector mesons with charge conjugation number -1 through their resonant effect in reactions

$$e^++e^-\to B^0\to\text{(final state)}, \tag{62}$$

where B^0 is the unstable meson. In this section we shall discuss reactions of the type

$$e^++e^-\to B+\bar{B}, \tag{63}$$

where B is a (charged or neutral) spin-one meson. Reactions of the kind (62) will be very suitable to detect vector mesons B^0 with $C=-1$ and zero strangeness. However, a vector meson K' with nonzero strangeness would not appear as intermediate state in (62), but it could be produced according to (63) or to reactions of the kind

$$e^++e^-\to K'+K, \tag{64}$$

conserving the total strangeness. The suggested strongly interacting vector mesons are all expected to be eminently unstable. Reactions like (63) would therefore be observed as many-body reactions, and the possibility of separating the over-all process into two stages, of which the first is the production process of the vector mesons, relies essentially on the hypothesis of a sufficiently long lifetime for the intermediate vector meson. When this hypothesis is not satisfied the separation of the process into two stages is less justified and can only lead to approximate results.

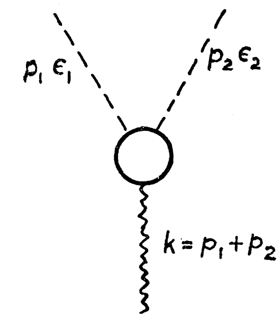

FIG. 6. Electromagnetic vertex for production of vector boson pair. The symbols are defined in the text.

6.2. We shall here examine in detail reaction (63), including also a discussion of the angular correlation at the decay of B, also in view of applications that we will consider in the next section to the verification of the intermediate meson theory of weak interactions. We shall first give the general form for the electromagnetic vertex of a vector boson on the basis of Lorentz-invariance, gauge invariance, and charge conjugation invariance. The vertex is described by three form factors. In the static limit they correspond to the charge, the magnetic moment, and the electric quadrupole moment.

In the electromagnetic vertex shown in Fig. 6, we call p_1^μ, ϵ_1^μ and p_2^μ, ϵ_2^μ the four-momenta and polarization four-vectors of the (physical) particles B and $\bar{B}$. They satisfy $p_1^2=p_2^2=-m_B^2$, $\epsilon_1^2=\epsilon_2^2=1$, and $(p_1\epsilon_1)=(p_2\epsilon_2)=0$. The matrix element J^μ of the electromagnetic current must be constructed out of p_1^μ, p_2^μ, ϵ_1^μ, and ϵ_2^μ. We take as independent vectors: $k^\mu=p_1^\mu+p_2^\mu$, $p^\mu=p_1^\mu-p_2^\mu$, ϵ_1^μ, and ϵ_2^μ. We note that $p^2=-k^2-4m_B^2$, $(kp)=0$, $(\epsilon_1p)=-(\epsilon_1k)$, $(\epsilon_2p)=-(\epsilon_2k)$. The only independent scalars are therefore: $k^2(\epsilon_1k)$, (ϵ_2k), and $(\epsilon_1\epsilon_2)$. The matrix element J_μ must transform like a vector and must depend linearly on each ϵ. We thus write

$$J^\mu=k^\mu[(\epsilon_1\epsilon_2)a(k^2)+(\epsilon_1k)(\epsilon_2k)b(k^2)]+p^\mu[(\epsilon_1\epsilon_2)c(k^2)+(\epsilon_1k)(\epsilon_2k)d(k^2)]+\epsilon_1^\mu(\epsilon_2k)e(k^2)+\epsilon_2^\mu(\epsilon_1k)f(k^2). \tag{65}$$

From the condition $(kJ)=0$ we obtain $a(k^2)=0$ and $-k^2b(k^2)=e(k^2)+f(k^2)$. We then make use of invariance

[27] C. N. Yang and R. Mills, Phys. Rev. **96**, 191 (1954).

under charge conjugation. The electromagnetic current operator j^μ transforms into $-j^\mu$ under charge conjugation. Such a condition requires that the matrix element J^μ transforms into $-J^\mu$ when $k^\mu \rightarrow k^\mu$, $p^\mu \rightarrow -p^\mu$ and $\epsilon_1{}^\mu \leftrightarrow \epsilon_2{}^\mu$. It follows that $e(k^2)=-f(k^2)$. The general form of J^μ is thus

$$J^\mu = p^\mu[(\epsilon_1\epsilon_2)c(k^2)+(\epsilon_1 k)(\epsilon_2 k)d(k^2)] + [\epsilon_1{}^\mu(\epsilon_2 k)-\epsilon_2{}^\mu(\epsilon_1 k)]e(k^2). \quad (66)$$

