the actual position of the group on each plate was not exactly the same. To illustrate the aging effect the groups have been displaced to roughly line up the extrapolated high-energy edges. The vertical scales were adjusted to give approximately equal peak heights. It is seen that in two days the low-energy "tail" increased noticeably and by five days the slope of the high-energy edge changed. Similar results were obtained at Rice.[2]

If the spectrograph calibration curve is used to calculate the energy of the alpha particles from each of the sources it is found that the maximum deviation from the average is 1.8 kev and the average deviation is 1.1 kev. The average of these energies is 0.047% below the value used to obtain the calibration curve; well within the uncertainties in the calibration. It can only be said from this that the sources used in the present work give the same energy as sources prepared previously, within the limits of reproducibility of the spectrograph field.

The conclusion may be drawn that for fresh sources of $\frac{1}{2}$-mm height the different source solutions, methods of preparation and shape of backing used here give no measurable difference in alpha-particle energy. For a $\frac{1}{4}$-mm source a small difference was observed which may be caused by the backing shape. This difference, however, is much smaller than the discrepancies in the various measurements.[2] Source age has an appreciable effect on the alpha-particle energy and may be an important cause of the discrepancies of some of the older measurements, as has been discussed before.[1]

The difference between the Notre Dame and the Rice values is 4.4 kev when the Rice value for the $\mathrm{Li}^7(p,n)\mathrm{Be}^7$ threshold is used with the Notre Dame data. The standard deviations in the two measurements are about 1.5 to 2.0 kev. It is concluded that the difference does not depend on the characteristics of the polonium sources. It should be pointed out that the Notre Dame value is the highest and the Rice value one of the lowest of the recent measurements.

PHYSICAL REVIEW VOLUME 124, NUMBER 5 DECEMBER 1, 1961

Distortion Effects in Deuteron Stripping Reactions with Low Q Values*

W. R. Gibbs
Physics Department, Rice University, Houston, Texas

AND

W. Tobocman
Physics Department, Case Institute of Technology, Cleveland, Ohio

(Received July 10, 1961)

A series of numerical calculations have been carried out to verify the hypothesis that distortion effects become small for deuteron stripping reactions with low bombarding energy when the Q value is sufficiently small. Our results do not support this hypothesis.

IT has been observed[1] that the angular distributions of protons from (d,p) reactions at low bombarding energy and low Q value correspond very closely to the predictions of the Butler theory[2] when the cutoff radius is adjusted appropriately. This result is at first glance rather surprising since the Butler theory is expected to work best when the bombarding energy is well above the Coulomb barrier. When the bombarding energy is not well above the Coulomb barrier, the Butler theory usually gives a poor fit to the observed deuteron stripping angular distribution.

The good fit to Butler theory at low bombarding energy and low Q value has been interpreted[1,3] as being the result of a diminution of distortion effects. When the Q value is equal to

$$Q_0=\left(\frac{M_I}{M_D+M_I}\right)\left(\frac{M_F M_P}{M_D M_I}-1\right)E_D\approx-\tfrac{1}{2}E_D$$

(where E_D is the incident energy, M_I is the mass of the target nucleus, M_F is the mass of the residual nucleus, M_D is the deuteron mass, and M_P is the proton mass), the situation is most favorable for the stripping to occur with the proton remaining a long distance from the target nucleus. This condition, $Q=Q_0$, is therefore regarded by some authors as the optimum condition for the validity of the Butler theory. Since the Q values for stripping reactions are seldom less than -1 Mev, these optimum conditions can only be achieved when the bombarding energy is low.

By resorting to numerical computation with high-speed digital computers, it has been possible to introduce into direct-reaction theory calculations for deuteron

* Supported in part by the National Science Foundation and the Atomic Energy Commission.

[1] D. H. Wilkinson, Phil. Mag. **3**, 1185 (1958); J. P. F. Sellschop, Phys. Rev. Letters **3**, 346 (1959); Phys. Rev. **119**, 251 (1960).

[2] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951); F. L. Friedman and W. Tobocman, Phys. Rev. **92**, 93 (1953).

[3] E. K. Warburton and L. F. Chase, Jr., Phys. Rev. **120**, 2095 (1960).

