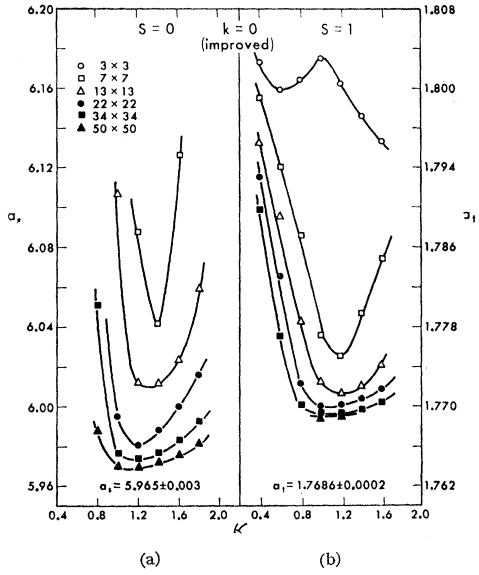

FIG. 4 (a) and (b). Improved zero-energy results.

mentioned that numerical inaccuracies [round-off errors accumulated in solving Eq. (4)] could be a serious problem. In particular, we could not determine the stationary value from our 50×50 matrix at zero energy (the old way) to any better than about 1%. However, with this improved calculation the numerical uncertainties were much reduced. There thus seems to be a correlation between good convergence and good numerical accuracy at each step.[8] This is a very interesting phenomenon for which we do not have any general theoretical understanding, but it appears to be a rather general property of variational calculations.

TABLE II. e^+-H phase shifts.

ka_0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
δ (radians)	0.151	0.188	0.168	0.120	0.062	0.007	−0.054

The computations reported here were carried out on the IBM 704 facility of the University of California Computation Center.

POSITRON SCATTERING

A few simple modifications of the programs allowed us to calculate elastic S-wave phase shifts for the scattering of positrons by atomic hydrogen. Since without the space symmetry we now need more terms in (3b) for each total power, $(l+m+n)$; the results do not converge as rapidly as for e^-. The results, shown in Table II, have probable errors of about ±0.001 radian. For the scattering length we find the upper bound $a_+ \leqq -2.10$; and from the apparent rate of convergence we believe that a_+ will not be as little[9] as −2.11.

[8] This behavior was also noted at the end of Appendix 2 of reference 4.

[9] Compare with previous results of L. Spruch and L. Rosenberg, Phys. Rev. **117**, 143 (1960).

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Continuous Photoelectric Absorption Cross Section of Helium*

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The continuous photoelectric absorption cross section of helium has been measured in the spectral region extending from 180 to 600 A with greater accuracy and the observations are found to agree with the calculations of Huang and Stewart and Wilkinson. A grazing incidence spectrometer with a photomultiplier was used for a single measurement at 180 A while the remaining measurements were carried out in a normal incidence spectrometer utilizing photographic techniques. Whereas in previous experiments the absorbing gas sample was allowed to fill the entire spectrometer chamber, in the current measurements the gas was confined to a small cell provided with sufficiently transparent windows. The use of an absoprtion cell reduces contamination and facilitates the measurement of gas pressures. The results indicate that the cross section varies from a value of 0.98±0.04 Mb at 180 A to a value of 7.7±0.3 Mb at the absorption edge located at 504 A.

I. INTRODUCTION

FOLLOWING recent success in the development of windows which are transparent to extreme ultraviolet radiation, we have carried out measurements of the continuous photoelectric absorption cross section of helium. The use of such windows in the construction of a gas cell permits confinement of the gas sample to a restricted volume. This technique has certain advantages over earlier methods[1,2] which used the entire

* Supported in part by the Office of Naval Research.

[1] P. Lee and G. L. Weissler, Phys. Rev. **99**, 540 (1955).

[2] N. Axelrod and M. P. Givens, Phys. Rev. **115**, 97 (1959).