It will be convenient to introduce form factors $G_1(k^2)$, $G_2(k^2)$, and $G_3(k^2)$ such that $eG_1(k^2)$, $\mu G_2(k^2)$, and $\epsilon G_3(k^2)$ describe in suitable linear combinations (for small spacelike k^2) the charge distribution, the magnetic moment distribution, and the electric quadrupole moment distribution. The new form factors are linearly related to $c(k^2)$, $d(k^2)$, and $e(k^2)$. We will thus write

$$J^\mu = (2\pi)^3\langle B\bar{B};\text{out}|j^\mu(0)|0\rangle = \frac{e}{(4\omega_1\omega_2)^{\frac{1}{2}}}\{G_1(k^2)(\epsilon_1\epsilon_2)p^\mu + [G_1(k^2)+\mu G_2(k^2)+\epsilon G_3(k^2)][(\epsilon_1 k)\epsilon_2{}^\mu-(\epsilon_2 k)\epsilon_1{}^\mu] + \epsilon G_3(k^2)m_B{}^{-2}[(k\epsilon_1)(k\epsilon_2)-\tfrac{1}{2}k^2(\epsilon_1\epsilon_2)]p^\mu\}, \quad (67)$$

where ω_1 and ω_2 are the center-of-mass energies of B and $\bar{B}$. The static anomalous magnetic moment is $\mu+\epsilon$; the static anomalous electric quadrupole moment is 2ϵ.

6.3. In a Lagrangian theory of vector mesons one would assume a Lagrangian

$$\mathfrak{L} = -\tfrac{1}{2}B_{\mu\nu}{}^\dagger B_{\mu\nu} - m_B{}^2 U_\mu{}^\dagger U_\mu, \quad (68)$$

where U_μ is the vector field, $B_{\mu\nu}=\partial_\mu U_\nu - \partial_\nu U_\mu$, with $\partial_\mu = (\partial/\partial x)_\mu - ieA_\mu$, and m_B the mass of the meson. The supplementary condition,

$$\partial_\mu U_\mu = (ie/2)F_{\mu\nu}B_{\mu\nu}, \quad (69)$$

follows from the field equations (if $m_B \neq 0$). The minimal electromagnetic current is thus

$$j_\mu = -ie[U_\nu{}^\dagger B_{\mu\nu} - U_\nu B_{\mu\nu}{}^\dagger]. \quad (70)$$

To such a current one can add nonminimal terms

$$j_\mu' = -ie\mu(\partial/\partial x_\nu)(U_\mu{}^\dagger U_\nu - U_\nu{}^\dagger U_\mu), \quad (71)$$

$$j_\mu'' = ie(\epsilon/m_B{}^2)(\partial/\partial x_\nu)(B_{\mu\lambda}{}^\dagger B_{\nu\lambda} - B_{\nu\lambda}{}^\dagger B_{\mu\lambda}), \quad (72)$$

The total current is then of the form (67) with $G_1(k^2) = G_2(k^2) = G_3(k^2) = 1$.

6.4. The cross section formula (7) reduces to

$$\sigma = \frac{\alpha}{(2\pi)}\frac{1}{16E^4}\int d^3p_1 d^3p_2\,\delta(\omega_1+\omega_2-2E) \times\delta^3(\mathbf{p}_1+\mathbf{p}_2)T_{mn}\sum_{1,2}R_{mn}, \quad (73)$$

where T_{mn} is given by (8) and R_{mn} by (9) and (67). Differential cross sections and cross sections for polarized final particles can be obtained from (73) by omitting the relevant integrations and spin summations.

We note that $T_{mn}\sum R_{mn}$ is a Lorentz invariant quantity. We want a complete description of one of the produced bosons, say of B, after averaging over the polarizations of the other. We first sum over the polarizations of $\bar{B}$, using

$$\textstyle\sum_2 \epsilon_2{}^\mu \epsilon_2{}^\mu = \delta_{\mu\nu} + p_{2\mu}p_{2\nu}/m_B{}^2,$$

and we write

$$T_{mn}\sum R_{mn} = R^{\rho\sigma}\epsilon_1{}^\rho \epsilon_1{}^\sigma. \quad (74)$$

Equation (74) defines the tensor $R_{\rho\sigma}$. The density matrix will be described in terms of the tensor

$$\tilde{R}_{\rho\sigma} = \Lambda_{\rho\tau}R_{\tau\omega}\Lambda_{\omega\sigma}, \quad (75)$$

where

$$\Lambda_{\mu\nu} = \delta_{\mu\nu} + p_{1\mu}p_{1\nu}/m_B{}^2 \quad (76)$$

is a projection operator such that $\tilde{\epsilon}_\mu{}^{(1)} = \Lambda_{\mu\nu}\epsilon_\nu{}^{(1)}$ always satisfies $(p^{(1)}\tilde{\epsilon}^{(1)})=0$.