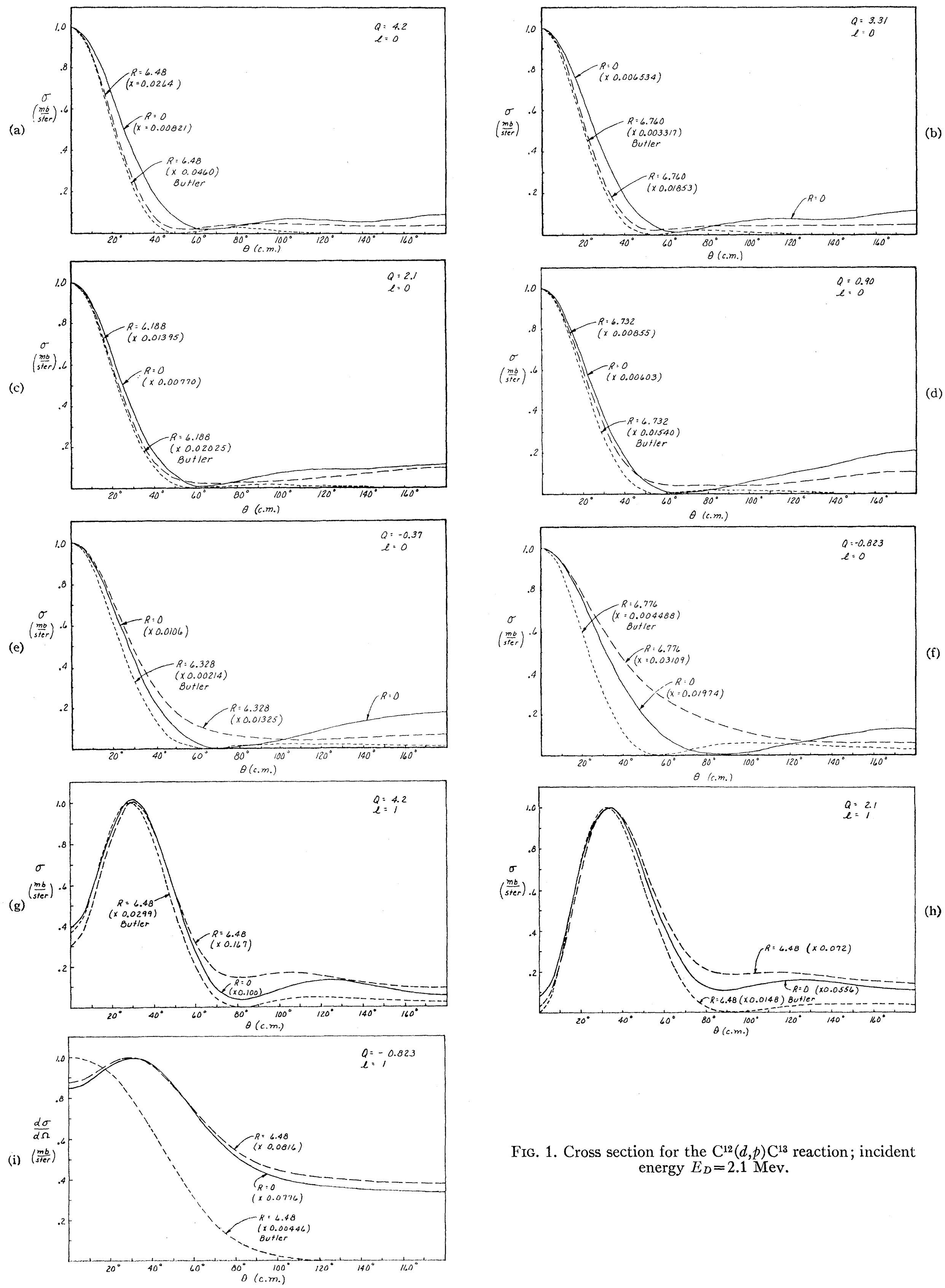

FIG. 1. Cross section for the $C^{12}(d,p)C^{13}$ reaction; incident energy $E_D=2.1$ Mev.