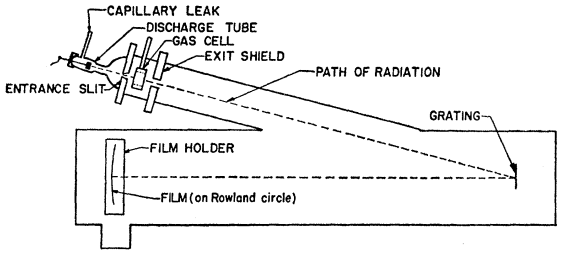

FIG. 1. The essential features of the normal incidence spectrograph used for measuring the photoelectric absorption cross section of helium.

optical path of a gas-filled spectrometer as the absorbing sample. One advantage accruing from the use of a gas cell is the possibility of carrying out absorption measurements utilizing a soft x-ray emission band as a source and a photomultiplier as a detector.[3] Such a combination provided an accurate attenuation measurement for helium at the short-wavelength end (180 A) of the spectral range of present interest. At all of the longer wavelengths (200 to 600 A), a condensed spark was used as the radiation source and detection was by photographic methods. An attempt was made to minimize the uncertainties in the measurements utilizing the spark discharge source and photographic recording by collecting a large number of data at various wavelengths. It is felt that these efforts have established the wavelength dependence and absolute value of the photoionization cross section with greater precision than has been previously achieved.

A number of theoretical evaluations of the cross section exist in the literature and in a later section a comparison will be made between the results of the present measurements and those of the various calculations.

II. EXPERIMENTAL

The absorption cross section of a gas is given by the relation

$$\sigma = (760/P)(T/273)(1/Nl)\ln(I_0/I), \qquad (1)$$

where I_0 is the incident intensity, I is the intensity transmitted by a column of gas l centimeters in length, P is the pressure of the gas in mm of Hg, T is the gas temperature in degrees absolute, and N is the number of particles per cm^3 (STP).

Observations of the radiation transmitted by a sample of helium gas were made in the spectral range extending from 180 A to 600 A. At the short-wavelength end of this range and for a single narrow wavelength interval, the measurements were carried out in a grazing incidence vacuum spectrometer. In this case the source consisted of a portion of the aluminum $L_{2,3}$ emission band and the detector was an open Be-Cu photomultiplier. The details of this instrument have been reported[3] upon previously and no further description need be given here other than to mention that a gas cell containing helium was inserted in the diffracted beam between the grating and the photomultiplier. Measurements at the longer wavelengths were all made with a normal incidence spectrograph and the experimental features of this work are described in some detail in the sections which follow.

A. Spectrograph

To evaluate the ratio of intensities at wavelengths longer than 200 A, we have used a normal incidence vacuum spectrograph with an aluminized grating of 30 000 lines per inch and with a radius of curvature of 1.62 m. Throughout the spectral range relevant to these measurements, the dispersion is essentially constant at a value of 5.26 A/mm. The photographic plate holder is mounted at the Rowland circle and can be moved so that four different exposures in the same wavelength range can be taken on a single plate. Kodak SWR spectroscopic film, one of the most sensitive emulsions in this spectral region, was used exclusively as the detector. Two mercury diffusion pumps were used to evacuate the main chamber of the spectrograph, and a third pumped directly on the sample chamber in which the gas cell was located. A diagram of the spectrograph is shown in Fig. 1.

B. Gas Cell

The cell used for these measurements is of the type described by Ederer *et al.*[4] with slight modification. The column of gas between the transparent plastic windows was 1.27 cm in length and had a diameter of 1.91 cm. A diagram showing the various pressurizing and exhaust connections attached to the cell is presented in Fig. 2. This combination of valves permitted evacuation of the cell concurrently with that of the main body of the spectrograph, and the windows of the cell proved sufficiently strong to withstand any pressure difference set up in this operation.

The windows of the cell were prepared by cementing electroformed mesh (obtained from Buckbee Mears Company[5]) of 100 lines per inch and 80% transmission

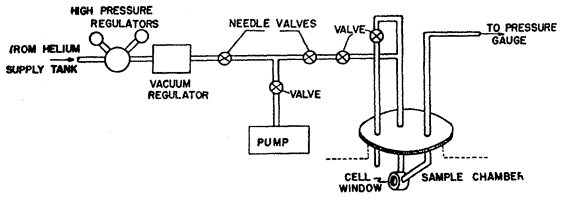

FIG. 2. The pressurizing, exhaust, and gauge connections to the absorption cell located in the sample chamber directly behind the entrance slit.