The differential cross section is given from (73), (74) and (75), by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\alpha}{32}\frac{1}{E^2}\beta\,\mathrm{Tr}[\tilde{R}], \quad (77)$$

where

$$\beta = (1 - m_B{}^2/E^2)^{\frac{1}{2}}$$

is the velocity of the produced bosons. The differential cross section can be evaluated directly from (77), (76), (75), and (74), or using the expression for $\tilde{R}$ that we give in the next section. Its expression is given by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{16}\alpha^2\lambda^2\beta^3\left\{2\left(\frac{E}{m_B}\right)^2|G_1(k^2) + \mu G_2(k^2) + \epsilon G_3(k^2)|^2(1+\cos^2\theta) + \sin^2\theta\left[2\left|G_1(k^2)+2\left(\frac{E}{m_B}\right)^2\epsilon G_3(k^2)\right|^2 + \left|G_1(k^2)+2\left(\frac{E}{m_B}\right)^2\mu G_2(k^2)\right|^2\right]\right\}. \quad (78)$$

The β^3 dependence in (78) for production near threshold is typical of P-state production. In our approximation of neglecting higher-order electromagnetic terms, the final mesons must be produced in a state of total angular momentum $J=1$, parity $P=-1$, and charge conjugation number $C=-1$. From angular momentum and parity conservation it follows that the final mesons can only be in ${}^1P_1$, ${}^3P_1$, ${}^5P_1$, and ${}^5F_1$. However, triplet states of odd orbital parity cannot be produced because they have $C=+1$, so we are left with ${}^1P_1$, ${}^5P_1$, and ${}^5F_1$ as the only permitted final states.

With $G_1=1$, $G_2=G_3=0$, the total cross section is

$$\sigma = m_B{}^{-2}(2.1\times10^{-32}\text{ cm}^2)\tfrac{3}{4}(1-u)^{\frac{3}{2}}(\tfrac{4}{3}+u), \quad (79)$$

where m_B is expressed in Bev and $u=(m/E)^2$. Therefore, e^+-e^- collisions may turn out to be very efficient for detecting possible unstable vector mesons.

6.5. The above cross section obtained with $G_1=1$, $G_2=G_3=0$ formally violates unitarity at high energies. For high energies (79) goes to a constant whereas it can be shown, on the basis of unitarity arguments, that the total reaction cross section must decrease proportionally to $\lambda\!\!\!^{-2}$.

Unitarity arguments are not very informative usually at relativistic energies. Electron-positron collisions present, however, an exceptional circumstance, that they go through one specified channel, the one-photon channel, as long as one neglects higher order electromagnetic terms. We shall present here a derivation of the upper limit to the reaction cross section required from unitarity for electron-positron collisions at the lowest electromagnetic order. For the derivation we shall employ the Jacob-Wick notation. Let us consider a process

$$a+b \rightarrow \text{(final state)}. \tag{80}$$

We shall denote by F a set of final states specified by the nature of the final products. The initial state is defined, for a given center-of-mass momentum of the colliding particles a and b, by their helicities λ_a and λ_b. We shall write

$$|i\rangle = |\lambda_a,\lambda_b\rangle. \tag{81}$$

The total cross section from such a state summing over the set F is then given, in the notations of Jacob and Wick, by

$$\sigma(\lambda_a\lambda_b;F)=(2\pi)^2\lambda\!\!\!^{-2}\langle\lambda_a\lambda_b|T(E)^\dagger P_F(E)T(E)|\lambda_a\lambda_b\rangle, \tag{82}$$

where $T(E)$ is the T matrix at energy E of each of the colliding particles and $P_F(E)$ is the projection operator into the states of total energy $2E$ of the set F. Both $T(E)$ and $P(E)$ are rotation invariant and therefore they commute with the total angular momentum J. The cross section σ can thus be written as a sum of σ_J belonging to the different J's,

$$\sigma_J(\lambda_a,\lambda_b;F)=\pi\lambda\!\!\!^{-2}(2J+1) \times\langle J,\lambda_a,\lambda_b|T_J^\dagger(E)P_J(E)T_J(E)|J,\lambda_a\lambda_b\rangle, \tag{83}$$

where $\langle J,\lambda_a,\lambda_b|$ is the J component of $\langle i|$. Now for a reaction (as opposed to scattering) we can substitute S for T, and using $S_J^\dagger(E)S_J(E)=1$, we obtain an upper limit for (77):

$$\sigma_J(\lambda_a,\lambda_b;F)\leq\pi\lambda\!\!\!^{-2}(2J+1). \tag{84}$$

We can apply this result to our reactions,

$$e^++e^-\rightarrow(\gamma)\rightarrow\text{(final state)}. \tag{85}$$

The initial e^+-e^- states must have $J=1$, $C=-1$, and $P=-1$. Two linear combinations of states (81) exist that have such quantum numbers, namely,

$$(1/\sqrt{2})(|1,1\rangle+|-1,-1\rangle),$$
$$(1/\sqrt{2})(|1,-1\rangle+|-1,+1\rangle),$$

both for $J=1$. Helicity $+1$ for a particle means that the spin is pointing in the direction of the momentum. However, only the second of such states participates to (85) in the limit when the electron mass can be neglected. In fact, the initial electron and positron appear in the combination $\bar{v}\gamma_\mu u$, which can be written $\bar{v}(\bar{a}\gamma_\mu a+a\gamma_\mu\bar{a})u$ where $a=\frac{1}{2}(1+\gamma_5)$ and $\bar{a}=\frac{1}{2}(1-\gamma_5)$ are the projection operators for negative and positive helicity. By averaging (84) over the initial polarizations we then find the upper limit

$$\tfrac{3}{4}\pi\lambda\!\!\!^{-2}$$

for the cross section of a reaction (85), neglecting the electron mass. The cross section (79), derived from (78) with the position $G_1=1$, $G_2=G_3=0$, is the same as would be given by the lowest order perturbation contribution to $e^++e^-\rightarrow B^++B^-$, ignoring any structure of B. The expression (79) violates unitarity at high energies. The violation, however, occurs at very high energies, of the order of $10^2 m_B$. At these high energies it is certainly inaccurate to neglect higher-order electromagnetic terms, and also structure effects due to other interactions of B, if they exist, would anyway be important.