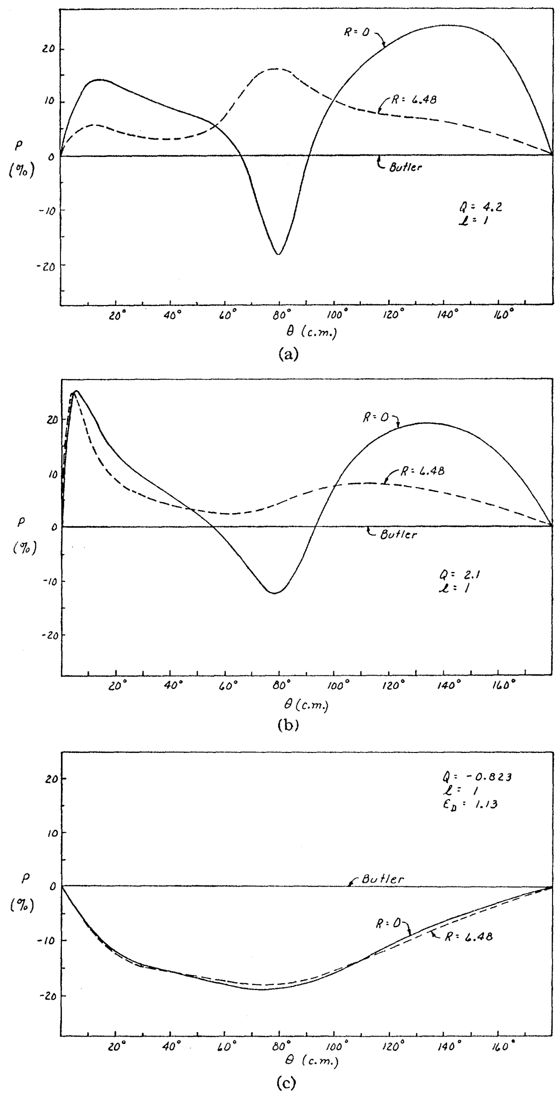

FIG. 2. Polarization for the $C^{12}(d,p)C^{13}$ reaction; incident energy equals 2.1 Mev.

stripping the distortion effects neglected in the Butler treatment.[4,5] When this is done one finds that the distortion effects neglected in the Butler treatment are not small. However, for the angular distribution there is a tendency for the Coulomb distortion effects to cancel the nuclear distortion effects.

We have carried out a series of calculations of the (d,p) cross section and polarization using the distorted-wave Born approximation. The calculation has been carried out for a series of Q values. The cross sections and polarizations have been compared with those given by the Butler treatment. Our purpose was to test the hypothesis that distortion effects are reduced as Q approaches Q_0.

The $C^{12}(d,p)C^{13}$ reaction for an incident deuteron energy of 2.1 Mev was calculated. We considered both the case where the neutron is captured with zero orbital angular momentum and the case where the neutron is captured with an orbital angular momentum of $1\hbar$. The cross section was calculated for Q values of 4.2, 3.3, 2.1, 0.9, −0.37, and −0.823 Mev for the orbital angular momentum zero cases. For orbital angular momentum $1\hbar$ we calculated both the (d,p) cross section and the polarization of the liberated protons for Q values of 4.2, 2.1, and −0.823 Mev. Q_0 for this reaction is −0.823 Mev. The optical potential used to distort the wave function representing the incident deuterons is a flat-bottomed Saxon well with the following parameters: $V=-44$ Mev, $W=-12$ Mev, $R=4.0$ fermi, and $a=0.75$ fermi. The optical potential for the protons is of the