[3] D. E. Bedo and D. H. Tomboulian, Rev. Sci. Instr. **32**, 184 (1961).

[4] D. L. Ederer, D. J. Baker, and D. E. Bedo, Bull. Am. Phys. Soc. **6**, 284 (1961).

[5] Buckbee Mears Company, Toni Building, St. Paul 1, Minnesota.

onto a stainless steel flange and then subsequently depositing a film of Formvar or Zapon onto the mesh. Our experience has indicated that it is extremely difficult to produce a leak-free window with a single layer of plastic of this thickness. However, tight windows can be made rather easily with a double layer of plastic. After sealing the edges of the plastic to the stainless steel with plastic-to-metal cement, the flange was mounted directly to the body of the cell with an *O*-ring seal.

The transmission of the windows is a function of the thickness of the plastic and of the wavelength of the incident radiation; decreasing with increasing wavelength. The most transparent of the various windows used in the course of this experiment had a transmission of about 30% at 350 A. All of these windows leaked gas to some degree. For the majority of the measurements, the helium pressures did not exceed 5 mm of Hg at which pressures the leak rate was tolerable. Two pairs of windows were sufficiently leak free so as not to change the pressure in the capillary discharge tube noticeably, but these were too thick to allow radiation of wavelength longer than about 375 A to pass through with measurable intensity. Windows thin enough to pass the longer wavelength radiation permitted sufficient gas to leak into the sample chamber to raise the pressure from 3×10^{-5} to 3×10^{-4} mm of Hg.

The use of a gas cell is imperative when the radiation is to be detected by an open photomultiplier since the multiplier responds readily to positive ions created in the interior of the spectrograph. However, because of the large range of intensities which can be detected by the photomultiplier, double layers of plastic could be used to produce leak tight windows and the ion counting rate could be kept to a negligible value.

In general, the gas cell presents a number of advantages over the earlier technique of filling the entire spectrograph with gas. Some of these features are as follows: (1) Gas purity can be maintained more readily in a clean cell where outgassing effects are small; (2) with the limited volume of the cell, the range of pressures necessary for measurable attenuation fall in the millimeter region where absolute and continuously recording manometers are available; (3) the small cell volume reduces the probability of pressure gradients; (4) the elimination of possible gaseous impurities by repeated filling and emptying of the cell becomes somewhat more feasible with a limited gas volume; and (5) the mass of gas irradiated is more easily determined in the case of the gas cell where consideration need not be given to the complicated optical path of the spectrograph.

One obvious disadvantage imposed by the use of a cell is the high attenuation of the windows. Furthermore, at pressures in excess of 5 mm of Hg the windows exhibit a transmission which is pressure dependent, apparently a result of mechanical distortion. This pressure dependence was not observed at pressures lower than 5 mm of

TABLE I. The continuous photoelectric absorption of helium in the soft x-ray region. The tabulation contains the wavelength and identification of spectral lines used in the photometric measurements of the cross section σ. The number of observations at each wavelength is also indicated. The quantity appearing in the third column is presumed to be independent of the pressure and was utilized in obtaining a least squares fit.