6.6. The matrix

$$\rho=\tilde{R}/\mathrm{Tr}[\tilde{R}], \tag{86}$$

gives complete information on the produced B, and has the transformation properties of a tensor. Its calculation is long but straightforward, using (9), (67), (74), (75), and (76). We give here the result:

$$\tilde{R}^{\mu\nu}=\frac{e^2}{8}\bigg\{B_1\delta_{\mu\nu}+B_2\frac{k^\mu k^\nu}{m_B{}^2}+B_3\frac{q^\mu q^\nu}{m_B{}^2}+B_4\frac{p^\mu p^\nu}{m_B{}^2} +B_5\frac{k^\mu q^\nu+k^\nu q^\mu}{m_B{}^2}+B_6\frac{k^\mu p^\nu+k^\nu p^\mu}{m_B{}^2}+B_7\frac{q^\mu p^\nu+q^\nu p^\mu}{m_B{}^2} +iB_8\beta x^2\cos\theta\bigg[\frac{p^\mu q^\nu-p^\nu q^\mu}{m_B{}^2}+\beta^2\frac{k^\mu q^\nu-k^\nu q^\mu}{m_B{}^2} -\beta\cos\theta\frac{k^\mu p^\nu-k^\nu p^\mu}{m_B{}^2}\bigg]\bigg\}, \tag{87}$$

where $q=q_+-q_-$, $x=E/m_B$, and the B's are given by

$$B_1=4\beta^2\sin^2\theta[|G_1+2x^2\epsilon G_3|^2+4x^2\beta^2|G_1+\mu G_2+\epsilon G_3|^2,$$

$$B_2=x^2\beta^2(1+\beta^2\cos^2\theta)|G_1+\mu G_2+\epsilon G_3|^2 +\beta^2\sin^2\theta\{|G_1+2x^2\mu G_2|^2 +4\,\mathrm{Re}[(G_1+x^2\mu G_2+x^2\epsilon G_3)(\epsilon G_3-\mu G_2)^*],$$

$$B_3=-\beta^2|G_1+\mu G_2+\epsilon G_3|^2,$$

$$B_4=x^2\beta^2(1+\cos^2\theta)|G_1+\mu G_2+\epsilon G_3|^2 +4x^2\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*] +\beta^2\sin^2\theta(|G_1|^2+4x^2\epsilon\,\mathrm{Re}G_1G_3^*+4x^4\mu^2|G_2|^2),$$

$$B_5=-2\beta^3x^2\cos\theta\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*],$$

$$B_6=x^2\beta^2(1+\cos^2\theta)|G_1+\mu G_2+\epsilon G_3|^2 +\beta^2\sin^2\theta|G_1+2x^2\mu G_2|^2 +2x^2\beta^2\cos^2\theta\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*],$$
$$B_7=\beta\cos\theta|G_1+\mu G_2+\epsilon G_3|^2 -2\beta x^2\cos\theta\,\mathrm{Re}[(G_1+\mu G_2+\epsilon G_3)(\mu G_2-\epsilon G_3)^*],$$
$$B_8=2\beta^2\,\mathrm{Im}G_1(\mu G_2+\epsilon G_3)^*. \tag{88}$$

The form factors are all taken at $k^2=-4E^2$.

6.7. The density matrix ρ contains a complete description of the produced B. If one knows the amplitude for a mode of decay of B, the angular correlations of the decay products with respect to the incident and final momenta in the production process can be calculated. Consider, for instance, a two-body decay of B. The decay amplitude will be of the form

$$\epsilon_1{}^\mu\mathcal{a}^\mu, \tag{89}$$

where $\mathcal{a}^\mu$ is a vector (or pseudovector, or a sum of both). The angular distribution of the secondaries in the rest system of the decaying B is then given by

$$\sum_{\text{spin}}(\mathcal{A}^\mu\rho^{\mu\nu}\mathcal{a}^\nu)d\Omega, \tag{90}$$

where $d\Omega$ is the solid angle in the B rest frame and $\mathcal{A}^\mu=(\mathcal{a}^{i*}-\mathcal{a}^{4*})$. The summation is extended over the final spin states. The quantity $\mathcal{A}^\mu\rho^{\mu\nu}\mathcal{a}^\nu$ is a scalar invariant and can be evaluated in the production center-of-mass system (system of the laboratory in a colliding beam experiment) using the expressions (87) and (89) that are valid in that system. The distribution in the laboratory system of the colliding beam experiment is thus given directly by

$$\sum_{\text{spin}}(\mathcal{A}^\mu\rho^{\mu\nu}\mathcal{a}^\nu)\frac{d\Omega}{d\Omega'}d\Omega', \tag{91}$$

where $d\Omega'$ is the decay solid angle in the laboratory system and $d\Omega/d\Omega'$ only depends on the decay angle with respect to the line of flight of B and on the velocity of B.