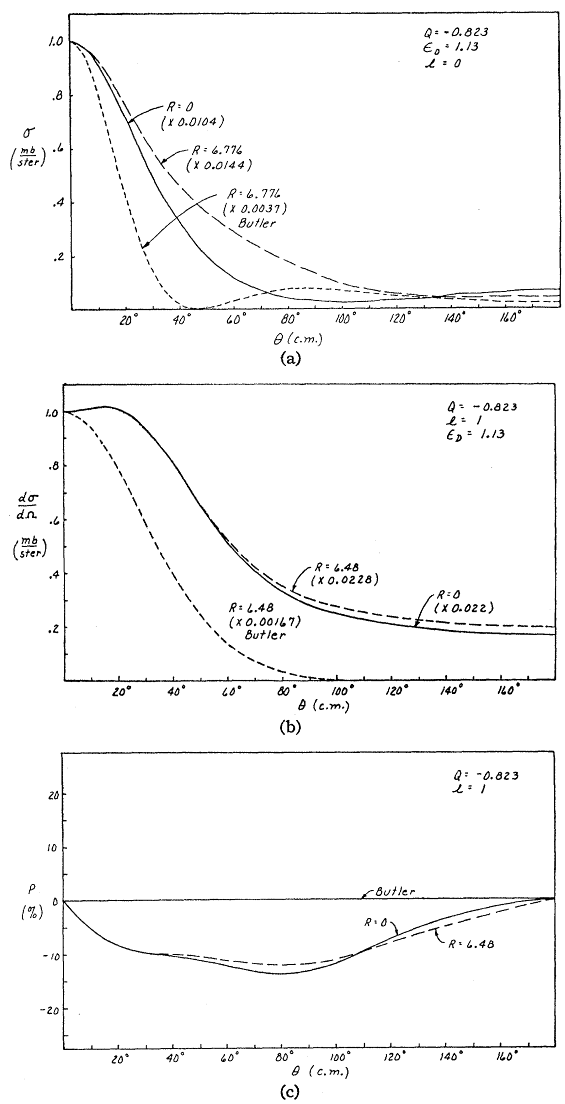

FIG. 3. Cross section and polarization for the $C^{12}(d,p)C^{13}$ reaction; incident energy equals 2.1 Mev and the deuteron binding energy equals 1.13 Mev.

[4] W. Tobocman and M. H. Kalos, Phys. Rev. 97, 132 (1955).
[5] W. Tobocman, Phys. Rev. 115, 98 (1959).

same type with the parameters $V=-52$ Mev, $W=-3.1$ Mev, $R=3.03$ fermi, and $a=0.52$ fermi.

We compare the results of three types of treatments: (1) the distorted-wave Born approximation, labeled "$R=0$"; (2) the cut-off distorted-wave Born approximation treatment, labeled "$R=\cdots$"; (3) the cut-off plane-wave Born approximation treatment, or Butler treatment, labeled "$R=\cdots$, Butler." (The cutoff radius R was chosen slightly differently for each value of Q for computational convenience.) For an account of the details of the calculation, see reference 5. The calculated cross sections and polarizations are shown in Figs. 1 and 2. The numbers appearing in the parentheses on the cross section graphs are the factors used to normalize the cross sections to one at the maximum.

Inspection of the calculated cross sections and polarizations reveals that there is no diminution of distortion effects as Q approaches Q_0. Indeed, at $Q=Q_0$ the discrepancy between the Butler treatment result and the distorted-wave treatment result is more pronounced than for any other value of Q tried. The reason for the large discrepancy is the fact that as Q is decreased beyond a certain point the Coulomb distortion effects come to predominate over the nuclear distortion effects.

The argument given by Warburton and Chase[3] to show that distortion effects are minimized when $Q=Q_0$ is based on Amado's[6] discussion of the importance of the pole in the transition amplitude. Amado pointed out that the transition amplitude for stripping has a pole when $k^2=(\mathbf{K}_P-\mathbf{K}_D M_P/M_D)^2=-\alpha^2$, where α is the decay length of the deuteron and $\mathbf{K}_D$ and $\mathbf{K}_P$ are the relative motion wave vectors in the incident and outgoing channels. Of course, this condition cannot be achieved, but it is most closely approached when $\mathbf{K}_P \| \mathbf{K}_D$ and $Q=Q_0$ so that k assumes its minimum value $k=0$.

When one is sufficiently close to this pole, distortion effects become negligible. By taking $Q=Q_0$ we minimize k and thus approach the pole as closely as we can. The question is whether this is sufficiently close. From the results shown in Figs. 1 and 2 we must conclude that we are not sufficiently close. In Fig. 3 is shown the result of moving a little closer to the pole by making $\epsilon_D=\hbar^2\alpha^2/2M_{NP}$ artificially small. Since distortion effects are still important, we must conclude that we are still too far from the pole to be able to apply Amado's analysis.

A possible explanation for the success of the Butler treatment in fitting low energy low Q-value (d,p) experiments is the following: At low energy and low Q value, the angular distribution has a very simple shape. This simplicity leads to a similarity in the predictions of the plane-wave and distorted-wave treatments. While the Butler angular distribution is not identical with the distorted-wave angular distribution, the similarity is close enough that the two can be made identical by suitably adjusting the cutoff radius.

ACKNOWLEDGMENT

In conclusion we would like to thank N. Austern for an enlightening discussion of Amado's work and for suggesting investigation of the effect of reducing ϵ_D.

[6] R. D. Amado, Phys. Rev. Letters 2, 399 (1959).