Wavelength in A	Identification	$(I_0/I)^{1/P}$	No. of obs.	Cross section (Mb)
202.3	O v	1.058	2	1.37
203.9	O v	1.047	2	1.12
207.2	O iv	1.050	1	1.19
209.3	N v	1.074	2	1.74
214.1	O iv	1.050	2	1.19
215.1	O v	1.073	3	1.71
220.4	O v	1.059	3	1.39
225.2	N iv	1.070	3	1.65
227.5	O v	1.043	1	1.03
231.2	O iv	1.088	5	2.05
233.5	O iv	1.088	9	2.05
238.5	O iv	1.088	8	2.05
240.1	O iv	1.071	1	1.67
243.5	O iii	1.043	1	1.03
247.6	N v	1.095	1	2.21
252.5	O iv	1.082	3	1.92
260.5	O iv	1.117	5	2.68
263.7	O iii	1.091	2	2.12
266.9	O iii, iv	1.104	2	2.41
272.2	O iv	1.098	6	2.27
275.4	O iii	1.085	2	1.98
279.8	O iv	1.113	9	2.60
285.8	O iv	1.091	2	2.12
289.1	C iv	1.085	1	1.98
290.0	O iv	1.085	2	1.98
291.3	C iii	1.090	1	2.10
294.1	Ar vi	1.106	7	2.46
295.7	O iii	1.141	3	3.19
297.7	Ar vii	1.141	5	3.19
299.8	O iv	1.119	1	2.78
301.2	C iii	1.105	2	2.43
303.6	He ii, O iii	1.148	5	3.36
305.7	O iii	1.144	6	3.26
304.9	N iii	1.130	1	2.97
306.8	O iv	1.142	3	3.24
308.3	Na v	1.104	2	2.41
311.8	O iv	1.130	1	2.97
312.4	C iv	1.163	3	3.68
314.9	N iii, iv	1.117	1	2.70
321.0	O iii	1.131	2	2.99
322.6	C iii, N iv	1.166	3	3.72
328.4	O iii	1.128	3	2.92
335.1	N iii	1.175	4	3.92
338.0	Ar v	1.145	2	3.28
350.9	Ar v	1.110	1	2.56
359.2	O iii	1.208	2	4.60
362.9	N iii	1.203	1	4.50
374.2	N iii	1.226	9	4.97
384.0	...	1.220	4	4.86
395.0	...	1.280	2	6.08
436.6	O ii	1.271	5	5.84
446.0	Ar v	1.287	5	6.13
446.9	Ar v	1.310	4	6.57
449.1	Ar v	1.238	1	5.18
451.9	N iii	1.269	1	5.80
457.0	Ar vi	1.267	1	5.74
459.3	Ar vi	1.267	2	5.74
464.3	Ar vi	1.358	6	7.45
475.7	Ar vii	1.325	5	6.84
479.4	Ar iv	1.370	4	7.66
487.0	Ar iii	1.390	6	8.00
501.3	Ar ii	1.372	2	7.69

Hg and all measurements reported here were made at pressures where this effect could be ignored.

C. Source

For all of the photographic measurements, the radiation source consisted of a condensed spark discharge in a glass capillary. A secondary spark gap, acting as a switch, was opened and closed at a constant rate to initiate the discharge in the capillary and to provide repeatable charging, discharging, and pressure conditions. A 1-μf capacitor bank was charged to 25 kv in 20 sec; the oscillatory discharge was effectively damped in about 10 μsec. To maintain adequate pressure in the discharge region, gas was admitted into the tube through a fine leak. Spectra have been examined with both air and argon providing the residual gas in the capillary necessary to start the discharge. Argon was found to produce a large number of lines in the region from 400 to 500 A, an important spectral region for the determination of the helium cross section. Listed in Table I are the spectral lines which were used in the course of these measurements.

The gas cell was located in a sample chamber a few centimeters behind the (narrow) defining slit of the spectrograph. To protect the fragile windows from the discharge, a screen of 1000 lines per inch was interposed between the discharge tube and the sample chamber. The pressure in the sample chamber could be maintained as low as 3×10^{-5} mm of Hg and the discharge would operate in a satisfactory manner. Where it was necessary to use thin windows which of necessity were not leak tight (at gas cell pressures of a few millimeters), the pressure rose by a factor of 10. For these higher pressures, an attempt was made to maintain sensibly constant discharge conditions by introducing helium into the sample chamber through a connection exterior to the gas cell. In this way, plate calibration, incident intensity, and transmitted intensity measurements were made with the ambient pressure in the discharge tube at the same value during all exposures. The length of individual runs varied from about 50 to 10 000 sparks at a sparking rate of 15 per min.