As an application we consider the decays $B\rightarrow\pi+\pi$ and $B\rightarrow\mu+\nu$, $B\rightarrow e+\nu$. The amplitude $\mathcal{a}^\mu$ for

$$B\rightarrow\pi+\pi,$$

has the general form $\mathcal{a}^\mu=a(s^2)p_1{}^\mu+b(s^2)s^\mu$ where s^μ is the difference of the two final four-momenta; $p_1{}^\mu$, the momentum of B, is their sum; and $a(s^2)$ and $b(s^2)$ are form factors. However, the first term in the above expression for $\mathcal{a}^\mu$ does not contribute in the decay of a physical B, because of $p_1{}^\mu\epsilon_1{}^\mu=0$. So we take the amplitude in the form

$$\mathcal{a}^\mu=b(s^2)s^\mu. \tag{92}$$

The decay correlation is thus given by

$$\rho^{\mu\nu}s^\mu s^\nu d\Omega. \tag{93}$$

We have calculated the angular correlation for B mesons produced close to threshold and assuming $\mu=0$, $\epsilon=0$, that is, neglecting any anomalous magnetic dipole or electric quadrupole moment. The angular correlation is given by

$$2-(\mathbf{i}\cdot\mathbf{f})^2-(\mathbf{i}\cdot\mathbf{d})^2+2(\mathbf{i}\cdot\mathbf{d})(\mathbf{d}\cdot\mathbf{f})(\mathbf{f}\cdot\mathbf{i}), \tag{94}$$

where **i**, **f**, and **d** are unit vectors in the direction, respectively, of the incoming momentum in the collision process, of the outgoing momentum in the collision process, and of the relative final momentum in the decay.

The amplitude $\mathcal{a}^\mu$ for $B\rightarrow\mu+\nu$ and $B\rightarrow e+\nu$, assuming that the leptons are produced locally in the $1+\gamma_5$ projection, is given by in general by

$$[\bar{l}\gamma_\mu(1+\gamma_5)\nu]c(s^2), \tag{95}$$

where l denotes either μ or e, and ν denotes the neutrino, and $c(s^2)$ is a form factor depending on the relative final four-momentum in the decay. The angular correlation can be obtained from (90) and is given by

$$\rho^{\mu\nu}[p_1{}^\mu p_1{}^\nu-s^\mu s^\nu+\delta^{\mu\nu}(m_B{}^2-m_l{}^2) +\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}(s^\rho p_1{}^\sigma-s^\sigma p_1{}^\rho)]. \tag{96}$$

Again we specialize to B production near the threshold and neglect μ and ϵ. The angular correlation is then given by

$$3+(\mathbf{i}\cdot\mathbf{d})^2-2(\mathbf{i}\cdot\mathbf{f})(\mathbf{f}\cdot\mathbf{d})(\mathbf{d}\cdot\mathbf{i}), \tag{97}$$

in terms of the same vectors defined before. We have neglected the mass of the final lepton m_l in comparison to the mass of B. General formulas can be easily derived from (93), (95) and their analogs, and the general expression for $\bar{R}^{\mu\nu}$ reported in (87) and (88), to cover all interesting cases.

7. EXPERIMENTS ON WEAK INTERACTIONS

7.1. Semiweakly interacting bosons have been suggested as intermediary agents of weak interactions.[28] A simplest scheme of weak interactions is based on charged weak currents only and can be reproduced by postulating only charged vector mesons. It is known that the absence of $\mu\rightarrow e+\gamma$ leads to a difficulty in a theory with intermediate vector bosons, and the usual suggestion to overcome such a difficulty is that there are two different neutrinos ν_e and ν_μ. Charged currents alone do not allow a simple incorporation of the $\Delta T=\frac{1}{2}$ rule in the theory of weak interactions. However, a coupling of neutral intermediate vector mesons to both the neutral strangeness nonconserving current and the neutral lepton current leads to contradictions with experimental data. Therefore it is probable that even if intermediate neutral vector mesons exist they do not couple to the neutral lepton currents and, in particular, to the initial electron-positron state of the reactions that we are discussing. A check of this supposition

[28] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); T. D. Lee and C. N. Yang, *ibid.* **119**, 1410 (1960).

could be carried out experimentally on the basis of the following remarks. If a B^0 exists which couples to e^+-e^-, $\mu^+-\mu^-$, etc., it would give rise to resonances in reactions of the kind

$$e^+ + e^- \to B^0 \to e^+ + e^-, \quad (99)$$

$$e^+ + e^- \to B^0 \to \mu^+ + \mu^-, \quad (100)$$

etc. It is remarkable that such resonances could lead to large observable effects in spite of the fact that two semiweak couplings are involved in reactions like (99) and (100). The mass of B^0 must be $> M_K$ in order to avoid a semiweak decay of K. We assume a width Γ appropriate to the semiweak decay couplings of B^0 of the order of 5×10^{17} sec^{-1}. We also assume for B^0 a mass of the order of the K mass, and we suppose that the branching ratio for its decay into $e^+ + e^-$ (and similarly into $\mu^+ + \mu^-$) is about one fifth. The width $\Gamma \cong 5\times 10^{17}$ sec^{-1} corresponds to a very sharp resonance extending over a few hundreds of ev and what will be actually measured is $\bar{\sigma}_R$ defined as in (22). For $\bar{\sigma}_R$ we find a value of $2.6\times 10^{-5}(2\pi\lambda\!\!\!^{-2})$ which is about three times bigger than the cross section for $e^+ + e^- \to \mu^+ + \mu^-$ at any energy $E \gg m_\mu$.

7.2. Intermediate charged vector mesons can be produced according to the reaction

$$e^+ + e^- \to B^+ + B^- \quad (63)$$

that we have discussed in the previous section. Of course, it seems perfectly consistent in this case of semiweakly interacting mesons to deal separately with their production processes and with their decay. Experimentally, reaction (63) would still appear as a many-body reaction, like for instance

$$e^+ + e^- \to (\mu^+ + \nu) + (e^- + \bar{\nu}). \quad (101)$$

An electromagnetic process like $e^+ + e^- \to \mu^+ + \mu^- + e^+ + e^-$, which could also originate a final μ^+ and e^-, is of higher order and would have a much smaller probability than (63) followed by the successive decay of B^+ and B^- into the final particles. The decay products of B^+ and B^- would exhibit specific angular correlations as we have already discussed in the previous sections.

In the absence of structure effects for B, the expression for the cross section obtained from (78) would violate unitarity at high energy. Inclusion of a point magnetic moment or of a point electric quadrupole moment does not change this situation. For instance, if a point magnetic moment μ_B is introduced, the cross section derived from (78) increases quadratically with E, making the unitarity violation worse.

Of course, the considerations that make possible the existence of the intermediate boson B, having no strong interactions, would also apply to a possible fermion with mass bigger than the K mass, which had no strong interactions. Such a fermion would hardly have been detected, if it existed, and $e^+ + e^-$ collisions may allow one to definitely exclude its existence. The cross section for production of a fermion-antifermion pair is given by (53) in the absence of structure effects.

One can also ask about the contribution of the known local weak interactions to electron-positron processes. If, for instance, a weak lepton interaction of the type $(\mu^+\mu^-)(e^+e^-)$ exists, there could be a weak amplitude of the form

$$(2\pi)^{-2}\sqrt{8}G[\bar{u}(\mu^-)\gamma_\mu\tfrac{1}{2}(1+\gamma_5)v(\mu^+)] \times[\bar{v}(e^+)\gamma_\mu\tfrac{1}{2}(1+\gamma_5)u(e^-)], \quad (102)$$

adding coherently to the electromagnetic amplitude for $e^+ + e^- \to \mu^+ + \mu^-$. The contribution from (102) is, however, very small though increasing very rapidly with energy. The cross-section for $e^+ + e^- \to \mu^+ + \mu^-$ obtained by adding the contribution from (102) to the lowest order electromagnetic amplitude is

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi}{8}\alpha^2\lambda\!\!\!^{-2}[(1+\cos^2\theta)(1+\epsilon+\epsilon^2) + 2(\epsilon+\epsilon^2)\cos\theta], \quad (103)$$

where $\epsilon = 6.2\times 10^{-4}(E/M_N)^2$, with M_N = nucleon mass. The numerical coefficient in the expression for ϵ has been calculated by taking for G the value of the β-decay coupling constant. The appearance of the $\cos\theta$ term is entirely due in (103) to the weak interaction (102). However, a $\cos\theta$ term in the differential cross section for $e^+ + e^- \to \mu^+ + \mu^-$ would also occur from the high order electromagnetic graphs (for instance, from a diagram with two gammas exchanged). The parity-nonconserving effects of (102) would constitute a more unique test of its presence. For instance, to the differential cross section (103) would be associated a longitudinal polarization

$$P^{\pm} = \pm(\epsilon+\epsilon^2)\frac{(1+\cos\theta)^2}{(1+\cos^2\theta)+(\epsilon+\epsilon^2)(1+\cos^2\theta)} \quad (104)$$

of the final $\mu^\pm$. For energies $E \sim 30$ Gev, ϵ becomes of the order unity and the polarization should be quite large. For colliding beam energies of the order of 1–2 Bev, effects of local weak interactions should be negligible. On the other hand, if intermediate mesons exist they would show off in various ways and electron-positron collisions would in fact constitute a good experimental means for their detection.