D. Purity of the Gas

The helium used in the experiment was obtained from the Matheson Company,[6] and was claimed to be 99.99% pure. No absorption was observed (within experimental error) at wavelengths well beyond the known edge of helium at 504 A and it was concluded that impurities were not present in sufficient amount to affect the measurements.

E. Pressure Measurements

The helium pressure in the gas cell was measured with an aneroid type absolute pressure gauge which was calibrated against an oil manometer. Since for the most part the windows were not leak tight, it was necessary to flow the helium into the cell and some fluctuations of pressure were unavoidable. Throughout each run, the pressure in the cell was recorded as a function of time and these values were averaged. Uncertainties in the determination of the pressure of the gas sample are believed to be of the order of 2%.

F. Method of Photometry

Microphotometer records of the plates established the values of photographic density as a function of wavelength. In order to convert the records from microphotometer deflections to relative intensities, a method similar to that of Woodruff and Givens[7] was employed. Two calibration spectra were taken on each plate. These two spectra differed only in that one represented an exposure to twice as many sparks as the other. Thus for every spectral line, the microphotometer traces of the two spectra furnished a pair of deflections, one of which was produced by an intensity I, the other by an intensity $2I$, if the intensity is assumed to be proportional to the number of sparks (no evidence of reciprocity failure has been observed). These pairs of deflections were then plotted to provide a d_I vs d_{2I} curve. If an arbitrary intensity value is assigned to one of the larger deflections, it is then possible to find a deflection corresponding to one-half of this intensity. This procedure is continued until a set of points is generated which relates the microphotometer deflections to relative intensities. Several sets of points are determined in the same way and are averaged to provide the final plate calibration. This calibration curve may then be used to evaluate the relative intensities of all spectral lines recorded on the plates. As a check of the validity of a calibration curve derived in this manner, the same calibration spectra are converted to intensities and the intensity ratio is computed at each wavelength. This ratio should have a value equal to two. The scatter of the computed ratios about the value two is generally found to be of the order of 5%. This scatter sets an upper limit to the precision to be expected from the photographic reductions.

In order to achieve repeatability, we have used only the linear portion of the plate calibration curve. The slope of this linear portion was found to be the same from plate to plate whereas the nonlinear regions seemed to be characteristic of a particular plate. This restriction on the acceptable values of the density reduced the number of spectral lines which could be used but did improve the reproducibility of the data. The calibration process which has been described is based on the assumption that the plate response is independent of wavelength. To test this assumption, two distinct calibration curves were determined; one for the 200- to 400-A region and one for the 500- to 700-A region. These calibration curves were found to be in agreement.

[6] The Matheson Company, Inc., East Rutherford, New Jersey.

[7] R. W. Woodruff and M. P. Givens, Phys. Rev. **97**, 52 (1955).

III. RESULTS

To arrive at an absorption spectrum characteristic of a sample of helium gas, it is necessary at each wavelength to determine the ratio of intensities incident upon and transmitted by the gas sample. At 180 A, where the photomultiplier was employed, counting rates were determined before and after filling the gas cell with helium. At the longer wavelengths, the photometric procedures just described were followed for the reduction of the two exposures made with and without helium in the cell.

With the present instrumentation, photoelectric detection was available only for a narrow band of wavelengths centered at 180 A and in this case a series of attenuation measurements were made as a function of gas pressure. In Fig. 3 the quantity $\ln(I_0/I)$ is plotted as a function of the helium pressure in the gas cell. The slope of the line drawn in the figure is related to the cross section as indicated in Eq. (1). The value of the cross section derived from these measurements is 0.98 ± 0.05 Mb.