8. EXPRESSION FOR THE VACUUM POLARIZATION DUE TO STRONG INTERACTING PARTICLES

The quantity

$$\Pi(k^2) = -\frac{(2\pi)^3}{3k^2}\sum_{p^{(z)}=k}\langle 0|j_\nu(0)|z\rangle\langle z|j_\nu(0)|0\rangle \quad (105)$$

is known to be of fundamental importance in quantum electrodynamics.[29] In (105), j_ν is the current operator and the sum is extended over all the physical states

[29] G. Källén, Helv. Phys. Acta **25**, 417 (1952).

with total four-momentum $p^{(z)}=k$. The Fourier transform of the photon propagator

$$D_{\mu\nu}^{F'}(x-x')=i\langle 0|P(A_\mu(x')A_\nu(x))|0\rangle,$$

where P is the chronological product and A_μ is the electromagnetic field, can be expressed in terms of $\Pi(k^2)$ as[29]

$$D_{\mu\nu}^{F'}(k)=\frac{\delta_{\mu\nu}}{k^2-i\epsilon}+\frac{k^2\delta_{\mu\nu}-k_\mu k_\nu}{k^2}\times\frac{\bar{\Pi}(0)-\bar{\Pi}(k^2)-i\pi\Pi(k^2)}{k^2-i\epsilon}. \quad (106)$$

In (106) $\bar{\Pi}(k^2)$ is defined as

$$\bar{\Pi}(k^2)=P\int_0^\infty \frac{\Pi(-a)}{k^2+a}da. \quad (107)$$

We show in this section that the experimentally measured cross sections for processes $e^+ + e^- \to \gamma \to F$, where F denotes a group of final states, is directly related to the contribution to (105) from the group of states F in the summation over the intermediate states z. This result will permit, for instance, calculation of the modifications of the photon propagator due to virtual strong interacting particles, directly from the measured cross sections.

A problem of this sort has been considered by Brown and Calogero,[30] who calculated the modifications to the photon propagator expected from intermediate two-pion states with resonant interaction. Here we shall determine the general relation between the modification to the photon propagator and the measured total cross sections for the annihilation processes.

We note that the matrix elements $\langle 0|j_\nu(0)|z\rangle$ occurring in (105) are proportional to the corresponding J_ν defined in (3). Therefore the total cross section for annihilations leading to the final states F in the center-of-mass system can be written, according to (7), as

$$\sigma_F(E)=-\frac{(2\pi)^5\alpha}{16E^4}T_{mn}\sum_{p_z=k}{}_F \langle 0|j_m(0)|z\rangle\langle z|j_n(0)|z\rangle. \quad (108)$$

Now we use gauge invariance to relate the sum in (108) to the analogous sum in (105). We have

$$(2\pi)^3\sum_{p_z=k}{}_F\langle 0|j_\mu(0)|z\rangle\langle z|j_\nu(0)|0\rangle =\Pi_F(k^2)(k_\mu k_\nu-k^2\delta_{\mu\nu}). \quad (109)$$

In (109) we have indicated by $\Pi_F(k^2)$ the contribution to $\Pi(k^2)$ from the group of intermediate states F. Substituting into (108) we obtain

$$\sigma_F(E)=(\pi^2\alpha/E^2)\Pi_F(-4E^2), \quad (110)$$

which gives the desidered connection. Note that integrals of the type

$$\int^\infty \frac{\Pi(-a)}{a^2}da, \quad (111)$$

must be convergent, as noted by Källén,[29] otherwise observable expressions would not be finite. It follows that for any group of states F, $\sigma_F(E)$ must be such that

$$\int^\infty \frac{\sigma_F(E)}{E}dE$$

converges. Such a condition is weaker than the one we derived in Sec. 6 from the unitarity requirement for the cross sections $\sigma_F(E)$. The integral

$$\bar{\Pi}(0)=P\int_0^\infty \frac{\Pi(-a)}{a}da$$

is connected to charge renormalization. If one wants it finite, $\int^\infty E\sigma_F(E)dE$ must be finite for any group of states F. If the cross sections decrease as λ^2, (111) is logarithmically divergent. Note that all the above statements about convergence only refer to the one-photon channel and they are not vigorous at all orders.

9. CONCLUSIONS

In high-energy electron-positron colliding beam experiments we see a possible field of spectacular developments for high-energy physics. Electron-positron experiments offer a unique possibility for a consistent and direct exploration of the electromagnetic properties of elementary particles. At the lowest electromagnetic order the annihilation proceeds through a virtual intermediate photon of timelike four-momentum which then disintegrates into the final products. The form factors of strongly interacting particles produced in the reaction are thus explored for negative values of the invariant four-momentum squared, k^2, inside the absorption cut in k^2 plane. The coupling to the one-photon intermediate state selects out of the incoming states a particular state with total angular momentum one, negative parity, and opposite helicity for the colliding relativistic particles. Pairs of spin-zero bosons, of positive relative parity, are produced in P state. Fermion-antifermion pairs are produced in $^3S_1$ and $^3D_1$ (or in $^1P_1$ and $^3P_1$ if the relative parity is negative). Pairs of spin-one bosons, of positive relative parity, are produced in $^1P_1$, $^5P_1$, and $^5F_1$. In Sec. 1 we have reported some general considerations relative to the most probable annihilations, occurring through one single photon. Radiative corrections do not substantially alter the single-photon picture as long as the experimental arrangements are symmetrical with respect to the produced charges. Annihilation into pions, $\pi^0+\gamma$, and K mesons should be the most important