The relative error in the cross sections is simply given by:

$$\left|\frac{\Delta\sigma}{\sigma}\right| = \left|\frac{\Delta P}{P}\right| + \left|\frac{1}{\ln(I_0/I)}\,\frac{\Delta(I_0/I)}{(I_0/I)}\right|. \qquad (2)$$

on the basis of Eq. (2), the use of somewhat higher pressures than those shown in Fig. 3 would appear to reduce the uncertainty in the determination of the cross section. Since at present, the relative error in the pressure determination arises predominantly from scale reading errors, higher pressure values would tend to reduce the first term on the right of Eq. (2). However, difficulty is encountered in carrying out a precise measurement of the intensity ratio because of the large attenuation factor introduced by the windows of the gas cell. Windows possessing a sufficiently high impedance to reduce the leak rate to a level not detectable by the photomultiplier produced an over-all attenuation factor of about 80. Admission of gas into the cell at pressures of a few millimeters of Hg reduced the photomultiplier counting rate to about 100 counts per second. Source instability and background noise make lower counting rates impractical.

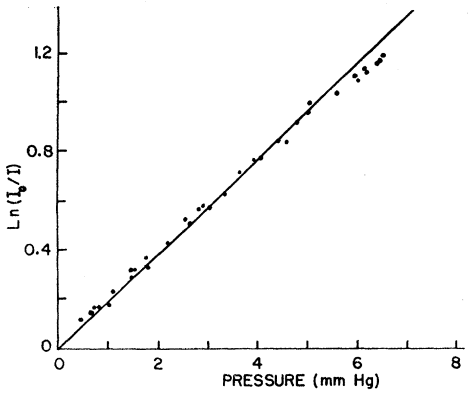

FIG. 3. A plot of $\ln(I_0/I)$ as a function of the helium pressure in the cell. At each pressure I_0 and I represent, respectively, the photon fluxes incident upon and emergent from the cell. The latter was irradiated by a narrow band of radiation centered at 180 A. The emergent beam was detected by a Cu-Be photomultiplier.

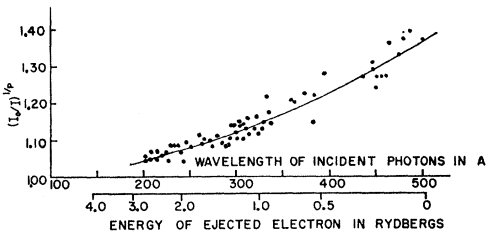

FIG. 4. For wavelengths greater than 200 A, the graph shows a plot of the quantity $(I_0/I)^{1/P}$ against the wavelength of incident photons. Here P is the gas pressure and the ratio of incident to transmitted intensities raised to the power $1/P$ is presumed to be independent of the pressure. The solid curve represents a three-parameter least squares fit to the individual determinations which at most exhibit a deviation of 5% from the solid curve. The energy (in rydbergs) of the ejected electron appears below the wavelength scale.

The uncertainty in the measured cross section at 180 A is believed to be about $\pm4\%$. It seems certain that with further refinement in the techniques of window manufacture and perhaps with the development of soft x-ray tubes capable of dissipating greater input power, that the photoelectric detection scheme may be expected to lead to results which are considerably more precise.

At wavelengths longer than 200 A, intensity measurements were made exclusively by photographic means. Because of the wavelength dependence of the transmission of the plastic windows and the distribution of intensity among the spectral lines of the spark discharge, the measurements may be divided into two groups. At wavelengths less than about 375 A, a leak tight gas cell (double layer of plastic in the windows) could be used and hence the pressure in the discharge tube was unaffected by admission of helium into the cell. In this spectral range there appeared to be no difficulty in obtaining consistent results from run to run. Thinner windows had to be used for recording wavelengths above 375 A. In this case the available pumping speed was insufficient (with helium in the gas cell) to maintain the discharge pressure at its lowest value. As mentioned previously, to realize uniform discharge conditions for all exposures helium was purposely admitted to the discharge region when the cell was evacuated. Adoption of this procedure improved the repeatability of the measurements. In those cases where thinner windows were necessary for adequate transmission of the long-wavelength radiation, intensity measurements could be made for every spectral line in the region of interest.