[30] L. M. Brown and F. Calogero, Phys. Rev. **120**, 653 (1960).

annihilation processes producing strongly interacting particles for not very high energies. Pion form factors can be directly explored along the absorptive cut on the k^2 plane and, as already discussed many times,[4,6,7] their values are directly related to the nature of forces among pions. A $T=1$, $J=1$ pion-pion resonance would be directly exhibited in the two-pion annihilation mode, and a $T=0$, $J=1$ three-pion bound state (or resonance) could dominate the amplitude for annihilation into three pions. Depending on the magnitude of the K^0, electromagnetic form factors for values of k^2 inside the physical region, pairs of neutral K mesons, in the combination $K_1^0+K_2^0$, could be produced. The electromagnetic form factor of the neutral pion can be explored, through the mode of annihilation into $\pi^0+\gamma$, for values of k^2 larger than one pion mass; two-pion and three-pion resonances (or bound states) may produce very large effects on the annihilation amplitude. A three-pion bound state would mostly decay into $\pi^0+\gamma$, or $2\pi+\gamma$, and give rise to a very sharp resonance, with a width presumably of a fraction of a Mev, in the $\pi^0+\gamma$ annihilation reaction. The annihilation cross section, averaged around the resonance, may possibly reach values of the order of 10^{-30} cm^2. In a theory of the π^0 electromagnetic form factor, one can tentatively assume the dominance of a two-pion resonance and a three-pion bound state, and introduce the suggested values for the π^0 lifetime and for the derivative of the form factor at the origin. Also these estimates lead to a very big annihilation cross section at the energy of the assumed bound state. From the assumed values of the derivative of the form factor near the origin one would also estimate a very big enhancement of the cross section at an energy corresponding to that of the assumed two-pion resonance. A discussion of the possible resonances is given in Sec. 4, based on general considerations of the relevant partial and total widths as compared to the experimental energy resolution. It is concluded that electron-positron collisions offer a very suitable mean for detecting intermediate neutral resonant states of total angular momentum one, negative charge conjugation quantum number and parity, and zero nucleonic number and strangeness. Other intermediate states are not expected to lead to observable effects. Annihilation into baryon-antibaryon pairs would allow exploration of the baryon form factors for the relevant negative values of k^2. Near the threshold the cross section is isotropic and rises proportionally to the final velocity. The form factors are complex in the physical region for the process and, as a consequence, the produced fermions are expected to have a polarization normal to the plane of production and proportional to the sine of the phase difference between the electric and the magnetic form factor (in contrast, for instance, to electron-nucleon scattering in which the final nucleon is unpolarized, excluding radiative correction terms). There is at present no information available on the form factors for the large negative values of k^2 of the experiment. If one assumes, quite arbitrarily, that the recently found core terms in the nucleon structure originate from contributions in the absorptive region above the nucleon-antinucleon threshold, one can then roughly expect cross sections for annihilation into nucleon plus antinucleon well above the perturbation theory estimates. The $\Sigma-\Lambda$ electromagnetic vertex is measured in annihilation into $\bar{\Sigma}+\Lambda$ and the processes show a strong dependence on the relative $\Sigma-\Lambda$ parity. Vector mesons have been suggested recently and shown formally to be connected to local conservation laws.[27] Pair production of spin-one mesons is discussed in Sec. 6, on the assumption that their lifetime is sufficiently long to allow a separation of the over-all process into a first stage of production of the vector mesons and a second stage in which they decay. Three form factors are needed to specify the electromagnetic interaction of a vector boson, corresponding to its charge, magnetic moment, and electric quadrupole moment. The perturbation theory cross section for annihilation into a pair of spin-one bosons increases to a value of the order (m_B in Bev)$^{-2}$(2.1 $\times 10^{-32}$ cm^2) at energies much larger than the boson mass m_B. The perturbation theory increase is certainly not valid at very high energies because it would lead to a direct violation of unitarity. For electron-positron annihilation through the one-photon channel, one can strictly state the unitarity limitation in the form of an upper limit to the reaction cross section, that must decrease not slower than $\lambdabar^2$. In Sec. 6 we also discuss the angular correlations that would be observed at the decay of vector bosons from electron-positron annihilations into their final products.

Vector bosons have also been suggested as intermediary agents of weak interactions.[28] Their production in pairs in electron-positron annihilation would be a convenient test for their existence. Neutral intermediary vector bosons can only be coupled to neutral lepton pairs provided they do not couple to the weak strangeness-nonconserving currents. If they existed and were coupled to leptons they would produce an evident resonance-like behavior in annihilation reactions. Particular effects, such as those arising from parity nonconservation, would most directly inform on the presence of weak interactions in a high-energy annihilation process. However, for a local weak interactions, such effects become large only at colliding beam energies greater than 10 Gev.

Quantum electrodynamics vacuum polarization is known to be affected by strong interactions. The effect is insignificant at the lower energies but its analysis is important for an examination of an eventual high-energy breakdown of the theory. In the last section of this paper we give the explicit relation between the strong interaction corrections to vacuum polarization (or, equivalently, modification of the photon propagator) and the cross section for electron-positron annihilation into strongly interacting particles.