The plates were carefully examined for internal consistency. In the many cases where exposures were taken expressly for intensity calibration, cross checks were carried out to certify the validity of the calibration curves and for all plates reductions were limited to

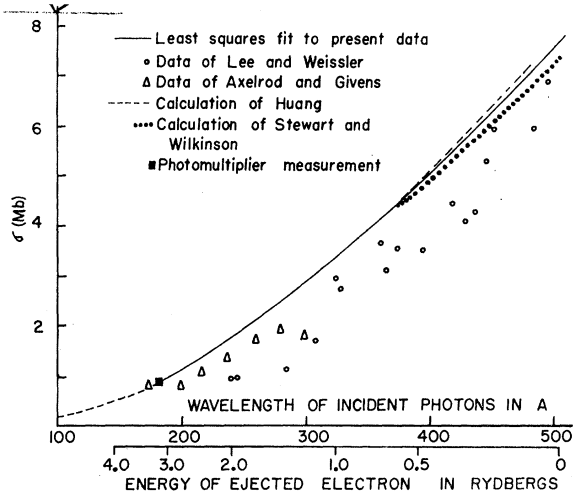

FIG. 5. The absorption cross section of helium as a function of the incident photon wavelength in angstroms and the ejected electron energy in rydbergs. The current data (solid line) are compared with recent theoretical calculations.

those spectral lines falling in the linear calibration region. Plates were checked also to ascertain that they predicted no absorption at wavelengths much longer than the ionization limit at 504 A.

As a result of the combination of the low optical transmission, the fragility, and the relatively low gas impedance of the windows, the gas cell pressure had to be kept less than about 5 mm of Hg. Hence the ratio of incident to transmitted intensities varied from values only slightly greater than one to a maximum of 2.5. In Fig. 4, $(I_0/I)^{1/P}$, a quantity presumed independent of the pressure, is plotted as a function of wavelength. The data for this plot is presented in Table I. The solid curve in Fig. 4 represents a three-parameter least squares fit to the individual data points which exhibit at most a deviation of about 5% from the solid curve. The cross section σ was calculated by multiplying the logarithm of the quantity $(I_0/I)^{1/P}$ by the constant factor 24.3×10^{-18} cm^2. The computed cross section at each wavelength is listed in Table I together with the number of observations made at each spectral line. The extreme short-wavelength determination represents data obtained from the photomultiplier measurements.

A number of theoretical evaluations of the cross section have been carried out by various authors. These differ in the choice of wave function for both the bound and free states and in the form of the interaction matrix element. The earliest calculation is that of Wheeler[8] who used Hylleraas' wave function for the bound state and a hydrogenic approximation for the free states. Vinti[9] carried out a similar calculation using a less elaborate ground-state wave function. Both authors used the dipole matrix formulation.

After Chandrasekhar[10] demonstrated that the momentum matrix element presents the more reliable formulation, Huang[11] carried out a calculation of the cross section based on this approximation. He chose the Hylleraas' wave function for the ground state and a hydrogenic approximation to the continuum states. More recently, Stewart and Wilkinson[12] have computed the cross section using the Hylleraas' ground-state wave function and a Hartree-Fock wave function for the free states. Stewart and Wilkinson have evaluated oscillator strengths based on both the dipole and momentum formulations.

In Fig. 5 the solid curve represents the least squares fit to the present experimental measurements of the cross section. The extreme short-wavelength determination represents data obtained from photomultiplier measurements. The dashed curve depicts the calculations of Huang while the dotted curve represents the results of Stewart and Wilkinson. As can be seen, the present measurements are in good agreement with the calculations, both being within the limits of experimental error. Also included in Fig. 5 are the earlier measurements of Lee and Weissler[1] and those of Axelrod and Givens.[2]

[8] J. A. Wheeler, Phys. Rev. **43**, 258 (1933).
[9] J. P. Vinti, Phys. Rev. **44**, 524 (1933).
[10] S. Chandrasekhar, Astrophysical J. **102**, 223 (1945).
[11] S. Huang, Astrophysical J. **108**, 354 (1948).
[12] A. L. Stewart and W. J. Wilkinson, Proc. Phys. Soc. (London) **75**, 796 (1960).